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Even-tempered atomic orbitals
in quantum chemical ab initio calculations:
Light atoms, heavy atoms, triatomic alkali ions

by

Richard Charles Raffenetti

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

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Approved:

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In Charge of Major Work

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1971

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INTRODUCTION

One aim of theoretical chemistry is to apply the laws of quantum physics to atoms and molecules and thereby, to form complete and accurate mathematical descriptions of these systems. An important intermediate step in this quest is the determination of rigorous solutions for the Schroedinger equation which describe electronic motion in the field of stationary nuclei. Recent advances in computer technology have put this goal within reach for small systems, i.e. those with, perhaps, up to a dozen electrons. As a result, theoretical chemists have indeed been able to make important contributions to the field of chemistry, not only with general theory but also with concrete accurate calculations.

Such calculations on systems with many electrons have been limited in number for several reasons. Usually very accurate techniques are difficult to extend to larger systems because of the mathematical complexities. Secondly, a large problem generates immense quantities of intermediate information which must be handled over and over. This information increases approximately as the fourth power of the number of electrons, causing a rapid explosion of data while the speed of calculation remains essentially constant. Computing machinery belonging to the current prototype has progressed so that the technical limit with respect to calculation speed is

very close. New prototypes (1) are being developed which promise to alleviate this bind, but there certainly will always be calculations on some molecules which will tax the new limit. Consequently the concerned theoretician must continue to look for improvements in his mathematical techniques which will facilitate the treatment of larger systems. It is the purpose of this investigation to point out one avenue of approach toward this goal.

Notable attempts at simplification exist, based on parametrization with the help of experimental data. Such data fitting techniques were originally devised in the face of a complete inability to obtain accurate ab initio results (2,3). More recently, these semi-empirical techniques have been used in order to defeat the data handling and data generation problems encountered with large systems (4,5). On the other hand, there are methods employing mathematical approximations rather than experimental data which are called non-empirical or semi-quantitative. Typically, they can be trusted only after repeated verification of success for similar systems.

The theory used for calculations contained in this work is a first principle or ab initio theory. Characteristic of ab initio theories is the assumption of some form for the wavefunction Ψ which is to form a solution to the Schrödinger equation. The wavefunction is constructed to be an eigen-

function of all symmetry operators which commute with the Hamiltonian operator \hat{H} . Hence the wavefunction has certain symmetries which are in accord with the postulates (in this case fermion antisymmetry) or with the physics of the problem (e.g. the spin operators S_z or S^2). In particular, the wavefunction is frequently written in terms of space and spin orbitals, i.e. one-electron space functions, $\phi_i(\underline{r})$ and spin functions α and β .

The space orbitals $\phi_i(\underline{r})$ are usually expanded in terms of a number of appropriate spatial basis functions $x_k(\underline{r})$

$$(1) \quad \phi_i(\underline{r}) = \sum_k x_k(\underline{r}) c_{ki}$$

which define a finite Hilbert space. Solution for Ψ implies determination of the matrix of linear coefficients C which define the best orbitals obtainable in terms of this basis. It is apparent that the quality of the basis is of critical importance for the quality of the wavefunction that can be obtained. Furthermore, reviewing the calculational procedure, one quickly realizes that the size of the basis is of critical importance for the amount of effort that must be expended. This investigation deals therefore with the basis, its form and selection, and ways of efficiently manipulating the data generated from it in connection with substantial atomic and molecular problems.

There are many kinds of ab initio approaches, each

dictated by a particular approximation to the wavefunction in terms of orbitals. In the following we adopt the Hartree-Fock approximation, in which the wavefunction is an antisymmetrized product of orthonormal spin orbitals, u_i , defined as products of pure space and pure spin orbitals,

$$(2) \quad \Psi = \Psi(1, 2, \dots, N) = \hat{A}\{u_1(1)u_2(2)\dots u_N(N)\}$$

where \hat{A} is the antisymmetrization operator. This product is formally equivalent to an N by N determinant and hence is also called a Slater determinant (6).

By virtue of the variational principle, one seeks the lowest energy possible for such a determinant, and this determines the Hartree-Fock orbitals. As is the case for the exact wavefunction, limitation to a finite Hilbert space by the expansion of Eq. 1, means that one obtains only an approximation to the Hartree-Fock approximation. Here too, the quality of the basis is essential for the quality of this approximation. Now, in all cases of 1S closed shell states, the Hartree-Fock wavefunction is a very good approximation to the true wavefunction. It is therefore a safe assumption that a basis which is effective for obtaining a good representation of the Hartree-Fock wavefunction will also be desirable for the construction of more sophisticated wavefunctions.

The Hartree-Fock approach has the advantage that compu-

tationally simple techniques exist so that calculations can be repeated a sufficient number of times with the objective to test the performance of various bases.

THE LCAO-MO-SCF METHOD

The Matrix Formulation

The LCAO-MO-SCF (linear-combination-of-atomic-orbitals molecular-orbital self-consistent-field) method is a technique for solving the Hartree-Fock equations,

$$(3) \quad \hat{F}\phi_i = e_i\phi_i .$$

The Fock operator, \hat{F} , is a pseudo-one-electron operator which depends on the system space orbitals ϕ_i implicit in Eq. 2. If each orbital ϕ_i , is expanded in a set of suitable symmetry adapted basis functions x_{ls} according to

$$(4) \quad \phi_{il} = \sum_s x_{ls} c_{sil} = |x\rangle c_{il}$$

the problem becomes one of matrix algebra. The solution of the matrix Hartree-Fock equations has been described by Roothaan (7,8) and Roothaan and Bagus (9). If the density matrices are defined

$$(5) \quad \begin{aligned} D_T(l) &= D_C(l) + D_O(l) \\ D_C(l) &= \sum_{i \in \text{closed}} N(il) c_{il} c_{il}^* \\ D_O(l) &= \sum_{i \in \text{open}} N(il) c_{il} c_{il}^* \end{aligned}$$

in terms of column vectors c_{il} of the coefficient matrix $C(l)$ and occupation numbers $N(il)$, corresponding to the i -th orbital of symmetry or irreducible representation l , then one may write the energy compactly as

$$(6) \quad E = \frac{1}{2} \sum_l \{ [2\bar{H}^+(l) + \bar{P}^+(l)] D_T(l) - \Omega^+(l) D_O(l) \}$$

where the lower triangular matrices $\bar{H}(l)$, $D_O(l)$, $D_T(l)$, $\bar{P}(l)$, and $\Omega(l)$ are considered supervectors, with the off-diagonal elements of the density matrices being doubled. The supervector $\bar{H}(l)$ represents a collection of one-electron integrals over the basis functions.

Application of the variation principle to the energy formula results in a set of coupled eigenvalue equations

$$(7) \quad F_Q(l) c_{il} = e_i S(l) c_{il}$$

$$(8) \quad F_C(l) c_{jl} = e_j S(l) c_{jl}$$

for each symmetry l , which must be solved for the coefficient vectors c_{il} and c_{jl} defining orbitals of Eq. 4. A set of initial orbitals is chosen to start the process. These are orthogonalized with respect to the metric matrix S and are used to construct the density matrices of Eq. 5. The Fock matrices F_C and F_O may then be formed. By virtue of their definition below, each has components from both closed- and

open-shells and from every symmetry. Now, each of these equations is treated as a general matrix eigenvalue problem

$$(9) \quad \underline{F} \underline{C} = \underline{S} \underline{C} \underline{e},$$

where \underline{e} is assumed diagonal. The problem is converted to an ordinary eigenvalue problem by finding a transformation matrix, \underline{U} , which brings \underline{S} to the identity,

$$\underline{U}^* \underline{S} \underline{U} = \underline{I}.$$

The eigenvectors which solve Eq. 9 are assumed to be of the form $\underline{C} = \underline{U} \underline{V}$ so that Eq. 9 becomes

$$\underline{F} \underline{U} \underline{V} = \underline{S} \underline{U} \underline{V} \underline{e}.$$

Multiplication on the left by \underline{U}^* gives

$$\underline{U}^* \underline{F} \underline{U} \underline{V} = \underline{U}^* \underline{S} \underline{U} \underline{V} \underline{e},$$

which is equivalent to

$$(10) \quad \underline{F}' \underline{V} = \underline{V} \underline{e},$$

where $\underline{F}' = \underline{U}^* \underline{F} \underline{U}$. Eq. 10 is solved by diagonalizing the real, symmetric matrix \underline{F}' . The orthogonal matrix which effects this is \underline{V} . The process is done first for closed-shells and then open. In this way one gets two solution matrices \underline{C} (open) and \underline{C} (closed). Certain vectors from each are chosen according to their associated eigenvalues e_i to be used as

occupied orbitals. Then new density matrices are constructed and the entire process is recycled until input orbitals equal output. If convergence of this iterative process is obtained, orbitals derived from Eqs. 7 and 8 will be mutually orthogonal. However, during early iterations this orthogonality must be forced.

If the problem is closed shell, $D_O(l)$, $\Omega(l)$, and $E_O(l)$ are not defined so that the problem is simpler, requiring solution of only the closed-shell equations.

The following equations serve to complete the definition of all quantities used above;

$$E_C(l) = H(l) + P(l) + R_O(l)$$

$$E_O(l) = H(l) + P(l) - \Omega(l) + R_C(l)$$

$$P(l) = \sum_{l'} D(l l') D_T(l')$$

$$\Omega(l) = \sum_{l'} Q(l l') D_O(l')$$

$$R_C(l) = [N_C(l)/(N_C(l)-N_O(l))] \sum_{iclosed} [\underline{v}_i(l) + \underline{v}_i^+(l)]$$

$$R_O(l) = [N_O(l)/(N_C(l)-N_O(l))] \sum_{icopen} [\underline{v}_i(l) + \underline{v}_i^+(l)]$$

$$V_i(l) = \underline{S}(l) c_{il} c^+_{il} Q^+(l)$$

$$N_O(l) = \sum_{i \text{ open}} N(il), \quad N_C(l) = \sum_{i \text{ closed}} N(il) .$$

The supermatrices $\underline{P}(ll')$ and $\underline{Q}(ll')$ contain the two-electron integrals arising from the electron-repulsion term of an atomic or molecular Hamiltonian and computed in terms of the basis functions.

Matrix Elements

One-electron integrals

The quantity $\underline{H}(l)$ mentioned above represents a supervector or lower triangular matrix of expectation values of the one-electron Hamiltonian terms computed over a symmetry adapted basis. The one-electron Hamiltonian operator is

$$\hat{H}_i = \sum_i \hat{h}(i)$$

where

$$\hat{h}(i) = -\hat{\Delta}_i/2 - \sum_{\alpha} (z_{\alpha}/r_{i\alpha}) ,$$

the i sum being over all electrons and α summing over all nuclei. The first term arises from the kinetic energy of the electrons with the operator

$$\hat{\Delta} = \hat{\nabla}^2 = (\partial/\partial x)^2 + (\partial/\partial y)^2 + (\partial/\partial z)^2 .$$

The second represents the Coulomb attraction of the nuclei for the electrons. If symmetry adapted basis functions x_{lr} are taken to be real, the integrals are defined such that

$$(11) \quad H(lrs) = \int dV_1 x_{lr}(1) h(1) x_{ls}(1) \\ = \langle x_{lr} | h | x_{ls} \rangle$$

where the second line is written in the bra-ket notation of Dirac (10). The overlap matrix $S(l)$ is computed with respect to the symmetry adapted functions such that

$$(12) \quad S(lrs) = \int dV_1 x_{lr}(1) x_{ls}(1) \\ = \langle x_{lr} | x_{ls} \rangle .$$

All one-electron integrals between symmetries l and l' are identically zero.

Two-electron integrals

The integral supermatrices $\mathcal{P}(ll')$ and $\mathcal{Q}(ll')$ are combinations of the more basic supermatrices $\mathcal{G}(ll')$ and $\mathcal{K}(ll')$. These in turn are formed from the basic electron repulsion integrals arising from the two-electron term of the molecular Hamiltonian operator

$$\hat{H}_2 = \sum_{i < j} r_{ij}^{-1}$$

where the sums are over the electrons. The basic integral is commonly written

$$(13) \quad [rs|tu] = \langle rt|r_{12}^{-1}|su\rangle \\ = \int dv_1 \int dv_2 \ X_r(1) \ X_t(2) (1/r_{12}) \ X_s(1) \ X_u(2)$$

with the general indices referring to basis functions, either symmetry adapted or not. The supermatrix derives its form from the three integral symmetry properties

$$[rs|tu] = [tu|rs]$$

$$(14) \quad [rs|tu] = [rs|ut]$$

$$[rs|tu] = [sr|tu] .$$

Unique integrals form a lower triangular supermatrix with rows and columns numbered by the (rs) pairs in the sequence $(rs) = (11), (21), (22), (31), (32), \dots$ etc. corresponding to row-wise lower triangular supervector elements. Hence there are $M(M+1)/2$ unique integrals where $M=N(N+1)/2$ and N is the number of basis functions X_s .

Elements of the Coulomb and exchange supermatrices $\mathcal{Q}(ll')$ and $\mathcal{K}(ll')$ are

$$(15) \quad \mathcal{Q}(lrs, l'tu) = [lr \ ls | l't \ l'u]$$

and

$$(16) \quad \mathcal{K}(lrs, l'tu) = \{[lr lt|l's l'u] + [lr lu|l's l't]\}/2 .$$

In terms of these, elements of $\mathcal{P}(ll')$ and $\mathcal{Q}(ll')$ are

$$(17) \quad \mathcal{P}(lrs, l'tu) = \mathcal{G}(lrs, l'tu) - (1/2)\mathcal{K}(lrs, l'tu)$$

and

$$(18) \quad \mathcal{Q}(lrs, l'tu) = A(ll')\mathcal{G}(lrs, l'tu) - (B(ll')/2)\mathcal{K}(lrs, l'tu) .$$

The quantities $A(ll')$ and $B(ll')$ are vector coupling constants peculiar to the degeneracy of a symmetry and are necessary in order to obtain an eigenfunction of correct spin state in open-shell systems. Constants used in this work are discussed in Appendix A. The symmetry classification (ll') of the constants is the reason for using symmetry adapted functions and integrals. It is convenient to insert them in the definition of $\mathcal{Q}(ll')$ if the basis is symmetry adapted. The closed-shell problem has no such requirement and is often carried out without explicit symmetry adaptation of integrals over the basis. A more detailed descriptions of the open-shell SCF procedure are given by Roothaan and Bagus (9) and Wahl et al. (11).

EVEN-TEMPERED ATOMIC ORBITAL BASES

The Role of the Basis

It seems clear that the selection of a basis rates high on the scale of important considerations. One must select a basis such that the function space of the system may be well represented. Naively one might suggest an infinite number of functions comprising a "complete set" capable of representing any function in any local function space. This is of course an impossible choice due to an inability to handle an infinite amount of data. The usual approach has been to select a finite set of functions, most frequently termed atomic orbitals (AO's), which are found separately for each atom of the molecular system, to combine them, and to use them as a basis for the calculation. The idea is that the electrostatic potential seen by the electrons near the atoms in the molecule is similar to that of the isolated atoms, so that the "atomic" functions provide a good description of that part of the wavefunction local to the atoms. To account for the modification of single atom potentials in interatomic regions, the set is normally augmented with extra functions the purpose of which is to better provide for this molecular polarization of the atoms. Various types of molecular basis have been tried. For example, Preuss (12) and Whitten (13,14) have used off-nucleus 1s Gaussians or "lobe functions" to

simulate atomic orbitals, and others have used single-centered bases (15) for molecules with one heavy atom. These have not met the practical success in terms of consistently attaining the accuracy of the nuclear-centered analytic atomic function basis used here. The lobe function bases have not yet been extended to include d-type functions. On the other hand, the single-centered bases require very many terms and still do not describe atoms away from the heavy nucleus very well. This investigation has been confined to nuclear-centered basis types.

Within this limitation, several considerations may be made. Although any finite basis could be handled successfully in principle, there are practical limits to be considered. Integrals must be computed which consist of expectation values of various terms of the molecular Hamiltonian \hat{H} over the basis functions. The occurrence of a two-electron term in \hat{H} requires computation of a quantity of unique two-electron integrals of the order of $N^4/8$ where N is the number of basis functions. This number gets large quite fast as N increases, implying the need to handle very large numbers of these integrals, a formidable data management problem. Moreover, these two-electron integrals are very time consuming to compute even by the very best available techniques so that a large basis implies a substantial amount of time just to get the integrals. This is not only a problem of practicality but of

expense considering the high cost of computer time.

Furthermore, the larger the basis, the larger the matrices which must be handled at one time. This fact first of all reflects the practicality with respect to the programming of matrix manipulations. Provision must be made to carry out intricate calculations as far as possible in the high-speed core rather than to make heavy use of either low-speed core or external storage. Consequently, the size of available high-speed core is a limitation. Secondly, the manipulation of large arrays increases the number of arithmetic operations, in turn affecting the degree of accuracy which may be maintained (considering truncation, roundoff, differencing, etc.). The larger the basis, the greater will be the accuracy required in for example the values of integrals in order that their subsequent handling will not diminish the accuracy of results below the desired level.

Definition of the Even-Tempered Basis

This investigation introduces a new concept in the selection of a basis to be used for the type of calculation described previously. Although the individual basis functions are not of a new analytic form, the fact that several basis functions are often needed prompted this author to consider them collectively. As a result, the "even-tempered" atomic basis is defined as a collection or group of pure exponential

or pure Gaussian functions multiplied by a real solid spherical harmonic. The latter consists of r^l multiplied by a normalized real spherical harmonic $S_l^m(\theta, \phi)$ given by,

$$S_l^m(\theta, \phi) = P_l^{|m|}(\cos\theta) \Phi_m(\phi)$$

$$\Phi_m(\phi) = \begin{cases} \pi^{-1/2} \cos m\phi & m > 0 \\ (2\pi)^{-1/2} & m = 0 \\ \pi^{-1/2} \sin |m|\phi & m < 0 \end{cases}$$

and $P_l^{|m|}$ are associated Legendre functions (16) which are normalized. A set of even-tempered atomic orbitals is defined by

$$(19) \quad X(klm) = N_l(\zeta_k) \exp(-\zeta_k r^p) r^l S_l^m(\theta, \phi)$$

with

$$p = \begin{cases} 1 & \text{exponential} \\ 2 & \text{Gaussian} \end{cases}$$

and the radial normalization factor

$$N_l(\zeta_k) = \begin{cases} (2\zeta_k)^{l+3/2} [(2l+2)!]^{-1/2} & p=1 \\ (2\zeta_k)^{(2l+3)/4} \pi^{-1/4} [(2l+1)!!]^{-1/2} & p=2 \end{cases}$$

Consequently, different even-tempered atomic orbitals for a given S_l^m do not differ in the power of r , but only in the values of the exponents ζ_k . This is in contrast to Slater-

type orbitals which, from the present point of view, form a "mixed" basis. In linear combinations of primitive functions $X(klm)$,

$$(20) \quad G_{lm}(\Sigma) = \sum_{k=1}^M c_k X(klm),$$

the solid harmonic may be factored, yielding,

$$(21) \quad G_{lm}(\Sigma) = r^l S_l^m(\theta, \phi) \cdot \sum_{k=1}^M c_k N_l(\zeta_k) \exp(-\zeta_k r).$$

A second condition, which suggests the name "even-tempered" governs the choice of the exponential parameters $\{\zeta_k : k=1, 2, \dots, M\}$. These are taken to form a geometric sequence dependent upon two parameters, α and β , by the relation

$$(22) \quad \zeta_k = \alpha \cdot \beta^k, \quad \alpha, \beta > 0, \quad \beta \neq 1, \quad k=1, 2, \dots, M.$$

The reason for choosing this progression, was that, for expansions of the type of Eq. 20 used for atomic calculations, independent optimal determination of all zeta's did yield a set of values which gave an almost linear plot of $\log(\zeta_k)$ vs. k . The next step then was the assumption of such a straight line determined by two parameters a , the intercept, and b ,

the slope, which is equivalent to selecting the zeta's in a geometric progression and is the restriction given by Eq. 6, since α and β may be taken as $\exp(a)$ and $\exp(b)$ respectively. The selection of exponential parameters in a geometric sequence had been previously recommended by Reeves (17) for Gaussian primitive functions. In 1963 Reeves and Harrison (18) presented calculations on ammonia in which the s Gaussian primitives were chosen in this way and optimized in terms of an α and β . Since that time, the concept of an even-tempered basis has not been used, even for Gaussians.

Adequacy of the Even-Tempered Basis

A first question of interest is the completeness of the radial part of the new basis. It is pertinent to discuss the Laplace-Stieltjes transform

$$(23) \quad f(r) = \int_0^{\infty} \exp(-\zeta r) du(\zeta),$$

which is a generalization of the Laplace transform for which

$$u(\zeta) = \int_0^{\zeta} c(s) ds.$$

This transform correctly represents a large class of functions $f(r)$. Representation of a single exponential is of course trivial, if $u(\zeta)$ is taken as the unit step function $u(\zeta-\zeta')$ such that

$$du(\zeta-\zeta') = \delta(\zeta-\zeta') d\zeta$$

and so

$$\begin{aligned} f(r) &= \int_0^\infty \exp(-\zeta r) \delta(\zeta - \zeta') d\zeta \\ &= \exp(-\zeta' r) \end{aligned}$$

where $\delta(\zeta - \zeta')$ is the delta function introduced by Dirac (10). Since exponential functions are known to be the best analytic functions representing atomic orbitals, it is natural to assume that the real atomic orbitals are also representable by such transforms. Moreover, the Laplace-Stieltjes transform generates a Dirichlet series (19,20)

$$(24) \quad f(r) = \sum_{k=1}^{\infty} a_k \exp(-\zeta_k r), \quad 0 \leq \zeta_1 < \zeta_2 < \zeta_3, \dots, \quad \lim_{k \rightarrow \infty} \zeta_k = \infty,$$

if

$$\begin{aligned} u(\zeta) &= 0 & -\infty < \zeta \leq \zeta_1 \\ &= s(n) & \zeta_n < \zeta \leq \zeta_{n+1} & n=1,2,\dots \infty \end{aligned}$$

with

$$s(n) = \sum_{k=1}^n a_k \quad n=1,2,\dots \infty .$$

Although infinite, this series clearly is of the form of the radial part of the linear combination of even-tempered expo-

nential functions in Eq. 21. Thus, the constraint of Eq. 22 defines a special class of Dirichlet series. We make the conjecture that the exponent selection prescribed by Eq. 22, with k ranging to infinity, would result in a set of exponentials which form a complete basis, just as the Laplace transform does. In that case, one could interpret Eq. 22 as a practical grid for a numerical approximation to the Laplace transform. If this is so, then there is no loss of generality in the restriction to a single power of r . Similar arguments may be made for Gaussian series generated from Gaussian transforms.

From a practical point of view, it is more important whether or not this type of basis is adequate to expand the type of radial functions occurring in atomic SCF orbitals. It must be proved that impractical summation limits need not be used to attain the necessary accuracy. Furthermore, the question of appropriateness with respect to molecular calculations must be answered. Experience described in this work shows that the basis is not only very practical in many ways but is appropriate in terms of the molecular results which may be obtained.

Advantages and Disadvantages
of the Even-Tempered Basis

Before attempting to show that the present basis is sufficient to replace other earlier bases, it is of interest to review the possible advantages of using such a basis. Even if the new basis is not strictly competitive with others, a careful evaluation of all points for and against is necessary in order that adoption might be considered. It is to this end that the following lists are given.

Advantages

1. The new basis, instead of having one optimizable parameter per basis function, has two such parameters per group of atomic functions belonging to the same symmetry species of an atom or in some cases per atom. In either case, the net result is a marked decrease in the number of optimizable parameters. Presently there is little possibility of subjecting any basis to full optimization in polyatomic molecule calculations. The crucial fact is that optimization methods are based on quadratic fits and are iterative, with each fit requiring $(n+1)(n+2)/2$ points, where n is the number of free parameters per independent group. Change of basis functions or their determining parameters implies the recomputation of integrals requiring a large fraction of the computation time. Together, these facts render the use of full basis optimiza-

tion impractical. It may be that with a smaller number of largely independent parameters, optimization will become feasible.

2. Restriction of the basis to fewer analytic forms means that programs to compute integrals over the functions will be simpler than for alternative cases. This will alleviate some of the difficult programming and will most likely mean that less of the fast core will be needed for the code instructions and certain constant arrays.

3. Another reason for using the even-tempered basis is connected with the analysis required to formulate expressions for integral evaluation. The limitation to a solid harmonic multiplied by an exponential or Gaussian is the key here. The evaluation of two-electron integrals is often accomplished using some expansion of r_{12}^{-1} (21-26). For a general four-centered integral the problem is to express the four basis functions in coordinates of two centers. It is desirable then to have explicit formulae, preferably in closed form (finite series), for this expression. It has been shown that the solid spherical harmonic translates to parallel coordinate systems simply and in closed form (27-30). A special property allows the product of two pure Gaussian primitives located at separate points to be expressed as a single Gaussian on a point along the line joining the original Gaussians. Translating the solid harmonics to the same point produces a

bipolar problem which coupled with a bipolar expansion of r_{12}^{-1} may be solved in closed form. There is no convenient property for simple exponentials but the zeta-function expansions of Barnett and Coulson (31,32) and others (33-35) involve the expansion of one exponential at the site of the other. Coupled with the solid-harmonic translation, the integrals could in principle be carried out. This has in fact been used with the more general exponential for four-centered integrals but resulting programs (34) are very slow and therefore impractical. Perhaps with the aforementioned restrictions and with more concentration on providing for fewer cases, a practical set of programs could be written. In summary then, it is this neat property of the solid harmonic which may be of practical aid in future integral analysis.

4. Since many basis functions are of the same symmetry subspecies (i.e. have the same quantum numbers l and m from the spherical harmonic), calculations may be set up so that large blocks of integrals of uniform type are computed at the same time. This is particularly useful if the molecular calculation is to be carried out in terms of "contracted" or linear combination of primitive basis functions (18). In this event, the integrals over the primitives need not be stored even temporarily since they may be used right away in forming contracted integrals. Efficient techniques for this process keep the storage area to a minimum. In this way a

large block of integrals is "condensed" into a smaller block. No other contribution to the smaller block is needed and these integrals may be written on a peripheral device for later use. This contraction procedure (Appendix C) is used here for all molecular calculations.

5. The even-tempered basis has a unique space covering feature as a result of the geometric sequence of exponents. The expressions for the overlap integral between two normalized Gaussians or two normalized exponentials $X(klm)$ and $X(k'lm)$ at the same point in space may be rearranged to reveal that the value of the overlap integral is a function of the ratio of the exponents ζ and ζ' . For an example, refer to Appendix A. Because of this, a set of even-tempered functions of the same symmetry type has a matrix of overlap integrals, the peculiar feature of which is that values along diagonal lines are constant. This feature suggests that the basis "covers" the Hilbert space evenly, or equivalently, that no part of the Hilbert space intercepted by the basis is covered better (i.e. may be represented better) than any other part except near the limits of the space. A feature such as this is not necessary but is valuable if one wishes to use the same basis over regions of space other than that for which the basis was originally designed by replacing the radial argument r by the "scaled" argument (tr) . This introduction of such a scale parameter t is an important device

when atomic functions are used for molecular calculations.

6. The uncontracted basis cannot become linearly dependent if the spacing parameter beta is forced to stay greater than one. This has been easy to avoid even with free optimization of beta since $\beta=1$ would imply only one function with one exponent, a situation which is clearly not as good as a collection of functions with several different exponents. If beta would approach unity, the energy would increase as would the possibility of linear dependence.

7. Because of the small number of optimizable parameters upon which the basis depends, it may be possible to formulate a set of rules allowing one to quickly determine parameters for any atom. The utility of such a set of rules is proven by the wide and successful usage of Slater's empirically derived rules (36) for minimal basis sets. There are no similar rules for larger basis sets. In essence these rules would be a further parametrization of the optimizable parameters which are now found by other means. Such a parametrization would be useful where the parameters could not otherwise be optimally determined or as a first guess in their determination.

8. The proper mixing of the basis need not be prescribed since no mixing is used. This in effect is also a removal of parameters from the specification of the basis.

Disadvantages

1. The limitation of the basis for atom and molecule calculations as a product of a solid harmonic times a Gaussian with no additional power of r is not new. O-ohata et al. (37) showed that a uniform Gaussian s-basis of this type is sufficient to expand either 1s, 2s, or 3s Slater-type orbitals. In fact, the mean square deviations for the 2s and 3s orbitals are shown to be smaller than for the 1s orbital. This is probably due to the absence of a cusp. Similarly, Slater-type 2p, 3p, and 3d orbitals were fit very well with 2p and 3d Gaussians. Subsequently, it has been common practice to limit a Gaussian-type basis in this way although perhaps some mixing of basis forms could give better results.

2. The same type of restriction as above has apparently never been used or conceived of before in the case of exponential-type bases. Early ab initio calculations on molecules used "minimal" bases of Slater-type atomic orbitals where the orbital exponents were fixed by Slater's rules (36). The word minimal is used to imply that just one basis function per fully or partially occupied atomic orbital in the atomic ground state configuration is used as the molecule basis. The minimal basis returns quite a good energy for diatomic molecules (38,39). It seems to have developed that the mixed Slater-type minimal basis was augmented by adding other functions of the same mixed variety, for example, the

mixed double-zeta basis invented by Richardson (40).

3. For Gaussian calculations, the practice of using the same functional exponents ζ , for all symmetries of the basis of one atom was adopted. This is a further restriction to be sure and separate ζ 's for the separate symmetries would have to be better as a consequence of the variational principle. It has been found, in determining Gaussian atomic bases, that the difference between least-square approximations to the same exponential atomic orbitals with and without this restriction is not overwhelmingly large, in most cases less than an order of magnitude.

4. Another disadvantage is that the primitives are not independently optimized which means that the best result per basis size is not attainable. This again is a consequence of the variational principle and certainly is true. One must weigh the small loss in quality against the enormous gain in feasibility of parameter optimization. An important question is how many more even-tempered basis functions are needed as compared to the standard mixed bases.

5. In order to apply constraints to the calculation to satisfy certain theoretical requirements, a loss in variational freedom is suffered. Because the number of free parameters is already small, this may have a deleterious effect on the results. Little work of this sort has been attempted here so the possibility remains untested. Constraints such

as the cusp constraint have been applied to mixed bases without affecting the energy measurably (41). However, more basis functions were needed to maintain the quality.

ATOM CALCULATIONS

Intention

The logical first test of the even-tempered basis concept is an application to atoms. Here a test of the "completeness" of the basis is easily possible since the symmetry classification of Hartree-Fock atomic orbitals is identical to the symmetry classification of the basis functions. Hence one does not need to be concerned that some other atomic functions of different symmetry might contribute to a given atomic orbital. For example, the helium 1S state is described by a single spherically symmetric atomic orbital. This means that only basis functions of spherical form, the s functions where $l=0$, need be used to describe it. Because of this type of limitation, the possibility of approaching the Hartree-Fock limit may be tested. In molecules, this is not generally possible. A diatomic orbital of sigma symmetry for instance, may have a contribution from all atomic function symmetries $l=0, 1, 2, \dots^\infty$.

Furthermore, atom calculations do not require extensive computer time since the integrals may be computed quickly in simple closed form with exponential functions as a basis. The exponentials exhibit proper analytic behavior at the origin (42) and at long range (43) and fewer of these functions are needed in comparison to Gaussian functions (44).

Several compilations of atomic SCF results are available with which to compare the new approach (45-48). These all employ a mixed basis, i.e. not restricted to 1s, 2p, 3d, etc., so that the effect of the restriction may be assessed. It is fair to compare the results of the present work to these results since the approach taken is similar.

The aim of the atom calculations was that the best possible even-tempered atomic orbitals would be found for a given basis size. To fulfill this purpose, the parameters of the basis, α and β , were varied automatically to minimize the SCF energy. Separate parameters were taken for each symmetry s, p, d, and f. Moreover, the primitive basis functions of Eq. 19 ($p=1$) were used as the variational functions of the SCF procedure. This means that the atomic orbitals which result have the form of an even-tempered function, that being a normalized linear combination of appropriately defined primitives. According to the SCF technique, these even-tempered basis atomic orbitals are the best atomic orbitals expressible in terms of the basis. It is therefore fair to compare the results of these calculations to the existing atomic SCF results since the approach is similar. In the other calculations, the primitive functions are also the variational functions and the orbital exponents ζ are usually optimized to some extent. A group of efficient programs were written to carry out the proposed calculations. These are described in

Appendix A.

In order to arrive at a good initial guess for the parameters α and β , a program was written to find the best weighted (r^{-1}) least-squares approximation of an even-tempered basis to some given mixed exponential atomic orbital or orbitals. This technique was used to get initial values whenever an atomic SCF calculation for a specific atom was available. Heavy atoms have often not been computed beforehand with analytic basis functions although calculations by numerical procedures for all atoms have been made. The resultant numerical orbitals are described by tables of values which the orbitals attain at various values of r , the distance from the nucleus. Although they are often the best results available, numerical procedures are not easily used for molecular calculations. Therefore analytic functions in the form of exponentials or Gaussians are used here in order that ultimate extension to molecular calculations may be made. For the heavy atoms extrapolation from lighter atoms was used to get values of α and β prior to optimization. Specific starting values of α and β were found to be of little importance since the optimization procedure automatically tests values about those given. Very bad starting values were avoided however.

Investigation of Basis Size for Adequacy and Accuracy

Because of restrictions placed on the composition of the even-tempered function as described in its definition, it was thought that more primitives would be needed in the even-tempered function to equal the results obtainable with a freer basis containing mixed analytic forms within a symmetry. Thus a group of preliminary calculations were made on the 1S ground states of helium, beryllium, and neon in order to find out just what the actual case is. Tables 1, 2, and 3 collect the results of these calculations. Several even-tempered basis results are given, followed by some results where basis mixing was allowed. All even-tempered calculations are optimum with respect to basis parameters α and β . The other results are also optimized at least in part with respect to individual exponential parameters.

The composition of the basis is given by telling the number of primitives of each standard analytic form which were used. The energy is the SCF energy, truncated to the number of figures shown. Truncation is used instead of round-off in order that a more credible value is available for comparison in for instance the checking of new programs. The quantity $|2-V/E|$ is a measure of the deviation of the virial coefficient V/E (potential energy divided by total energy) from its theoretical value of two. This is an indication of

how well the virial theorem is satisfied and is a measure of the optimization of the wavefunction (49). For complete optimization, that is when the minimum energy with respect to the parameter space of the wavefunction which is being searched is found, the virial theorem is satisfied. The value 4.9(-8) means that the deviation is 4.9 in the eighth decimal place while 0.0(-10) means that it is exactly zero to the ten decimals printed in the program output. The column or columns headed "Cusp" contain values of the computed orbital cusp (42).

$$\left. \frac{\partial \phi_{nlm}(r)}{\partial r} \right|_{r=0} \neq \phi_{nlm}(0) ,$$

which theoretically assumes the value $-Z/(l+1)$ for each atomic orbital $\phi_{nlm}(r)$, n being the atomic orbital number analogous to the principal quantum number n in the hydrogen atom wavefunctions.

All quantities in these tables and in other tables included in this work are in atomic units (50 or 51) unless otherwise specified.

Helium

The first group of calculations in Table 1 represents the even-tempered results with the comparison values following. The three term basis of Gilbert and Bagus (45) surpass-

es the three-term even-tempered energy but the difference is hardly significant. It is of interest that the four-term even-tempered energy is lower than the four-term energy of Clementi (46). This should not be true since the basis is composed of similar analytic functions (four 1s functions) with the even-tempered basis more restricted. It has to be that optimization of the unrestricted basis is incomplete or that the optimization leads to a false or relative minimum.

In all even-tempered results, the virial coefficient is close to the theoretical value while the other calculations are not as good. It is perhaps difficult to compare these though, since the calculations may have been made with different numbers of significant figures carried by separate machines. Of further interest is the value of the cusp. This property of the wavefunction at the nucleus is an even finer test of the basis than the energy or other properties calculated from the wavefunction. The even-tempered basis is capable of representing the proper behavior even though several basis functions are required.

Beryllium

The general features of Table 2 are the same as Table 1. Notable are the energies for the four-term calculations. It is seen that the energy for the even-tempered basis falls between the two energies from four-term mixed bases. This

would suggest that the mixed basis could be best in certain cases if the mixing of the basis is appropriate. However no prescription exists for selection of the analytic form of the primitive basis functions and some trial and error procedure might be necessary. With an increase in the number of basis functions, the number of mixing arrangements becomes large and as a consequence, determination of the best mixed basis would be highly impractical. The even-tempered basis avoids this problem since the mixing question is not left open. Furthermore, the result obtained is not significantly worse, at least for the four-term wavefunction. The two five-term comparison functions attain a better energy than the five-term even-tempered function although at worst, the difference is only five ten-thousandths of a percent. Again, the virial theorem is easily satisfied and the cusp is represented reasonably well.

Neon

The neon energies are displayed in Table 3 according to the same format as above. It is somewhat difficult to compare the bases here since all comparison calculations used primitives only of 2p analytic form. These are the same as the even-tempered functions and therefore, difference effects of the p parts of the wavefunctions are minimal and difficult to identify. However, general trends in for example, the 2p

cusp suggest that the p part of the wavefunction is as adequately represented as the s part. Again it is seen that the even-tempered basis lags the more traditional mixed basis with respect to energy lowering but the difference between energies computed with bases of similar size is not really significant.

Summary

In summary, the results for helium, beryllium, and neon indicate that the even-tempered basis is an adequate basis for the representation of Hartree-Fock atomic orbitals. The standards of low energy, accurate cusp values, and satisfaction of the virial theorem are all met successfully. This suggests that the basis set is complete, at least to the same degree as the mixed bases used for comparison. A general fact that emerges is that more often than not, more even-tempered exponentials would be needed to attain the accuracy obtained with a good mixed basis. With optimization however, this must be a minor consideration since the total time of calculation is of sole import. For helium and beryllium, the number of optimizable parameters is reduced to just two, and for neon, just four. Bases good enough to attain the Hartree-Fock accuracy limit require large numbers of basis functions. Consequently, widely differing numbers of optimizable parameters are required for the two types of basis, implying an

even-tempered advantage. For example, the neon eighteen primitive mixed basis of Table 3 requires optimization of ten parameters as opposed to four.

It is difficult to identify the effects of the individual even-tempered basis restrictions because those bases available for comparison are not designed to bring out the differences. The programs used here could not be used to make other mixed basis calculations because of the built-in restrictions.

It was deemed worthwhile nevertheless, to carry out an extensive group of calculations on several atoms with basis sizes governed by these preliminary results and by practical considerations. In this way some good even-tempered atomic bases would be made available and furthermore, this new basis might be proved useful in other ways.

Double-Zeta-Type SCF Calculations

Because of computational limitations, some limit to the size of a computational basis must be imposed. Although molecular calculations are not widely carried out with exponential-type basis functions, the "double-zeta" (DZ) exponential basis has been proposed as a basis of proper and adequate size for molecular work (40,52). In fact, atomic Gaussian bases which duplicate atomic double-zeta SCF energy results are often chosen to be used for molecular calculations. The

double-zeta basis is therefore a kind of benchmark or standard of accuracy.

By definition, the double-zeta basis is a mixed basis, which is made up of two non-orthogonal Slater-type primitives of the same designation as the corresponding atomic orbital which is to be represented. For example, neon has a ground state configuration $1s^2 2s^2 2p_x^2 2p_y^2 2p_z^2$ or $1s^2 2s^2 2p^6$. Consequently, the double-zeta basis for neon consists of two 1s type functions, two 2s type, and two each of $2p_x$, $2p_y$, and $2p_z$. Recent optimum double-zeta bases for helium through argon are given by Huzinaga and Arnaud (53). The exponents of each of these primitive functions were carefully varied to attain best energy minima. These bases are found sufficient to obtain a large part of the Hartree-Fock atomic energy and contain enough functions so that each electron could be represented by one analytic basis function. This property means that the basis has enough variational freedom for use in configuration interaction theories. Minimal basis sets cannot be used for that purpose.

As a comparison we have found the even-tempered double-zeta (ETDZ) bases for the atoms lithium through argon. The even-tempered double-zeta basis is an even-tempered basis which is of the same size (same number of primitives) as the ordinary double-zeta basis but with the even-tempered restrictions. For example, neon requires four 1s primitives

and two each of the $2p_x$, $2p_y$, and $2p_z$ primitives. Because there is no basis mixing within a symmetry, a notation similar to that introduced for Gaussians by Neumann and Moskowitz (54) may be conveniently used. The symbol (ks,lp,md) is used to denote the basis of k s functions, l independent p functions, and m independent d functions. Each symmetry is degenerate of degree $2l+1$ so that, for instance, each independent p function implies three primitives; p_x , p_y , and p_z . The parentheses further mean that each function is a variational function in the theoretical procedure. The neon basis above is $(4s,2p)$. Square brackets denote the number of contracted functions, a concept to be treated at length in the following sections on molecular calculations. The letters s, p, d, or f, or the commas may be left out in order to be more compact.

The energies from these calculations are collected in Table 4. Some basis parameters are presented in Table 9 along with corresponding coefficients describing the even-tempered SCF atomic orbitals. This table contains basis information for only the atoms boron through magnesium since the even-tempered double-zeta basis is identical to the even-tempered augmented double-zeta (ETADZ) basis to be described below. Basis information for the remaining atoms is given under the ETADZ classification.

In Table 4 we find that the even-tempered double zeta

basis gives a lower energy for beryllium and boron. For all other atoms the regular double-zeta basis gives a lower energy although either way, the difference is smaller than two-hundredths of one-percent. Therefore, the even-tempered double-zeta basis is quite good in comparison, and in fact would probably retain the same standard of accuracy in a molecular calculation. This contention has however not been tested at this time. A simple test would be to use both of the optimum double-zeta bases for calculations on a group of first- and second-row diatomic molecules.

Augmented Basis SCF Calculations

In order to provide other fully optimized atomic bases which might be more suitable for future molecule calculations, especially for heavy atoms, two bases other than the double-zeta size were considered. First of all, we have taken what we call the even-tempered augmented double-zeta basis which is near double-zeta size. The difference is that whenever only two basis functions are used to represent a symmetry for the double-zeta case, the basis for that symmetry is augmented by one function. For example, neon, which requires two 2p functions in the double-zeta basis, would be given three in the augmented basis, i.e. (4s,3p) as opposed to (4s,2p). For argon, the basis is (6s,4p), so the augmented basis is identical to the double-zeta basis.

The reason for this augmentation is to achieve a "balance" (55) among the bases of all symmetries. It is reasonable to have each symmetry represented equally well so that for example, the addition of a single function to any symmetry will result upon optimization in an approximately equal energy lowering. In the case of double-zeta neon, only two basis functions can represent the 2p atomic orbital, whereas in argon, all four p basis functions share in the representation of the 2p and 3p atomic orbitals. The extra function of the augmented basis adds extra potential linear variation for the neon p orbital.

As a second basis we have chosen the even-tempered augmented triple-zeta (ETATZ) basis. This is a triple-zeta sized basis (i.e. three primitives per occupied or partially occupied orbital) of even-tempered variety which is augmented in the same way as the double-zeta basis above. Here neon would have a (6s,4p) basis while that for argon would be (9s,6p).

In order to evaluate the results obtained with the augmented bases it is convenient to divide them into the separate classifications of light atoms and heavy atoms. The division is chosen to be at atomic number Z=21, which is the first atom requiring d orbitals in its ground state configuration.

Light atoms

A complete set of results has been compiled for the light atoms in both of the augmented bases. The energies are presented in Table 5 along with accurate SCF values approaching Hartree-Fock (47,48) for comparison.

By a comparison of Table 5 with Table 4, one sees that for the atoms boron through neon, for which the double-zeta and augmented double-zeta bases are different, a substantial difference is observed. In fact, this difference is greater than that between the two augmented bases of Table 5. Thus, it would appear that the augmented double-zeta basis is a significant improvement over the ordinary double-zeta basis and that consequently it may be a good computational choice for molecules.

Of further interest is the accuracy characteristic of the triple-zeta energies. The energies obtained for silicon, phosphorus, sulfur, and potassium are lower than the previous accurate values while the others remain higher but close. The important point may be reiterated that just four parameters were varied to obtain these optimum energies whereas, for example, the accurate argon wavefunction, a cusp-constrained function, depends on fourteen non-linear variational parameters.

Heavy atoms

The heavy atoms, as defined here require d and/or f orbitals for the ground state configuration. It was not possible to try each atom in the transition or lanthanide series because the proper coupling coefficients are not available with which to form the correct ground states. However, results given include both open and closed d- and f-shells and should be sufficient to test the performance of the new basis, since prior results for light atoms show no dependence on the state.

For these heavy atoms, the basis used for the most part was the smaller augmented double-zeta basis, purely for economic reasons. In instances where the triple-zeta basis was tried, the energy comparison with accurate results is very good, the difference being a very small percentage of the total energy. The energies are displayed along with light atom energies in Tables 5, 10, and 11.

The calculations for the series rubidium through lanthanum represent the first tabulation of analytic basis SCF calculations for this group of atoms. Furthermore, these analytic bases are designed to have a balanced composition, with no symmetry emphasized. In the past, some SCF calculations were made on heavy atoms, especially the lanthanide series (56), in which the outer f-shells were emphasized. A basis which could be considered nearly minimal (viz. one basis

function per pair of electrons) was used for s, p, and d electrons. Four or five f basis functions were then used for each f orbital. It may be argued that the symmetries are largely independent so this may be done. However they are not completely independent and just how much so is an important question if one is to justify reaching any practical conclusions on the basis of the results. This is especially true here since the core orbitals have a large shielding effect on outer orbitals. This kind of calculation most probably was done because of a lack of computing power and general methods for the selection of good bases.

Although present calculations obtained for heavy atoms past krypton cannot be compared to other analytic basis SCF calculations, it appears that there is no loss of computing ability with the extension to greater numbers of electrons or higher symmetry. The fifth period of the table involves a second occupied d-level. Moreover, we include two calculations involving f orbitals, those of gadolinium ion and radon. The linear coefficients for the f orbitals shown in Table 10 are quite similar to corresponding cases where a new symmetry was occupied for the first time, for example, the d shell of krypton or the p shell of neon. Also the optimal slope parameter β is consistent as expected.

A comparison of energies may be made with the numerical SCF results of Fischer (57) or Mann (58,59). Mann's calcula-

tions do not treat the orthogonality of open-shells properly so that in that sense they do not exactly correspond to the Hartree-Fock solutions. The calculations of Fischer apparently do treat this criterion correctly. Both sets of numerical calculation apparently were done with low precision arithmetic so that due to accumulation of machine error, the latter figures of the energy values might be considered dubious. Nevertheless, Fischer records an energy of -21866.79 Hartrees for radon while the present calculation yielded a value of -21866.62648 Hartrees. This is quite a good comparison. A similar result for Gd⁺³ was not found. Numerical results are said to be very close to exact Hartree-Fock since no problem of only partial representation of the Hilbert space of an atom exists. One in effect solves for the solution space directly. If these numerical results are indeed well done, then the even-tempered basis has also performed very well.

Local minima

Although no serious problems occurred while running the programs, it was found that local minima in the energy surfaces do occur. A scan of Tables 10 and 11 reveals that the slope or spacing parameter, β , is often approximately constant through a similar group of atoms within one type of basis (e.g. boron through neon). This holds for the augmen-

ted triple-zeta bases in Table 11 but it is not strictly true for the augmented double-zeta basis as seen in Table 10. If now the coefficients in Table 10 corresponding to the latter are examined, one finds a change of the primitive function which has the largest coefficient in the 1s and 2s atomic orbitals. In particular, the largest coefficient of the 2s orbital is always the first function, but in the sequence boron to neon, the largest coefficient of the 1s atomic orbital changes at carbon from the third primitive to the fourth. This coincides with the change in the slope parameter. It was possible to find a set of optimum bases where the parameter β was more constant throughout this group of atoms but bases other than those displayed produced a slightly higher energy. Such minima were true local minima since extensive optimization about the point did not reveal a false minimum or saddle point. In fact, the virial coefficient was close to two at these points.

Although these local minima can occur, it is sometimes possible to spot irregularities in sequences such as that described above so that the true minimum may be found. For example, if the β parameter switched back in the sequence discussed above, one would expect that something is amiss and look to resolve it.

It is hoped that none of the wavefunctions given represents a local minimum of the energy surface but no assurance

can be given that all are true optima. For the most part, extensive trials were made, especially for the light atoms in the smaller bases. Often it was found that even though the basis parameters were quite different for separate minima, the corresponding energy values were equal to a large number of significant figures. Thus the results given here may be considered a good indication of the capability of the even-tempered basis.

We feel that apart from the problem of possible local minima, the even-tempered basis has been demonstrated to be quite useful for atomic calculations. It seems to be at least adequate, and may be used conveniently to attain high standards of accuracy. It is hoped furthermore, that the even-tempered basis approach contains improvements which will prove useful to cope with problems arising in atomic as well as molecular problems.

Properties

A compilation of properties computed in each basis for all atoms listed in this work is included in Table 12. The table is arranged by atomic number and within this classification by basis size, the smaller bases taken first. The atomic number and name of the atom or ion along with the spectroscopic state and orbital configuration precedes the basis composition. The quantity $|2-V/E|$ described before is

given along with $\|1\text{-Norm}\|$ which is the deviation from the value one of the computed overlap, $\langle \Psi | \Psi \rangle$, of the total wavefunction Ψ . Values of the cusp summed over all orbitals of the same symmetry are given next. Finally expectation values of several operators, $\langle \phi_i | O | \phi_i \rangle$, are given for each atomic orbital ϕ_i and also summed over all occupied orbitals

$$\langle \Psi | O | \Psi \rangle = \sum_i^{\text{occ.}} N(i) \langle \phi_i | O | \phi_i \rangle$$

for the total wavefunction, with occupation numbers $N(i)$. The total is listed just below the operator heading with the orbital values following according to the conventional atomic orbital notation. The operators are in order: T, kinetic energy; V(1), one-electron potential energy; V(2), two-electron potential energy; V(T), total potential energy; E, total energy or $T+V(T)$; e, orbital energy (the total represents $[T+V(1)]+2\cdot V(2)$); -cusp, orbital values; r max, the value of r at which the orbital distribution attains a maximum; and r^{-3} , r^{-2} , r^{-1} , r , r^2 , r^4 , r^6 , various powers of the operator r. The cusp and r max are not expectation values but orbital properties so that a total is not given. The expectation value of r^{-3} is not defined for s orbitals and therefore the total is not possible. The ordering of the orbitals in this table is according to a numerical sequencing

of the orbital eigenvalues e. Table 12 is computer printed and as a consequence, all values printed here have been automatically rounded off, including the total energy.

Table 1. SCF ground state helium calculations

Number of Basis Functions			Energy	$ 2-V/E $	-Cusp	Ref.
1s	2s	Total				
3	-	3	-2.861679036686	5.0 (-10)	2.015	a
4	-	4	-2.861679875316	1.3 (-7)	2.0013	a
5	-	5	-2.861679986833	4.0 (-8)	2.0004	a
6	-	6	-2.861679987495	1.0 (-10)	2.0011	a
7	-	7	-2.861679994968	4.9 (-8)	2.00060	a
8	-	8	-2.861679995610	2.0 (-10)	2.000029	a
12	-	12	-2.861679995615	3.7 (-8)	1.999968	a
2	-	2	-2.8616725			b
1	2	3	-2.861680	7.0 (-6)	2.012	c
4	-	4	-2.8616785	1.2 (-6)		d
5	-	5	-2.8616799	2.3 (-6)		d

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a. This work
 b. Huzinaga and Arnau (53)
 c. Bagus and Gilbert (45)
 d. Clementi (46)

Table 2. SCF ground state beryllium calculations

Number of Basis Functions				Energy	2-V/E	-Cusp		Ref.
1s	2s	3s	Total			1s	2s	
4	-	-	4	-14.57282754	2.9 (-7)	4.08	4.03	a
5	-	-	5	-14.57294014	1.7 (-8)	3.95	4.11	a
6	-	-	6	-14.57301487	5.0 (-10)	4.03	4.05	a
7	-	-	7	-14.57302122	7.2 (-9)	3.993	3.96	a
8	-	-	8	-14.57302308	4.6 (-9)	4.003	4.006	a
12	-	-	12	-14.57302316	2.0 (-9)	3.9998	4.0001	a
2	2	-	4	-14.572371				b
2	1	1	4	-14.57299	1.3 (-5)	4.02	3.95	c
2	3	-	5	-14.573014	1.2 (-6)			d
2	2	1	5	-14.57302	0.0 (-6)	4.02	4.03	c
2	4	-	6	-14.573020	1.5 (-6)			d

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a. This work
 b. Huzinaga and Arnaud (53)
 c. Bagus and Gilbert (45)
 d. Clementi (46)

Table 3. SCF ground state neon calculations

Number of Basis Functions					Energy	$ 2-V/E $	1s	-Cusp 2s	2p	Ref.
1s	2s	3s	2p	Total						
4	-	-	2	10	-128.5342215	5.7(-8)	9.87	9.91	4.16	a
4	-	-	3	13	-128.5449241	3.1(-8)	9.87	9.90	4.62	a
5	-	-	3	14	-128.5465597	2.0(-10)	10.02	9.97	4.62	a
5	-	-	4	17	-128.5469298	2.5(-8)	10.02	9.97	4.76	a
6	-	-	4	18	-128.5469510	1.5(-8)	10.02	10.12	4.75	a
6	-	-	5	21	-128.5470677	4.0(-8)	10.02	10.12	5.10	a
8	-	-	6	26	-128.5470968	3.8(-8)	10.001	9.98	5.07	a
2	2	-	2	10	-128.53508					b
2	1	1	3	13	-128.54648	8.0(-6)	10.01	10.01	4.68	c
2	3	-	4	17	-128.54698	3.2(-6)				d
2	2	1	4	17	-128.54709	2.0(-6)	10.02	10.05	5.0003	c
2	4	-	4	18	-128.54701	4.9(-6)				d

a. This work

b. Huzinaga and Arnaud (53)

c. Bagus and Gilbert (45)

d. Clementi (46)

Table 4. Double-zeta vs. even-tempered double-zeta energies

Atomic State	This work	E n e r g y Reference (53)	Percent Diff.
Li(² S)	-7.431888	-7.4327223	0.0112
Be(¹ S)	-14.572827	-14.572371	0.0031
B(² P)	-24.528240	-24.527921	0.0013
C(³ P)	-37.686548	-37.686751	0.0005
N(⁴ S)	-54.397139	-54.397950	0.0014
O(³ P)	-74.803506	-74.804316	0.0010
F(² P)	-99.400464	-99.401302	0.0008
Ne(¹ S)	-128.534221	-128.53508	0.0006
Na(² S)	-161.848025	-161.85002	0.0012
Mg(¹ S)	-199.604884	-199.60702	0.0010
Al(² P)	-241.865719	-241.87320	0.0030
Si(³ P)	-288.844336	-288.85120	0.0024
P(⁴ S)	-340.709037	-340.71597	0.0020
S(³ P)	-397.491350	-397.50230	0.0209
Cl(² P)	-459.458310	-459.47962	0.0046
Ar(¹ S)	-526.795062	-526.81512	0.0038

Table 5. Augmented double- and triple-zeta energies

Atomic State	ADZ	Energy ATZ	Reference (48)
He (¹ S)	-2.861679	-2.861679	-2.8616799
Li (² S)	-7.431888	-7.432720	-7.4327257
Be (¹ S)	-14.572827	-14.573014	-14.573020
B (² P)	-24.528484	-24.529049	-24.529052
C (³ P)	-37.687365	-37.688598	-37.688611
N (⁴ S)	-54.398981	-54.400895	-54.400911
O (³ P)	-74.807525	-74.809286	-74.809359
F (² P)	-99.407396	-99.409204	-99.409284
Ne (¹ S)	-128.544924	-128.546951	-128.54701
Na (² S)	-161.855877	-161.858803	-161.85889
Mg (¹ S)	-199.606331	-199.614546	-199.61458
Al (² P)	-241.865719	-241.876625	-241.87665
Si (³ P)	-288.844336	-288.854277	-288.85426
P (⁴ S)	-340.709037	-340.718696	-340.71866
S (³ P)	-397.491350	-397.504766	-397.50475
Cl (² P)	-459.458310	-459.481799	-459.48187
Ar (¹ S)	-526.795052	-526.817019	-526.81734
K (² S)	-599.147486	-599.164566	-599.16446
Ca (¹ S)	-676.743435	-676.757145	-676.75803
Sc (² D)	-759.717952		-759.73555
Cr (⁷ S)	-1042.333876		-1043.3539
Mn (⁶ S)	-1149.834817		-1149.8651
Cu (² S)	-1638.880127		-1638.9614
Zn (¹ S)	-1777.780153		-1777.8470

Table 5. (Continued)

Atomic State	ADZ	E n e r g y ATZ	Reference (48)
Ga (² P)	-1923.158789	-1923.259911	-1923.2602
Ge (³ P)	-2075.252892		-2075.3590
As (⁴ S)	-2234.127989		-2234.2382
Se (³ P)	-2399.794470		-2399.8669
Br (² P)	-2572.371868		-2572.4406
Kr (¹ S)	-2751.989840		-2752.0543
Rb (² S)	-2938.301898		
Sr (¹ S)	-3131.497207		
I (² D)	-3331.580682		
Mo (⁷ S)	-3975.348771		
Tc (⁶ S)	-4203.747941		
Pd (¹ S)	-4937.518502		
Aq (² S)	-5197.389750		
Cd (¹ S)	-5464.891360		
In (² P)	-5739.955617		
Sn (³ P)	-6022.778861		
Sb (⁴ S)	-6313.332880		
Te (³ P)	-6611.644773		
I (² P)	-6917.851512		
Xe (¹ S)	-7232.015542	-7232.136903	
Cs (² S)	-7553.835452		
Ba (¹ S)	-7883.462981		
La (² D)	-8220.974409		
Gd ⁺³ (⁸ S)	-10819.244854		
Rn (¹ S)	-21866.626489		

Table 6. Even-tempered helium orbitals

Num. of Primitives	α/β	Coefficients 1s	Num. of Primitives	α/β	Coefficients 1s
3	0.932625 1.517207	0.754230 0.176473 0.094288	8	0.791863 1.326345	0.002257 0.670548 0.195043 0.077427
4	0.852996 1.662827	0.770410 0.212485 0.044500 -0.002390			0.069819 0.011532 -0.000622 -0.000911
5	0.520556 1.654836	0.003783 0.772507 0.204780 0.046655 -0.002620	12	0.886077 1.250257	0.002039 0.632834 0.188303 0.077297 0.070527 0.039531
6	0.520505 1.655188	0.003942 0.772942 0.204117 0.046742 -0.002718 0.000021			0.019261 -0.005271 0.001719 -0.001360 0.000240 -0.000027
7	0.743184 1.373390	0.003370 0.699106 0.181136 0.092362 0.048028 0.002940 -0.001848			

Table 7. Even-tempered beryllium orbitals

Num. of Primitives	α/β	Coefficients	
		1s	2s
5	0.341735	-0.003323	1.423143
	2.181110	0.020668	-0.600065
		0.944531	-0.135170
		0.050992	-0.004535
		-0.001715	-0.000186
7	0.445690	0.000086	1.466503
	1.668493	-0.001087	-0.322507
		0.012748	-0.370989
		0.894730	-0.069240
		0.096131	-0.017014
		0.009841	-0.002451
		-0.001086	0.000432
8	0.514111	-0.000914	1.601002
	1.472859	0.005444	-0.433816
		-0.019970	-0.124325
		0.086934	-0.343648
		0.864124	0.025367
		0.034619	-0.037930
		0.045149	-0.002806
		-0.004028	0.009515
12	0.582434	-0.000584	1.789901
	1.318837	0.004106	-0.612197
		-0.015097	0.030074
		0.040205	-0.198415
		-0.096614	-0.261399
		0.568058	-0.040633
		0.512672	0.014270
		-0.071786	-0.034889
		0.071893	-0.001886
		-0.000247	-0.001052
		-0.000896	0.000503
		-0.000347	0.000015

Table 8. Even-tempered neon orbitals

	Primitive s orbs.	α/β	Coefficients		Primitive p orbs.	α/β	Coefficients
			1s	2s			2p
6	1.183392	0.001522		1.085652	5	0.945886	0.379104
	1.683379	-0.002025		0.317739		1.730264	0.471489
	0.041120		-0.626813				0.232068
	0.924005		-0.126793				0.014380
	0.043768		0.002019				0.001433
	-0.000197		-0.001120				
7	1.186983	0.001692		1.096454	5	0.945945	0.379168
	1.683024	-0.002824		0.298667		1.730272	0.471472
	0.044342		-0.615346				0.232028
	0.924064		-0.133471				0.014369
	0.040441		0.007127				0.001433
	0.000642		-0.003291				
	-0.000161		0.000475				
8	1.328402	-0.000079		1.016811	6	1.016690	0.274020
	1.487490	0.005844		0.357148		1.518533	0.376742
	-0.012690		-0.173826				0.289370
	0.095221		-0.485263				0.149715
	0.884190		-0.061623				0.005168
	0.025912		0.001422				0.003801
	0.011006		-0.004812				
	-0.001232		0.000744				

Table 9. Even-tempered double-zeta atomic orbitals

	Li (² S)	Be (¹ S)	B (² P)	C (³ P)	N (⁴ S)	O (³ P)	F (² P)	Ne (¹ S)
α	0.307856	0.340407	0.473478	0.596363	0.831171	0.955814	1.073954	1.187882
β	1.819795	2.191497	2.130513	2.107092	1.718128	1.713781	1.713069	1.714098
1s	0.040383 -0.160234 0.629373 0.507880	-0.005416 0.030455 0.945497 0.040503	-0.004106 0.029201 0.950047 0.034552	-0.003359 0.029763 0.953092 0.029537	0.023833 -0.092634 0.298659 0.779664	0.021349 -0.081961 0.267849 0.801869	0.019280 -0.073120 0.243306 0.819366	0.017584 -0.065909 0.223818 0.833149
2s	1.747934 -1.005712 0.023828 -0.078399	1.422549 -0.604531 -0.130205 -0.004865	1.447173 -0.619446 -0.160126 -0.001243	1.451785 -0.613568 -0.182008 0.001841	1.331797 -0.115896 -0.480427 -0.077345	1.270778 -0.002848 -0.534955 -0.078874	1.216481 0.091891 -0.575754 -0.081021	1.169547 0.170392 -0.606555 -0.083446
α			0.455209	0.577616	0.690710	0.743435	0.817103	0.900784
β			2.203933	2.172201	2.165283	2.226565	2.260341	2.278920
2p			0.839385 0.216592	0.801671 0.260567	0.775851 0.290777	0.742980 0.334277	0.726153 0.356594	0.715755 0.370114

Table 9. (Continued)

	Na (² S)	Mg (¹ S)
α	0.290947	0.379323
β	1.836890	1.784254
1s	0.005192	0.005290
	-0.019411	-0.020411
	0.043093	0.046080
	-0.076456	-0.082053
	0.182226	0.194774
	0.873338	0.863933
2s	0.043694	0.055187
	-0.187687	-0.239363
	0.782709	0.944856
	0.886052	0.775013
	-0.785336	-0.821832
	-0.103060	-0.090682
3s	1.849830	1.791271
	-1.212085	-0.971433
	0.108614	-0.208562
	-0.168976	-0.075757
	0.134383	0.151598
	0.012730	0.017434
α	1.196692	1.490216
β	2.136139	2.040478
2p	0.732854	0.744486
	0.338102	0.316733

Table 10. Even-tempered augmented double-zeta atomic orbitals

	B (² P)	C (³ P)	N (⁴ S)	O (³ P)	F (² P)	Ne (¹ S)
α	0.473537	0.596677	0.831533	0.956576	1.075176	1.187882
β	2.130513	2.106704	1.717951	1.713445	1.712584	1.714098
	-0.004163	-0.003360	0.023842	0.021355	0.019289	0.017610
1s	0.029415	0.029770	-0.092683	-0.082012	-0.073182	-0.066011
	0.949949	0.953064	0.298839	0.268055	0.243536	0.224006
	0.034492	0.029560	0.779524	0.801706	0.819184	0.833035
	1.447285	1.452187	1.331992	1.271281	1.217363	1.167772
2s	-0.619675	-0.614147	-0.115889	-0.003089	0.091253	0.173780
	-0.160039	-0.181890	-0.480841	-0.535558	-0.576435	-0.608672
	-0.001245	0.001822	-0.077219	-0.078675	-0.080769	-0.083075
α	0.506643	0.628198	0.735915	0.759646	0.814197	0.890710
β	1.895047	1.876172	1.877756	1.943099	1.966034	1.911663
	0.755251	0.687456	0.638197	0.568813	0.521448	0.452051
2p	0.274521	0.340733	0.388061	0.458217	0.498248	0.524900
	0.030547	0.039495	0.046753	0.059188	0.073570	0.120834

Table 10. (Continued)

	Na (² S)	Mg (¹ S)	Al (² P)	Si (³ P)	P (⁴ S)	S (³ P)	Cl (² P)	Ar (¹ S)
α	0.290910	0.379303	0.504009	0.547043	0.638558	0.739189	0.829412	0.924977
β	1.836890	1.784253	1.726074	1.903839	1.871659	1.841115	1.821166	1.802225
	0.005162	0.005277	0.005667	-0.001608	-0.001312	-0.000893	-0.000622	-0.000260
1s	-0.019299	-0.020357	-0.022709	0.006014	0.005015	0.003539	0.002532	0.001182
	0.042873	0.045977	0.052237	-0.013969	-0.012121	-0.009234	-0.007276	-0.004619
	-0.076147	-0.081919	-0.093048	0.057151	0.054256	0.049355	0.045786	0.040994
	0.181775	0.194579	0.217325	0.944044	0.944929	0.946737	0.948365	0.950625
	0.873611	0.864052	0.847797	0.015146	0.015724	0.016729	0.017209	0.017843
	0.043408	0.054991	0.065350	0.002533	-0.000843	-0.005723	-0.006021	-0.006498
2s	-0.185634	-0.237936	-0.288134	0.006915	0.021625	0.044535	0.046725	0.052602
	0.778505	0.941960	1.185580	1.662561	1.698594	1.725791	1.759736	1.788861
	0.890045	0.777793	0.550223	-0.894349	-0.960646	-1.024543	-1.073387	-1.120724
	-0.786947	-0.823033	-0.823494	-0.180903	-0.167682	-0.153533	-0.144777	-0.135235
	-0.102902	-0.090526	-0.076834	0.005518	0.004248	0.002805	0.002077	0.001208
	1.840723	1.785693	1.920029	1.890233	1.940176	2.010165	2.044823	2.089977
3s	-1.191602	-0.959045	-1.090592	-1.304205	-1.355841	-1.453003	-1.484125	-1.540341
	0.092869	-0.218280	-0.341121	-0.206467	-0.264487	-0.286746	-0.337346	-0.366725
	-0.164919	-0.073213	0.000693	0.239128	0.295459	0.340587	0.388375	0.428762
	0.134227	0.151571	0.170622	0.042501	0.035822	0.030396	0.023386	0.017262
	0.013055	0.017604	0.018545	-0.001327	-0.000255	0.000570	0.001551	0.002383
α	1.238510	1.578058	0.455807	0.541613	0.603840	0.662097	0.797545	0.820346
β	1.840941	1.764896	1.991736	1.955839	2.016551	2.028799	1.857083	1.993139
	0.540021	0.575075	0.007284	0.010654	-0.005755	-0.007839	0.027044	-0.006503
2p	0.466062	0.429179	0.008828	-0.005489	0.066672	0.076309	-0.071825	0.064570
	0.072052	0.061991	0.773604	0.789900	0.842321	0.856230	0.782553	0.871163
			0.270127	0.258736	0.144255	0.119140	0.305895	0.108307
			1.197201	1.166101	1.144724	1.113911	1.069998	1.089682
3d			-0.270025	-0.197767	-0.172111	-0.117159	0.009000	-0.059845
			-0.125751	-0.181131	-0.223382	-0.260169	-0.305873	-0.311454
			-0.056780	-0.056241	-0.031696	-0.024701	-0.070805	-0.020545

Table 10. (Continued)

	K (2S)	Ca (1S)	Sc (2D)	Cr (7S)	Mn (6S)	Cu (2S)	Zn (1S)
α	0.193922	0.343262	0.263126	0.352324	0.396031	0.409456	0.459794
β	1.776909	1.782463	1.732148	1.697936	1.662137	1.705810	1.688416
	0.001510	-0.000122	0.001333	0.000877	0.001019	0.000868	0.000740
	-0.005877	0.000529	-0.005384	-0.003694	-0.004329	-0.003609	-0.003147
	0.013277	-0.001467	0.012504	0.008958	0.010497	0.008617	0.007686
1s	-0.023803	0.003224	-0.022889	-0.017225	-0.019978	-0.016232	-0.014847
	0.038226	-0.007957	0.037317	0.029662	0.033726	0.027341	0.025670
	-0.060646	0.045935	-0.060140	-0.051085	-0.056273	-0.046173	-0.044541
	0.141405	0.949088	0.142526	0.130212	0.136674	0.112723	0.111783
	0.901427	0.016114	0.899830	0.906835	0.903067	0.920337	0.920410
	0.014434	-0.002772	0.021732	0.025638	0.028621	0.014281	0.018589
	-0.055543	0.010291	-0.085961	-0.102785	-0.117095	-0.057106	-0.075363
	0.123336	-0.019607	0.193596	0.234065	0.268449	0.128113	0.170765
2s	-0.215179	0.051404	-0.339109	-0.409435	-0.470466	-0.216323	-0.291955
	0.405278	1.843742	0.626299	0.743353	0.860758	0.380755	0.516169
	1.550034	-1.187484	1.427626	1.394319	1.311645	1.745317	1.655284
	-1.159450	-0.123644	-1.205952	-1.268999	-1.275244	-1.397954	-1.396131
	-0.085620	0.000909	-0.067517	-0.052852	-0.045229	-0.052245	-0.043688
	-0.017668	0.096609	-0.016847	0.025497	0.007222	-0.055971	-0.044386
	0.059015	-0.413305	0.056707	-0.075448	-0.003769	0.207085	0.159487
	-0.070583	2.621276	-0.072400	0.024230	-0.198710	-0.358089	-0.231886
3s	2.355684	-1.760550	2.601548	-2.667596	-2.480177	2.866494	2.854218
	-2.031462	-0.483729	-2.490041	2.727879	2.780368	-2.449012	-2.651293
	-0.081624	0.543374	0.111532	-0.193055	-0.280755	-0.187467	-0.041954
	0.384082	0.001088	0.392877	-0.426529	-0.411272	0.578318	0.554905
	0.020907	0.004647	0.019001	-0.013029	-0.012428	-0.002049	-0.001607
	1.168058	2.312155	-0.987379	1.449794	1.939448	2.133012	1.961376
	0.325305	-2.024376	-0.784649	-0.079027	-1.086542	-1.703447	-1.237000
	-1.069121	0.029374	1.448903	-0.889710	-0.204049	0.603518	0.127024
4s	0.072696	0.298300	0.019375	-0.278211	-0.456106	-0.825651	-0.685636
	0.229964	0.168087	-0.468728	0.625212	0.662520	0.630139	0.676927
	0.067890	-0.153660	-0.007577	-0.047152	-0.089357	-0.041710	-0.047061
	-0.088391	0.005718	0.105073	-0.104334	-0.084806	-0.082656	-0.099969
	-0.001339	-0.001939	0.002223	-0.001861	-0.002670	-0.001418	-0.000569
α	1.018843	1.215501	1.335553	1.641460	1.725853	2.035471	2.185093
β	1.889991	1.815976	1.797167	1.686892	1.781532	1.795653	1.777885
	-0.003524	-0.001962	-0.000891	0.030727	0.000693	0.001314	0.001049
2p	0.053256	0.045521	0.042040	-0.102286	0.042390	0.044676	0.047170
	0.858100	0.846153	0.849390	0.747169	0.870057	0.897799	0.885729
	0.127654	0.143792	0.141478	0.354573	0.114971	0.092026	0.091320
	1.140557	1.187755	1.193261	1.025141	1.183573	1.147546	1.171331
3p	-0.110759	-0.156268	-0.153111	0.195680	-0.119911	-0.061098	-0.090996
	-0.334388	-0.353141	-0.371087	-0.502079	-0.419698	-0.455758	-0.455737
	-0.027322	-0.033332	-0.032300	-0.090356	-0.021931	-0.011773	-0.012069
α			0.658687	0.533402	0.978752	0.945582	1.150641
β			2.124709	2.208727	2.018390	2.040090	1.972380
			0.537951	0.388789	0.454829	0.343518	0.345979
3d			0.524132	0.558042	0.568905	0.568746	0.578565
			0.097099	0.274431	0.123918	0.268914	0.236307

Table 10. (Continued)

	Ga (² P)	Ge (³ P)	As (⁴ S)	Se (³ P)	Br (² P)	Kr (¹ S)
α	0.538402	0.637392	0.753626	0.823619	0.897175	1.018984
β	1.662350	1.634359	1.711996	1.697318	1.683338	1.565084
1s	0.000594	0.000519	0.000419	0.000613	0.000810	0.000801
	-0.002626	-0.002399	-0.001605	-0.002400	-0.003227	-0.003906
	0.006685	0.006377	0.003302	0.005107	0.007007	0.010648
	-0.013503	-0.013429	-0.004864	-0.007929	-0.011172	-0.022162
	0.024374	0.025053	0.003821	0.008235	0.012889	0.039405
	-0.043923	-0.046008	0.021131	0.014803	0.008196	-0.066169
2s	0.113811	0.119842	0.967091	0.969810	0.972666	0.151044
	0.918237	0.913602	0.014241	0.015219	0.016211	0.893596
	0.026019	-0.033916	0.004126	0.000776	-0.002454	0.047925
	-0.107784	0.143936	-0.017620	-0.004164	0.009030	-0.217307
	0.248016	-0.336867	0.046351	0.014892	-0.016425	0.532882
	-0.430678	0.593340	-0.077425	-0.011650	0.055163	-0.972850
3s	0.771636	-1.087023	2.134996	2.109655	2.081600	1.954400
	1.464603	-1.209114	-1.453893	-1.486330	-1.516792	0.409936
	-1.387456	1.357751	-0.088028	-0.074376	-0.060995	-1.208611
	-0.031748	0.020350	0.001129	-0.000303	-0.001721	0.000952
	0.022474	-0.004357	0.120974	0.117151	0.112834	-0.069543
	-0.068252	0.051396	-0.520805	-0.510239	-0.496229	0.364056
4s	-0.047971	-0.452221	2.890218	3.004752	3.114914	-1.754039
	-2.716076	-2.408991	-1.849784	-2.082617	-2.317919	-1.036775
	2.925693	3.143605	-0.872210	-0.754357	-0.629177	3.298353
	-0.209249	-0.484533	0.869794	0.861242	0.848565	-1.114128
	-0.501000	-0.433224	-0.055865	-0.057001	-0.057165	-0.256261
	-0.000729	-0.004138	0.010376	0.010408	0.010308	-0.013268
5s	1.958264	2.105454	2.484167	2.499421	2.562312	2.668671
	-1.032649	-1.231747	-2.182328	-2.125493	-2.196173	-1.926050
	-0.213144	-0.380281	-0.187986	-0.432291	-0.538898	-0.868508
	-0.553456	-0.434572	0.474025	0.712864	0.879170	0.194731
	0.750437	0.856339	0.272673	0.193359	0.132183	0.987681
	-0.090157	-0.162035	-0.265268	-0.260240	-0.258552	-0.383412
6s	-0.107311	-0.103266	0.022686	0.019499	0.018168	-0.076837
	-0.000387	-0.001066	-0.003813	-0.003360	-0.003165	-0.003959

Table 10. (Continued)

	Ga (2P)	Ge (3P)	As (4S)	Se (3P)	Br (2P)	Kr (1S)
α	0.696547	0.708026	0.751527	0.760648	0.804871	0.854333
β	1.790710	1.798003	1.789553	1.682593	1.675264	1.667047
	0.001338	0.001585	0.001636	0.012598	0.012419	0.012314
	-0.003363	-0.004203	-0.004402	-0.044987	-0.044657	-0.044513
2p	0.003446	0.004949	0.005179	0.096888	0.096449	0.096434
	0.049649	0.046022	0.046117	-0.172268	-0.171786	-0.172025
	0.890622	0.895697	0.896177	0.519131	0.516286	0.516074
	0.083028	0.080155	0.078989	0.612056	0.614191	0.614139
	0.033406	0.041236	0.044169	0.021931	0.024379	0.024583
	-0.054644	-0.099708	-0.119834	-0.089491	-0.096116	-0.102101
3p	1.186480	1.207956	1.232700	0.554248	0.545453	0.543674
	-0.067981	-0.045200	-0.052915	0.951674	0.979383	1.000540
	-0.470229	-0.486533	-0.494815	-0.640964	-0.660154	-0.680477
	-0.008045	-0.006097	-0.005305	-0.202177	-0.203783	-0.204053
	1.590420	1.487097	1.436202	1.235484	1.146865	1.074870
	-0.932807	-0.711794	-0.591404	-0.176584	-0.006362	0.133899
4p	0.058034	-0.091049	-0.193453	-0.253685	-0.354746	-0.441498
	-0.102057	-0.057367	-0.026484	-0.248919	-0.251233	-0.253324
	0.135952	0.130336	0.131882	0.174378	0.187269	0.200053
	-0.004586	-0.002634	-0.001557	0.046768	0.051041	0.054034
α	1.362749	1.585101	1.809814	2.052312	2.288205	2.523336
β	1.920683	1.879020	1.845455	1.813346	1.784316	1.756748
	0.353956	0.371114	0.390883	0.418014	0.437820	0.454058
3d	0.585103	0.584422	0.579264	0.565091	0.552328	0.539606
	0.205666	0.176308	0.151289	0.129318	0.114781	0.104889

Table 10. (Continued)

	Pb (² S)	Sr (¹ S)	T (² D)	Bo (⁷ S)	Tc (⁶ S)	Pd (¹ S)	Aq (² S)	Cd (¹ S)
α	0.307132	0.374861	0.386023	0.419244	0.441493	1.929641	0.467969	0.506288
β	1.616878	1.589542	1.588910	1.587330	1.582854	1.491100	1.587379	1.578295
	0.000198	0.000266	0.000210	0.000084	0.000064	0.001903	-0.000083	-0.000110
	-0.000913	-0.001258	-0.001002	-0.000431	-0.000343	-0.009630	0.000331	0.000455
	0.002388	0.003331	0.002697	0.001279	0.001069	0.026022	-0.000626	-0.000923
	-0.004854	-0.006760	-0.005596	-0.002986	-0.002623	-0.050784	0.000543	0.001047
1s	0.008807	0.012057	0.010276	0.006258	0.005752	0.080469	0.000777	0.000126
	-0.015322	-0.020320	-0.017901	-0.012397	-0.011797	-0.114106	-0.004792	-0.004167
	0.026031	0.033065	0.030072	0.023181	0.022559	0.204194	0.013507	0.013160
	-0.045743	-0.054901	-0.051471	-0.043400	-0.042821	0.864605	-0.031816	-0.032002
	0.114796	0.128786	0.123932	0.111894	0.111051		0.094167	0.095336
	0.917768	0.908831	0.911811	0.919356	0.919871		0.930560	0.929605
	-0.019320	0.023584	0.023404	0.022806	0.023318	0.049069	0.021556	0.023015
	0.084928	-0.106513	-0.105774	-0.103301	-0.106048	-0.241289	-0.097572	-0.105160
	-0.205726	0.263037	0.261342	0.255560	0.263220	0.626543	0.241348	0.261866
	0.372441	-0.481601	-0.478605	-0.468257	-0.483276	-1.194944	-0.442101	-0.481550
2s	-0.570412	0.741085	0.736245	0.719942	0.743569	2.869590	0.679083	0.740732
	0.796338	-1.036860	-1.029623	-1.006326	-1.039575	-0.643181	-0.948566	-1.034999
	-1.256815	1.673559	1.653550	1.595138	1.648014	-0.939224	1.470557	1.612085
	-1.148335	0.791321	0.820012	0.901034	0.862908	0.012551	1.052182	0.937367
	1.390255	-1.320011	-1.335223	-1.377688	-1.376135		-1.446251	-1.427066
	0.010592	-0.002451	-0.001045	0.002975	0.005132		0.008214	0.011495
	0.003616	-0.011715	-0.011256	-0.007446	-0.008871	0.134636	0.006079	-0.000290
	-0.012850	0.056819	0.054357	0.035998	0.042375	-0.716163	-0.027064	0.001764
	0.017919	-0.159553	-0.151222	-0.099553	-0.114796	3.698719	0.066091	-0.005798
	0.006096	0.359929	0.334625	0.211874	0.238129	-1.647614	-0.131723	0.004280
3s	-0.221076	-0.989408	-0.881307	-0.522164	-0.556294	-2.616428	0.211378	-0.036185
	-2.948795	-2.258424	-2.440256	-2.979628	-3.004148	1.612942	-3.898602	-3.730558
	3.707173	3.760757	3.887910	4.247592	4.349073	0.074360	4.765807	4.859235
	-0.688357	-0.974463	-1.000943	-1.072454	-1.139695	0.021097	-1.131967	-1.264978
	-0.439569	-0.352691	-0.358753	-0.376635	-0.365899		-0.419133	-0.383364
	-0.002019	-0.006841	-0.006291	-0.004472	-0.004903		0.000139	-0.002087
	-0.060759	-0.082695	-0.066618	0.014115	0.007199	3.474169	0.066245	0.064151
	0.305509	0.429899	0.347714	0.014024	-0.007442	-2.682641	-0.278024	-0.269682
	-1.321321	-1.828830	-1.443778	-0.438872	-0.224558	-2.574620	0.541496	0.538948
	-1.352999	-0.941078	-1.604310	-2.998929	-3.455601	2.579404	-4.108974	-4.337397
	2.728129	3.170587	3.633125	4.325090	4.756638	0.302524	4.256258	4.789715
	-0.143454	-0.742762	-0.803375	-0.572708	-0.705275	-0.503633	0.495048	0.104555
	-1.133686	-1.013719	-1.098202	-1.500140	-1.563149	-0.060831	-2.371953	-2.312149
	0.247337	0.304470	0.335028	0.477249	0.520763	-0.003591	0.753877	0.788848
	0.151309	0.140685	0.143958	0.135720	0.133741		0.109561	0.102366
	-0.000876	-0.000916	-0.000749	0.001640	0.002010		0.006595	0.006788

Table 10. (Continued)

	Rb (² S)	Sr (¹ S)	T (² D)	Mo (⁷ S)	Tc (⁶ S)	Pd (¹ S)	Xq (² S)	Cd (¹ S)
5s	-3.194113	-3.154585	-3.032932	-3.180161	2.823796		-3.458394	-3.042873
	3.845138	3.476460	3.199840	3.782551	-2.806829		4.681154	3.500282
	-1.879896	-1.103242	-0.971842	-2.103449	1.070159		-3.404981	-2.107325
	1.371329	0.923190	1.023321	2.162100	-1.651990		3.057069	2.461434
	-1.173604	-1.277949	-1.389747	-2.013035	1.870772		-2.211301	-2.139089
	0.300569	0.428401	0.445252	0.632523	-0.517937		0.545159	0.467971
	0.141409	0.176174	0.195874	0.162109	-0.277849		0.222777	0.358795
	-0.021597	-0.053082	-0.061089	-0.052559	0.098894		-0.071454	-0.130842
	-0.039478	-0.042447	-0.042495	-0.045757	0.040822		-0.039306	-0.034751
	0.001274	0.001135	0.001046	0.001664	-0.000594		0.001342	0.000167
α	1.023906	1.191272	1.287009	1.474807	1.637720	1.670979	1.829600	1.987744
β	1.630316	1.601883	1.589930	1.573549	1.556146	1.562669	1.547345	1.533964
2p	0.012727	0.012892	0.012812	0.012307	-0.012323	0.011511	0.011562	0.011652
	-0.047344	-0.049071	-0.049306	-0.048327	-0.049126	-0.046014	-0.046896	-0.047806
	0.103854	0.108697	0.109700	0.108537	0.110919	0.104365	0.106934	0.109305
	-0.183786	-0.190552	-0.191900	-0.191599	-0.193910	-0.187752	-0.191382	-0.194174
	0.575375	0.628200	0.642656	0.633285	0.671524	0.597800	0.627438	0.655220
3p	0.561147	0.511356	0.497118	0.505620	0.467714	0.538436	0.510363	0.483483
	0.032755	0.036247	0.039069	0.046722	0.047942	0.051773	0.055423	0.057403
	-0.134666	-0.149023	-0.160387	-0.192409	-0.198413	-0.210618	-0.226888	-0.235567
	0.745754	0.915844	0.971635	0.986747	1.115500	0.906893	1.028921	1.137067
	0.81253*	0.628477	0.579033	0.625130	0.469951	0.789305	0.657770	0.531315
4p	-0.703852	-0.707642	-0.717057	-0.765643	-0.754537	-0.840072	-0.834359	-0.823992
	-0.175962	-0.152890	-0.146450	-0.147660	-0.131652	-0.157723	-0.144191	-0.132173
	1.191289	1.318392	1.328904	1.191639	1.307477	-0.964707	-1.085978	-1.200615
	0.026710	-0.119684	-0.098718	0.219019	0.081312	-0.658824	-0.517935	-0.377480
	-0.561326	-0.638219	-0.715952	-0.969871	-1.036272	1.202524	1.263331	1.313243
3d	-0.182257	-0.113642	-0.083428	-0.052841	0.030139	0.101794	0.019648	-0.059335
	0.216127	0.226979	0.235687	0.265410	0.262298	-0.322736	-0.316684	-0.308855
	0.052247	0.049409	0.049496	0.054007	0.051176	-0.058876	-0.055995	-0.053572
α	2.772643	3.023167	0.996917	1.052897	1.129936	1.170472	1.320666	1.420105
β	1.729781	1.705237	1.882927	1.876034	1.858816	1.853921	1.823712	1.934544
4d	0.471783	0.487714	-0.000727	0.017486	0.017649	0.030084	0.026950	-0.008796
	0.524489	0.510043	0.068351	-0.025520	-0.028064	-0.082243	-0.069431	0.092627
	0.096309	0.089549	0.735244	0.721726	0.724899	0.661313	0.694828	0.843816
			0.282185	0.361807	0.358007	0.457880	0.412088	0.132062
			1.057302	0.908072	0.891383	0.754083	0.771840	0.901271
			-0.055639	0.187460	0.216665	0.401425	0.391817	0.210577
			-0.179931	-0.232786	-0.251901	-0.249631	-0.291849	-0.358635
			-0.077331	-0.099087	-0.102826	-0.141108	-0.132908	-0.038568

Table 10. (Continued)

	In (2P)	Sn (3P)	Sb (4S)	Te (3P)	I (2P)	Te (1S)
α	0.596136	0.693171	0.771089	0.847500	0.917865	0.984008
β	1.556172	1.536291	1.523249	1.512065	1.503076	1.495581
	-0.000068	0.000074	0.000216	0.000368	0.000501	0.000622
	0.000258	-0.000429	-0.001133	-0.001903	-0.002593	-0.003228
	-0.000396	0.001416	0.003294	0.005380	0.007274	0.009037
	-0.000056	-0.003582	-0.007230	-0.011300	-0.015019	-0.018498
1s	0.002151	0.007882	0.013701	0.020155	0.026048	0.031551
	-0.007575	-0.015826	-0.023896	-0.032707	-0.040678	-0.048065
	0.018356	0.029031	0.038943	0.049498	0.058888	0.067466
	-0.039157	-0.051659	-0.062529	-0.073730	-0.083459	-0.092161
	0.106474	0.122491	0.135084	0.147455	0.157793	0.166715
	0.922502	0.913059	0.905972	0.899170	0.893597	0.888879
	0.026317	0.028473	0.029466	0.029881	0.029955	0.029842
	-0.123114	-0.136181	-0.143038	-0.146969	-0.148921	-0.149714
	0.311891	0.350698	0.372443	0.386476	0.394812	0.399679
	-0.579632	-0.658556	-0.704245	-0.735469	-0.755352	-0.768178
2s	0.895990	1.023245	1.097700	1.150058	1.184349	1.207269
	-1.255094	-1.437648	-1.544638	-1.621451	-1.672589	-1.707452
	2.004557	2.366605	2.600432	2.794295	2.943494	3.062755
	0.585901	0.235855	-0.003038	-0.213747	-0.384144	-0.526471
	-1.345896	-1.254572	-1.188597	-1.127297	-1.076213	-1.032588
	0.016277	0.018668	0.019502	0.019641	0.019480	0.019197
	-0.021406	-0.038061	-0.047630	-0.054087	-0.058223	0.060991
	0.101307	0.184520	0.234776	0.270716	0.295167	-0.312616
	-0.263174	-0.489752	-0.631364	-0.737504	-0.812620	0.868240
	0.508205	0.975323	1.274858	1.509624	1.681239	-1.811969
3s	-1.001570	-1.999123	-2.686603	-3.284310	-3.760409	4.152427
	-2.855162	-1.807552	-1.025467	-0.292407	0.323393	-0.852195
	4.810302	4.562510	4.314061	4.039484	3.787771	-3.558940
	-1.541968	-1.725712	-1.812805	-1.864448	-1.892940	1.908798
	-0.284493	-0.203982	-0.158068	-0.122514	-0.096692	0.076846
	-0.009027	-0.014182	-0.016708	-0.018440	-0.019519	0.020225
	0.036445	0.002594	-0.022775	-0.046159	-0.066131	-0.083779
	-0.137821	0.032644	0.162102	0.283290	0.387021	0.478514
	0.058061	-0.604974	-1.114450	-1.611286	-2.037002	-2.408277
	-4.220711	-3.711910	-3.226817	-2.687349	-2.201048	-1.767555
4s	5.895779	6.652784	7.026228	7.258333	7.408493	7.520528
	-1.180564	-2.391529	-3.159223	-3.819499	-4.353647	-4.810199
	-1.842860	-1.305494	-0.943251	-0.604666	-0.320222	-0.071122
	0.774467	0.716529	0.674024	0.626075	0.583137	0.543604
	0.091834	0.090397	0.090798	0.093389	0.096579	0.100147
	0.005923	0.004140	0.002888	0.001563	0.000394	-0.000676
	-3.194492	-3.397819	-3.602381	-3.731879	-3.825485	-3.895039
	3.628940	3.853820	4.177341	4.287997	4.323199	4.309593
	-1.863862	-1.593495	-1.556783	-1.244099	-0.874900	-0.475082
	2.434131	2.250173	2.160127	1.826669	1.430913	1.002870
5s	-2.606173	-3.005207	-3.291894	-3.423168	-3.468115	-3.459587
	0.856009	1.302861	1.656259	1.943018	2.171137	2.360086
	0.287152	0.155822	0.026865	-0.082241	-0.175548	-0.258206
	-0.146707	-0.144563	-0.130946	-0.122600	-0.115666	-0.109469
	-0.036434	-0.040595	-0.045146	-0.048495	-0.051071	-0.053167
	0.000291	0.000836	0.001511	0.001987	0.002362	0.002674

Table 10. (Continued)

	In (² P)	Sn (³ P)	Sb (⁴ S)	Te (³ P)	I (² P)	Xe (¹ S)
α	0.738113	0.637502	0.781632	0.811162	0.849520	0.892860
β	1.625032	1.589438	1.621584	1.617733	1.611646	1.604594
	-0.000569	0.005728	-0.000393	-0.000307	-0.000214	-0.000102
	0.002269	-0.022440	0.001622	0.001309	0.000970	0.000545
	-0.005567	0.050578	-0.004175	-0.003509	-0.002790	-0.001874
2p	0.011174	-0.088625	0.008871	0.007777	0.006593	0.005076
	-0.024691	0.136240	-0.021593	-0.020112	-0.018486	-0.016358
	0.113459	-0.204312	0.106996	0.104138	0.101287	0.097669
	0.860333	0.529581	0.865099	0.866720	0.867737	0.868794
	0.060751	0.610148	0.060164	0.060308	0.060967	0.062064
	0.021407	0.012804	0.023879	0.024504	0.024598	0.024551
	-0.079708	-0.050292	-0.089494	-0.092356	-0.093422	-0.093773
	0.175729	0.115266	0.197974	0.204871	0.208037	0.209639
3p	-0.297971	-0.212640	-0.343352	-0.357605	-0.364382	-0.368105
	1.698318	0.541075	1.736380	1.754461	1.772800	1.791185
	-0.512005	1.231647	-0.512630	-0.519731	-0.534196	-0.551221
	-0.473121	-0.912385	-0.486604	-0.490742	-0.492287	-0.493039
	-0.002281	-0.197968	-0.000574	-0.000132	-0.000145	-0.000406
	0.034626	-0.011699	-0.048208	-0.055903	-0.062542	-0.064563
	-0.093046	0.029071	0.181289	0.214492	0.240861	0.261859
	1.389693	0.230873	-1.409678	-1.420738	-1.445209	-1.477260
4p	0.021718	1.669219	-0.169828	-0.223055	-0.244604	-0.251144
	-1.279660	-1.333063	1.442662	1.515836	1.576734	1.633045
	0.464395	-0.410269	-0.517464	-0.546376	-0.577023	-0.608916
	0.143637	0.432307	-0.147414	-0.147884	-0.147133	-0.145897
	0.007094	0.072975	-0.007607	-0.008033	-0.008625	-0.009331
	2.197670	1.682106	1.904592	1.787340	1.699310	1.630805
	-2.084051	-0.970845	-1.357313	-1.073850	-0.852569	-0.672462
	0.806230	0.290734	0.144563	-0.106789	-0.320027	-0.504372
5p	-0.612565	-0.680870	-0.330214	-0.246509	-0.173659	-0.106701
	0.605440	0.474802	0.550648	0.560252	0.576017	0.590431
	-0.223798	0.043928	-0.201581	-0.206583	-0.216021	-0.226273
	-0.003787	-0.088626	-0.022784	-0.028601	-0.033054	-0.036200
	-0.006636	-0.018641	-0.004585	-0.004256	-0.004138	-0.004125
α	1.608219	1.754309	1.890998	2.031747	2.175617	2.322247
β	1.783317	1.772571	1.762978	1.744809	1.725615	1.707339
	0.019569	0.013827	0.008420	0.005292	0.002972	0.000890
3d	-0.036824	-0.009889	0.015257	0.028634	0.038239	0.046945
	0.749158	0.776423	0.791021	0.790946	0.787957	0.784467
	0.328438	0.277950	0.241628	0.229670	0.223736	0.219068
	0.815810	0.850590	0.877970	0.894557	0.909562	0.925450
4d	0.355216	0.315328	0.282577	0.266043	0.252146	0.236914
	-0.360600	-0.388986	-0.409901	-0.425764	-0.439449	-0.451452
	-0.112133	-0.096107	-0.084222	-0.081308	-0.080491	-0.079985

Table 10. (Continued)

	Cs (² S)	Ba (¹ S)	La (² D)		Cs (² S)	Ba (¹ S)	La (² D)
α	0.378072	0.411680	0.428286				
β	1.516492	1.508160	1.505417				
	0.000172	0.000256	0.000261		0.005082	-0.003706	-0.004426
	-0.000879	-0.001309	-0.001341		-0.020470	0.023193	0.026483
	0.002449	0.003639	0.003738		0.032507	-0.085860	-0.092796
	-0.005069	-0.007461	-0.007676		0.002268	0.245396	0.248817
	0.008897	0.012851	0.013225		-0.395551	-0.893512	-0.835168
1s	-0.014232	-0.019957	-0.020519	4s	-4.766528	-4.370596	-4.606281
	0.021713	0.029211	0.029966		8.630773	8.928735	9.313634
	-0.032338	-0.041327	-0.042236		-3.883651	-4.492032	-4.768607
	0.046927	0.056736	0.057694		-0.954258	-0.657879	-0.590481
	-0.068938	-0.078612	-0.079451		0.770311	0.729007	0.735966
	0.137666	0.147505	0.147870		0.081361	0.083538	0.082460
	0.905904	0.900711	0.900678		0.003731	0.002686	0.002610
	0.018223	0.018467	0.018551		-0.075283	-0.095512	-0.082418
	-0.090595	-0.092801	-0.093556		0.438730	0.564352	0.490228
	0.242663	0.250665	0.253418		-2.717219	-2.818330	-2.403824
	-0.473984	-0.492433	-0.498831		0.205439	-0.226251	-1.139720
	0.765187	0.797450	0.808738		3.172317	4.252471	5.180043
2s	-1.090633	-1.137476	-1.153982	5s	-0.151614	-0.992172	-1.334012
	1.427816	1.487555	1.508682		-3.053750	-2.982409	-3.132777
	-1.800675	-1.873591	-1.899504		1.620384	1.826732	1.995402
	2.805501	2.961057	3.007351		0.319809	0.256686	0.230450
	-0.118407	-0.281531	-0.323545		-0.297857	-0.309607	-0.320006
	-1.173071	-1.125875	-1.116969		-0.024998	-0.023863	-0.024387
	0.021750	0.021792	0.022454		-0.001970	-0.002105	-0.002095
	-0.027622	-0.030667	-0.031562		-6.025698	-5.236979	-5.160674
	0.137525	0.154388	0.159438		11.477281	8.748098	8.543513
	-0.369873	-0.418969	-0.433800		-10.501892	-6.561709	-6.490014
	0.730804	0.833447	0.864284		8.178001	5.105928	5.369003
	-1.210484	-1.387699	-1.439449		-6.116963	-4.578641	-4.947416
3s	1.830849	2.111645	2.189480	6s	3.108905	2.273148	2.455405
	-3.108248	-3.646581	-3.770534		-0.832219	-0.171046	-0.170952
	-0.905765	-0.304156	-0.200637		0.329304	-0.058125	-0.088627
	4.469795	4.250304	4.252867		-0.366560	-0.252474	-0.250858
	-1.941364	-1.981280	-2.018625		0.168703	0.147104	0.151768
	-0.139568	-0.112110	-0.101680		-0.018149	-0.008538	-0.008880
	-0.016617	-0.017947	-0.018498		0.003630	0.002555	0.002608

Table 10. (Continued)

	Cs (² S)	Ba (¹ S)	La (² D)		Cs (² S)	Ba (¹ S)	La (² D)
α	1.004893	1.116925	1.193349	α	2.477668	2.637982	1.000774
β	1.582588	1.563309	1.552705	β	1.689604	1.672721	1.665982
	-0.000282	-0.000378	-0.000360		-0.001454	-0.003755	-0.000496
	0.001256	0.001659	0.001601		0.057266	0.067744	0.002375
	-0.003482	-0.004421	-0.004312	3d	0.780712	0.776685	-0.010187
2p	0.007812	0.009388	0.009189		0.213370	0.207820	0.087442
	-0.020522	-0.022838	-0.022565				0.777806
	0.107035	0.112587	0.112539				0.190464
	0.860073	0.853319	0.851390				
	0.063664	0.065990	0.067588		0.946200	0.969100	0.027577
	0.020256	0.016507	0.015042	4d	0.214308	0.188231	-0.031515
	-0.079051	-0.065658	-0.060531		-0.461115	-0.468879	1.009931
	0.178747	0.149756	0.138706		-0.079025	-0.078050	0.151631
3p	-0.304771	-0.243064	-0.219001				-0.474833
	1.808138	1.815171	1.825718				-0.071713
	-0.637244	-0.708011	-0.744390				1.273093
	-0.466012	-0.443894	-0.435423				-0.325780
	-0.003587	-0.006548	-0.007876	5d			-0.253139
	0.065586	0.063912	0.066382				-0.098867
	-0.267869	-0.261995	-0.274523				0.160888
	1.672328	1.837347	1.930378				0.017603
4p	-0.047633	-0.324041	-0.447467				
	-1.557895	-1.476235	-1.462399				
	0.647319	0.674991	0.700839				
	0.135192	0.127729	0.124117				
	0.010775	0.011938	0.012736				
	1.694391	1.775499	1.791256				
	-0.696980	-0.764421	-0.731257				
	-0.713291	-0.884503	-1.038378				
5p	0.079291	0.263978	0.383766				
	0.575679	0.548838	0.539526				
	-0.246498	-0.262110	-0.274665				
	-0.039039	-0.041912	-0.043830				
	-0.004410	-0.004635	-0.004810				

Table 10. (Continued)

Gd (⁸ S)			
α	1.366622		
β	1.525425		
	0.003240	-3.972376	1.770812
	-0.015694	4.251203	-0.644411
	0.040708	1.538433	-0.668959
	-0.076182	-3.385796	5p -0.510139
1s	0.115873	5s 0.075855	0.991252
	-0.151944	1.637424	-0.134210
	0.176712	-0.707212	-0.152170
	-0.186066	0.034869	-0.020633
	1.058417	-0.022318	
	0.036982	0.002622	α 3.261845
			β 1.653774
	0.016423	α 1.410400	
	-0.079933	β 1.491535	0.004196
	0.209632		3d 0.038130
	-0.402592	0.005825	0.802800
2s	0.663780	-0.025630	0.198270
	-1.190996	0.062285	
	-1.491098	2p -0.112941	0.913966
	1.810278	0.172581	4d 0.296103
	-0.078978	-0.248287	-0.541993
	0.014203	0.655874	-0.073977
		0.503867	
	-0.012322		α 1.874408
	0.065924	0.028027	β 1.903542
	-0.201449	-0.121363	
	0.462202	0.292063	0.392601
3s	-4.956003	3p -0.535915	4f 0.595891
	6.286138	1.315230	0.181809
	-1.753547	0.640777	
	-0.442338	-0.989806	
	0.009959	-0.126864	
	-0.002504		
		-0.008992	
	0.197269	0.061776	
	-1.030268	-0.790264	
	5.763025	4p -1.551073	
	-5.650291	2.362691	
4s	-1.519904	-0.232153	
	4.515749	-0.432518	
	-1.836269	-0.057441	
	0.104077		
	-0.063734		
	0.007729		

Table 10. (Continued)

Rn (¹ S)			
	α	β	α
	1.017676		0.841669
	1.449446		1.489038
	0.000466	-0.024845	0.002845
	-0.002576	0.145763	-0.012782
	0.007579	-0.467440	0.031906
	-0.016047	1.139391	-0.059613
	0.027735	-3.160704	0.094202
1s	-0.041866	4s	-0.134208
	0.057502	-2.929483	0.179594
	-0.073595	11.879178	-0.243222
	0.088633	-8.795040	0.520566
	-0.104406	0.934302	0.631058
	0.157524	0.789093	0.006520
	0.900588	0.019838	-0.028889
	0.019295	0.004078	0.070733
	-0.105241	-0.100137	-0.129113
	0.303445	0.588960	0.198391
	-0.624378	-2.961369	-0.283844
	1.038865	-2.341382	0.575116
2s	-1.495459	5s	10.669775
	1.946786	-6.595263	1.514425
	-2.433675	-2.606659	-1.275925
	4.049441	3.579441	-0.165326
	-1.345022	-0.298870	-0.020870
	-0.860008	-0.450378	0.090362
	0.028973	0.016792	-0.214984
	0.046604	-0.005116	0.379545
	-0.253836	-5.375386	4p
	0.732717	7.841727	-0.494401
	-1.519581	-3.911756	3.029392
	2.582968	4.377012	-2.495463
3s	-3.953163	6s	-7.624111
	6.983192	5.270034	-0.473784
	-3.127942	-0.535025	0.774931
	-3.456980	-0.678666	0.054525
	2.588508	-0.143392	
	-0.115965	0.252462	
	0.029135	-0.027153	
		0.004506	

Table 11. Even-tempered augmented triple-zeta atomic orbitals

	Li (² S)	Be (¹ S)	B (² P)	C (³ P)	N (⁴ S)	O (³ P)	F (² P)	Ne (¹ S)
α	0.351242	0.447163	0.581716	0.704515	0.822525	0.946188	1.066337	1.183685
β	1.604735	1.659528	1.659501	1.666496	1.673719	1.676262	1.679609	1.683251
1s	-0.002603	-0.001260	-0.000400	0.000221	0.000668	0.001026	0.001306	0.001517
	0.017558	0.005646	0.003377	0.001614	0.000311	-0.000632	-0.001403	-0.001997
	-0.053651	-0.011621	0.003385	0.015836	0.025324	0.031520	0.036751	0.041033
	0.834283	0.888665	0.904948	0.913218	0.917787	0.920815	0.922698	0.924008
	0.198973	0.124440	0.095751	0.076914	0.064100	0.055540	0.048989	0.043833
2s	0.021222	0.005473	0.002870	0.001443	0.000667	0.000266	-0.000009	-0.000200
	1.934306	1.457189	1.334494	1.251604	1.193151	1.147665	1.113605	1.086141
	-1.104538	-0.295732	-0.072032	0.061520	0.149710	0.224605	0.277007	0.317050
	-0.022867	-0.388255	-0.497893	-0.545656	-0.570568	-0.600146	-0.616571	-0.626564
	-0.083137	-0.064645	-0.075986	-0.091830	-0.105520	-0.112710	-0.120123	-0.126861
2p	-0.029565	-0.022989	-0.017079	-0.010847	-0.005894	-0.002973	-0.000278	0.002025
	-0.003024	-0.000747	-0.000610	-0.000721	-0.000872	-0.000929	-0.001025	-0.001121
	α		0.509473	0.624700	0.731294	0.758132	0.848776	0.929874
	β		1.802049	1.805955	1.813350	1.896120	1.736217	1.733414
			0.669543	0.607656	0.565058	0.525330	0.381974	0.360763
			0.355289	0.409494	0.443993	0.483657	0.492795	0.486189
			0.030349	0.046316	0.060523	0.076099	0.203134	0.231366
			0.008344	0.006817	0.005474	0.002142	0.017047	0.020451

Table 11. (Continued)

	Na (² S)	Mg (¹ S)	Al (² P)	Si (³ P)	P (⁴ S)	S (³ P)	Cl (² P)	Ar (¹ S)
α	0.325177	0.409171	0.527348	0.630010	0.727210	0.820042	0.911962	1.003757
β	1.643118	1.610974	1.572014	1.549692	1.534194	1.523061	1.514069	1.506496
	-0.000090	-0.000081	0.000010	0.000005	-0.000048	-0.000151	-0.000247	-0.000328
	0.000421	0.000409	-0.000011	-0.000014	0.000213	0.000685	0.001127	0.001509
	-0.001215	-0.001236	-0.000256	-0.000308	-0.000961	-0.002248	-0.003480	-0.004571
	0.003677	0.003783	0.002155	0.002321	0.003576	0.005994	0.008322	0.010398
1s	-0.004762	-0.005085	-0.002645	-0.003184	-0.005495	-0.009658	-0.013694	-0.017339
	0.048284	0.052070	0.051613	0.055191	0.061189	0.069851	0.077997	0.085242
	0.918725	0.915683	0.912845	0.910458	0.907867	0.904637	0.901703	0.899156
	0.042671	0.041732	0.042995	0.041760	0.039462	0.036292	0.033323	0.030679
	0.000030	0.000219	0.000453	0.000615	0.000745	0.000869	0.000964	0.001031
	-0.001123	-0.004976	-0.003068	-0.004152	-0.006156	-0.008423	-0.010648	-0.012719
	0.010621	0.026888	0.020255	0.028224	0.040793	0.054744	0.068400	0.081187
	-0.048214	-0.097127	-0.077618	-0.098510	-0.131103	-0.168046	-0.203205	-0.235328
	1.265600	1.428293	1.585429	1.715391	1.828621	1.926844	2.012284	2.086895
2s	0.158835	0.018277	-0.199331	-0.340963	-0.448990	-0.532021	-0.600178	-0.657333
	-0.632270	-0.638238	-0.605834	-0.593061	-0.589250	-0.592858	-0.599646	-0.608349
	-0.119693	-0.112260	-0.110311	-0.106013	-0.099540	-0.090823	-0.081366	-0.071606
	0.001765	0.001270	0.002406	0.002611	0.002140	0.001063	-0.000330	-0.001915
	-0.001101	-0.001039	-0.001266	-0.001278	-0.001165	-0.000952	-0.000697	-0.000420
	2.008829	1.728457	1.666489	1.603998	1.562918	1.500894	-1.468466	-1.455493
	-1.270181	-0.584350	-0.264302	-0.019534	0.141431	0.324185	-0.427913	-0.481230
	0.045634	-0.495880	-0.910442	-1.165152	-1.306623	-1.437804	1.490799	1.498042
	-0.148866	-0.072906	0.007920	-0.007626	-0.081694	-0.163840	0.276721	0.401786
3s	-0.024873	-0.018102	0.006927	0.079330	0.176808	0.275729	-0.383826	-0.492435
	0.101444	0.130688	0.152125	0.145846	0.125689	0.103835	-0.076654	-0.048138
	0.016661	0.020498	0.023194	0.031041	0.040938	0.049974	-0.059429	-0.068406
	-0.000476	-0.000852	-0.001243	-0.003130	-0.005577	-0.007810	0.010124	0.012287
	0.000208	0.000307	0.000420	0.000698	0.001025	0.001306	-0.001587	-0.001841
α	0.702393	0.934504	0.805759	0.515606	0.613755	0.639211	0.689268	0.751677
β	1.884022	1.799327	1.727817	1.681310	1.657403	1.688668	1.698387	1.697587
	0.034229	0.039008	0.002372	0.002318	0.001886	0.002391	0.002762	0.001564
	0.617761	0.644457	-0.008599	-0.008072	-0.007500	-0.007621	-0.007320	-0.007440
2p	0.392288	0.352757	0.066053	0.065525	0.065192	0.067889	0.069170	0.067194
	0.036026	0.032324	0.665414	0.680044	0.699030	0.738971	0.762545	0.778620
			0.305688	0.285048	0.264310	0.229179	0.205392	0.190815
			0.028763	0.028353	0.025077	0.013775	0.009332	0.007287
			0.851748	0.798531	0.753145	0.639134	0.580502	0.545963
			0.291913	0.388353	0.468941	0.585501	0.646028	0.681742
3p			-0.182731	-0.227329	-0.255889	-0.233414	-0.218062	-0.206214
			-0.102031	-0.127815	-0.152356	-0.199921	-0.234151	-0.259852
			-0.059216	-0.064698	-0.065087	-0.053686	-0.046034	-0.040274
			-0.004472	-0.005347	-0.005364	-0.003905	-0.003541	-0.003555

Table 11. (Continued)

	K (² S)	Ca (¹ S)	Ga (² P)		K (² S)	Ca (¹ S)	Ga (² P)
α	0.344127	0.415657	0.626256				
β	1.431709	1.412682	1.422508				
	0.000692	0.000859	0.000294		0.024248	0.026060	0.007195
	-0.003925	-0.005036	-0.001687		-0.141268	-0.145778	-0.037191
	0.011767	0.015521	0.005101		0.428800	0.432117	0.089260
	-0.025186	-0.033957	-0.010955		-1.148394	-1.101725	-0.137111
	0.043546	0.059069	0.018888		-1.098396	-1.528933	-1.870855
1s	-0.065312	-0.090353	-0.028032	3s	0.185153	0.749602	-0.292703
	0.087392	0.121465	0.036338		1.930174	1.846766	2.150993
	-0.106302	-0.148003	-0.041832		-0.403768	-0.456729	0.512898
	0.124703	0.171786	0.044118		-0.215116	-0.252148	-0.959373
	-0.105490	-0.160685	-0.003447		-0.148320	-0.138708	-0.015756
	0.955232	0.968641	0.942562		-0.032653	-0.037291	-0.046376
	0.088442	0.105445	0.042369		-0.000088	0.000099	0.003076
	0.005446	0.006490	-0.004135		3.245425	2.744139	1.911767
	-0.029818	-0.036536	0.023664		-3.180614	-2.019746	-0.674825
	0.084759	0.106512	-0.071482		1.037371	0.505968	0.080229
	-0.167966	-0.215496	0.154181		-0.839436	-1.451081	-1.203780
	0.266072	0.345997	-0.268099		0.426321	1.003725	0.498019
2s	-0.338044	-0.448641	0.426957	4s	-0.453330	-0.728483	-0.516610
	0.462808	0.635336	-0.575922		0.722308	1.009223	0.786645
	1.850473	1.809734	2.848532		-0.254985	-0.384358	0.013041
	-0.988226	-1.086456	-1.384317		0.020028	0.031873	-0.214374
	-0.466848	-0.437525	-0.557700		-0.046367	-0.058283	-0.001435
	-0.097289	-0.101106	-0.035505		-0.002331	-0.003541	-0.011109
	-0.004776	-0.005360	-0.004422		-0.000470	-0.000558	0.000773

Table 11. (Continued)

	K (² S)	Ca (¹ S)	Ga (² P)
α	0.996699	0.860080	0.484890
β	1.607603	1.701137	1.596937
	0.001040	0.000043	-0.000052
	-0.005290	-0.002769	0.000297
	0.064294	0.051364	-0.001106
	0.754980	0.806402	0.004071
2p	0.205056	0.173608	-0.009351
	0.015565	0.004221	0.092071
			0.822570
			0.113551
			0.002556
	0.634364	0.222882	-0.000309
	0.648581	1.080168	0.006413
	-0.287611	-0.290656	-0.027035
	-0.257837	-0.308059	0.456448
3p	-0.050662	-0.043142	0.924291
	-0.005765	-0.002594	-0.340715
			-0.382806
			-0.016599
			-0.003414
			1.056243
			-0.032286
			0.038001
			-0.215719
4p			-0.106264
			0.036132
			0.074251
			-0.000661
			0.001069
α			1.272385
β			1.801427
			0.232149
3d			0.522349
			0.356762
			0.039480

Table 11. (Continued)

Xe (¹ S)			
α	0.985547	α	0.806807
β	1.362618	β	1.411697
-0.000244	-0.031416	0.000301	-0.912624
0.001578	0.223955	-0.001552	-0.154501
-0.005301	-0.766626	0.004347	-0.550929
0.012472	2.504756	-0.008934	1.159031
-0.023267	2.483870	0.015219	0.000687
0.036909	-4.633794	2p -0.024117	5p 0.150349
-0.051783	-3.677212	0.036534	-0.681576
1s 0.065903	4s 5.950003	-0.059181	0.182969
-0.078884	-1.391547	0.255573	0.058860
0.091292	0.428162	0.736198	0.047685
-0.106792	-0.679573	0.054345	-0.001429
0.196972	0.120670	0.007087	0.000688
0.861843	-0.038746		
-0.000445	0.006193	0.006215	α 2.003283
0.002072	-0.000612	-0.031637	β 1.549698
		0.086987	
-0.000195	-3.130750	-0.175421	0.000035
0.001261	2.360574	0.299870	-0.003509
-0.003961	-2.002153	3p -0.456712	3d 0.084378
0.008412	6.563380	1.413939	0.700356
-0.013307	-5.842315	0.465814	0.251216
0.011262	3.185441	-0.833649	0.018680
0.007887	-4.823678	-0.234090	
2s -0.069082	5s 4.605616	-0.021982	0.322112
0.525315	-1.896770	-0.001531	0.817320
2.486635	0.804098		4d -0.007650
-2.268725	-0.514305	0.006861	-0.388754
-0.105909	0.143409	-0.035284	-0.091938
-0.050255	-0.043181	0.086805	-0.009658
0.005575	0.008351	-0.332268	
-0.000876	-0.000894	-1.456535	
		4p 0.143827	
0.016813		1.665899	
-0.109053		-0.340972	
0.365076		-0.258708	
-0.864145		-0.137973	
1.675289		0.000612	
-2.832154		-0.001789	
6.834554			
3s -3.708565			
-2.751227			
1.273534			
0.584744			
0.063158			
0.001480			
0.001947			
-0.000187			

Table 12. Atom properties

2 Helium Atoms		Singlet S State			1s(2) Configuration			(3s) Basis	
		$ 2-V/Z = 3.9D-10$			$ 1-Norm = 1.1D-15$				
				Total S Cusp:	-2.014980				
	T 2.861679	V(1) -6.749128	V(2) 1.025770	V(T) -5.723358	Z -2.861679	e -1.835909	-Cusp	r max	
1s	1.430840	-3.374564	0.512885	-2.861679	-1.430840	-0.917955	2.014980	0.569172	
	r^{-3} 11.99103	r^{-2} 3.374564	r^{-1} 1.854516	r^0 2.369346	r^1 7.764691	r^2 50.26258			
1s	5.996716	1.687282	0.9272581	1.184673	3.882345	25.13129			
2 Helium Atoms		Singlet S State			1s(2) Configuration			(4s) Basis	
		$ 2-V/Z = 1.3D-07$			$ 1-Norm = 6.7D-16$				
				Total S Cusp:	-2.001257				
	T 2.861680	V(1) -6.749128	V(2) 1.025769	V(T) -5.723359	Z -2.861680	e -1.835911	-Cusp	r max	
1s	1.430840	-3.374564	0.512885	-2.861680	-1.430840	-0.917955	2.001257	0.569151	
	r^{-3} 11.99113	r^{-2} 3.374564	r^{-1} 1.854518	r^0 2.369399	r^1 7.766994	r^2 50.31303			
1s	5.995563	1.687282	0.9272592	1.184700	3.883897	25.15651			
2 Helium Atoms		Singlet S State			1s(2) Configuration			(5s) Basis	
		$ 2-V/Z = 4.8D-08$			$ 1-Norm = 1.8D-15$				
				Total S Cusp:	-2.000351				
	T 2.861680	V(1) -6.749129	V(2) 1.025769	V(T) -5.723360	Z -2.861680	e -1.835911	-Cusp	r max	
1s	1.430840	-3.374564	0.512884	-2.861680	-1.430840	-0.917956	2.000351	0.569149	
	r^{-3} 11.99100	r^{-2} 3.374564	r^{-1} 1.854547	r^0 2.369655	r^1 7.775827	r^2 50.56506			
1s	5.995498	1.687282	0.9272735	1.184828	3.887914	25.28253			
2 Helium Atoms		Singlet S State			1s(2) Configuration			(6s) Basis	
		$ 2-V/Z = 1.7D-08$			$ 1-Norm = 2.2D-15$				
				Total S Cusp:	-2.001078				
	T 2.861680	V(1) -6.749129	V(2) 1.025769	V(T) -5.723360	Z -2.861680	e -1.835911	-Cusp	r max	
1s	1.430840	-3.374564	0.512884	-2.861680	-1.430840	-0.917956	2.001078	0.569140	
	r^{-3} 11.99105	r^{-2} 3.374564	r^{-1} 1.854546	r^0 2.369653	r^1 7.775896	r^2 50.56977			
1s	5.995525	1.687282	0.9272732	1.184827	3.887948	25.28488			
2 Helium Atoms		Singlet S State			1s(2) Configuration			(7s) Basis	
		$ 2-V/Z = 7.5D-08$			$ 1-Norm = 4.4D-16$				
				Total S Cusp:	-2.000596				
	T 2.861680	V(1) -6.749129	V(2) 1.025769	V(T) -5.723360	Z -2.861680	e -1.835911	-Cusp	r max	
1s	1.430840	-3.374564	0.512884	-2.861680	-1.430840	-0.917956	2.000596	0.569139	
	r^{-3} 11.99104	r^{-2} 3.374564	r^{-1} 1.854547	r^0 2.369657	r^1 7.775824	r^2 50.55414			
1s	5.995520	1.687282	0.9272735	1.184829	3.887912	25.27707			
2 Helium Atoms		Singlet S State			1s(2) Configuration			(8s) Basis	
		$ 2-V/Z = 1.4D-08$			$ 1-Norm = 2.2D-16$				
				Total S Cusp:	-2.000029				
	T 2.861680	V(1) -6.749129	V(2) 1.025769	V(T) -5.723360	Z -2.861680	e -1.835911	-Cusp	r max	
1s	1.430840	-3.374564	0.512884	-2.861680	-1.430840	-0.917956	2.000029	0.569145	
	r^{-3} 11.99101	r^{-2} 3.374564	r^{-1} 1.854547	r^0 2.369657	r^1 7.775896	r^2 50.55694			
1s	5.995508	1.687282	0.9272734	1.184829	3.887948	25.27847			

Table 12. (Continued)

2	Helium Atom	Singlet S State				1s(2) Configuration		(12s) Basis	
		$ 2-V/Z = 3.7D-08$				$ 1-Norm = 9.2D-13$			
			Total S Cusp: -1.999968						
		T 2.861680	V(1) -6.789129	V(2) 1.025769	V(T) -5.723360	E -2.861680	e -1.835911	-Cusp	r max
1s	1.430840	-3.374564	0.512884	-2.861680	-1.430840	-0.917956	1.999968	0.569145	
		r^{-3} 11.99101	r^{-2} 3.374564	r^{-1} 1.854547	r^0 2.369657	r^1 7.775898	r^2 50.55682		
1s		5.995503	1.687282	0.9272735	1.184829	3.887949	25.27841		
3	Lithium Atom	Doublet S State				1s(2)2s(1) Configuration		(4s) Basis	
		$ 2-V/Z = 5.5D-08$				$ 1-Norm = 2.2D-16$			
			Total S Cusp: -2.925391						
		T 7.431889	V(1) -17.148921	V(2) 2.285144	V(T) -14.863778	E -7.431889	e -5.106705	-Cusp	r max
1s	3.609675	-8.052189	0.983641	-7.068548	-3.458873	-2.475231	2.923866	0.361917	
2s	0.212539	-1.044544	0.317861	-0.726682	-0.514143	-0.196282	2.986008	3.085527	
		r^{-3} 30.10711	r^{-2} 5.716307	r^{-1} 4.981412	r^0 18.24392	r^1 542.8181	r^2 27438.38		
1s		14.83276	2.684063	0.5740390	0.4502143	0.6366592	9.373911		
2s		0.4415863	0.3481812	3.833338	17.34349	541.1488	27419.63		
3	Lithium Atom	Doublet S State				1s(2)2s(1) Configuration		(6s) Basis	
		$ 2-V/Z = 1.9D-08$				$ 1-Norm = 0.0$			
			Total S Cusp: -3.026972						
		T 7.432721	V(1) -17.146387	V(2) 2.280945	V(T) -14.865442	E -7.432721	e -5.151776	-Cusp	r max
1s	3.611943	-8.055086	0.982708	-7.072378	-3.460435	-2.477727	3.026489	0.363808	
2s	0.208836	-1.036215	0.315529	-0.720686	-0.511850	-0.196321	3.046704	3.102348	
		r^{-3} 30.22036	r^{-2} 5.715662	r^{-1} 5.020426	r^0 18.64596	r^1 570.2094	r^2 29525.39		
1s		14.89233	2.685029	0.5731307	0.4466134	0.5322173	1.251309		
2s		0.4357004	0.3454049	3.874165	17.75233	569.1450	29522.89		
4	Beryllium Atom	Singlet S State				1s(2)2s(2) Configuration		(4s) Basis	
		$ 2-V/Z = 3.0D-07$				$ 1-Norm = 1.3D-15$			
			Total S Cusp: -4.082137						
		T 14.572823	V(1) -33.636816	V(2) 4.491166	V(T) -29.145651	E -14.572828	e -10.081662	-Cusp	r max
1s	6.784751	-14.726838	1.605086	-13.121752	-6.337001	-4.731916	4.083918	0.267840	
2s	0.501661	-2.091570	0.640497	-1.451073	-0.949413	-0.308915	4.028147	2.064675	
		r^{-3} 57.68049	r^{-2} 8.409204	r^{-1} 6.115663	r^0 17.20395	r^1 265.6462	r^2 7104.469		
1s		27.78412	3.681709	0.4148970	0.2328330	0.1433224	0.2058896		
2s		1.056124	0.5228926	2.642935	8.369144	132.6798	3552.029		
4	Beryllium Atom	Singlet S State				1s(2)2s(2) Configuration		(5s) Basis	
		$ 2-V/Z = 2.5D-08$				$ 1-Norm = 2.2D-15$			
			Total S Cusp: -3.951671						
		T 14.572940	V(1) -33.636162	V(2) 4.490282	V(T) -29.145880	E -14.572940	e -10.082658	-Cusp	r max
1s	6.785122	-14.727219	1.604887	-13.122332	-6.337211	-4.732324	3.946357	0.267719	
2s	0.501348	-2.090862	0.640254	-1.450608	-0.949259	-0.309006	4.110219	2.064781	
		r^{-3} 57.60666	r^{-2} 8.409040	r^{-1} 6.118865	r^0 17.22709	r^1 266.5261	r^2 7142.737		
1s		27.74689	3.681805	0.4149581	0.2328961	0.1425037	0.1813852		
2s		1.056442	0.5227154	2.644474	8.380649	133.1205	3571.187		

Table 12. (Continued)

4		Rutherford Atom		Singlet S State		1s(2) 2s(2) Configuration		(6s) Basis	
				$ 2-V/Z = 3.4D-09$		$ \text{Norm} = 3.1D-15$			
				Total S Cusp:		-4.029301			
		T	V(1)	V(2)	V(T)	E	e	-Cusp	r max
		14.573015	-33.635161	4.489131	-29.146030	-14.573015	-10.083884		
1s	6.785385	-14.727517	1.604728	-13.122789	-6.337400	-4.732672	4.028746	0.267112	
2s	0.501116	-2.090063	0.639837	-1.450226	-0.949108	-0.309270	4.048468	2.052478	
		E^{-3}	E^{-2}	E^{-1}	E	E^2	E^*	E^*	
		57.62992	8.408790	6.129706	17.33243	272.1175	7432.327		
1s	27.75891	3.681879	0.4149965	0.2329608	0.1420663	0.1675701			
2s	1.056044	0.5225158	2.649856	8.433256	135.9167	3715.996			
4		Beryllium Atom		Singlet S State		1s(2) 2s(2) Configuration		(7s) Basis	
				$ 2-V/Z = 2.7D-09$		$ \text{Norm} = 1.1D-15$			
				Total S Cusp:		-3.991817			
		T	V(1)	V(2)	V(T)	E	e	-Cusp	r max
		14.573021	-33.635179	4.489136	-29.146042	-14.573021	-10.083885		
1s	6.785385	-14.727508	1.604725	-13.122783	-6.337398	-4.732673	3.992821	0.267081	
2s	0.501125	-2.090081	0.639843	-1.450238	-0.949112	-0.309269	3.961536	2.052365	
		E^{-3}	E^{-2}	E^{-1}	E	E^2	E^*	E^*	
		57.61672	8.408795	6.129809	17.32909	271.8251	7410.553		
1s	27.75283	3.681877	0.4149944	0.2329552	0.1420581	0.1666988			
2s	1.055531	0.5225203	2.649730	8.431592	135.7705	3705.110			
4		Beryllium Atom		Singlet S State		1s(2) 2s(2) Configuration		(8s) Basis	
				$ 2-V/Z = 3.7D-09$		$ \text{Norm} = 6.7D-16$			
				Total S Cusp:		-4.003358			
		T	V(1)	V(2)	V(T)	E	e	-Cusp	r max
		14.573021	-33.635190	4.489144	-29.146046	-14.573023	-10.083879		
1s	6.785384	-14.727507	1.604726	-13.122780	-6.337396	-4.732670	4.003260	0.267040	
2s	0.501127	-2.090088	0.639846	-1.450243	-0.949115	-0.309270	4.006305	2.050472	
		E^{-3}	E^{-2}	E^{-1}	E	E^2	E^*	E^*	
		57.61869	8.408798	6.128857	17.31960	270.7910	7321.707		
1s	27.75367	3.681877	0.4149941	0.2329548	0.1420613	0.1668065			
2s	1.055674	0.5225221	2.649835	8.426843	135.2534	3660.687			
4		Beryllium Atom		Singlet S State		1s(2) 2s(2) Configuration		(12s) Basis	
				$ 2-V/Z = 4.4D-09$		$ \text{Norm} = 1.9D-15$			
				Total S Cusp:		-3.999834			
		T	V(1)	V(2)	V(T)	E	e	-Cusp	r max
		14.573023	-33.635191	4.489144	-29.146046	-14.573023	-10.083879		
1s	6.785384	-14.727507	1.604726	-13.122781	-6.337396	-4.732670	3.999825	0.267051	
2s	0.501127	-2.090088	0.639846	-1.450243	-0.949115	-0.309270	4.000114	2.050592	
		E^{-3}	E^{-2}	E^{-1}	E	E^2	E^*	E^*	
		57.61801	8.408798	6.128816	17.31878	270.6595	7306.160		
1s	27.75336	3.681877	0.4149941	0.2329547	0.1420594	0.1667011			
2s	1.055648	0.5225221	2.649814	8.426436	135.1877	3652.914			
5		Boron Atom		Doublet P State		1s(2) 2s(2) 2p(1) Configuration		(4s, 2p) Basis	
				$ 2-V/Z = 5.4D-07$		$ \text{Norm} = 0.0$			
				Total S Cusp:		-5.073734		Total P Cusp:	
		T	V(1)	V(2)	V(T)	E	e	-Cusp	r max
		24.528227	-56.901364	7.864895	-49.056468	-24.528241	-16.683345		
1s	10.93172E	-23.370658	2.372494	-20.998164	-10.066436	-7.693942	5.080380	0.211418	
2s	0.959937	-3.567523	1.057021	-2.510502	-1.550566	-0.493544	4.907958	1.583332	
2p	0.74489E	-3.025002	0.985865	-2.039136	-1.294238	-0.308373	1.788777	1.582409	
		E^{-3}	E^{-2}	E^{-1}	E	E^2	E^*	E^*	
		93.72012	11.38027	6.762485	15.60899	158.4529	2862.890		
1s	44.57464	4.674132	0.3259172	0.1433045	5.3829164D-02	4.2222384D-02			
2s	2.022015	0.7135047	1.967759	4.639374	40.60129	597.6724			
2p	0.750090E	0.5268124	0.6050003	2.195333	6.043637	77.14266	1667.461		

Table 12. (Continued)

5 Boron Atom		Doublet P State			1s(2)2s(2)2p(1) Configuration			(4s, 3p) Basis	
		$ 2-\nabla/E = 6.40-06$			$ 1-Norm = 2.20-16$				
		Total S Cusp: -5.073434			Total P Cusp: -2.017699				
		T	V(1)	V(2)	V(T)	E	C	-Cusp	r max
		24.528642	-56.900835	7.843708	-49.057127	-24.528485	-16.684777		
1s	10.931650	-23.370576	2.372336	-20.998239	-10.066589	-7.694252	5.080064	0.211418	
2s	0.960103	-3.567802	1.056929	-2.510873	-1.550770	-0.493842	4.908107	1.543256	
2p	0.745135	-3.024079	0.985178	-2.038902	-1.293767	-0.308589	2.017699	1.588736	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ^0	Σ^1	Σ^2		
		93.72128	11.38017	6.789022	15.66995	161.6712	3026.200		
1s	44.57398	4.674115	0.3258195	0.1433085	5.38395130-02	4.2319145D-02			
2s	2.022352	0.7135604	1.967606	4.638598	40.58605	597.3098			
2p	0.7635403	0.5286142	0.6048159	2.202172	6.10e136	80.39144	1831.496		
5 Boron Atom		Doublet P State			1s(2)2s(2)2p(1) Configuration			(6s, 4p) Basis	
		$ 2-\nabla/E = 4.5D-08$			$ 1-Norm = 1.8D-16$				
		Total S Cusp: -5.028306			Total P Cusp: -2.069785				
		T	V(1)	V(2)	V(T)	E	C	-Cusp	r max
		24.529051	-56.897230	7.839130	-49.058100	-24.529050	-16.69920		
1s	10.932716	-23.371710	2.371831	-20.99879	-10.067163	-7.695332	5.027266	0.210997	
2s	0.958989	-3.564389	1.055370	-2.509019	-1.550070	-0.694700	5.053762	1.526133	
2p	0.745720	-3.025032	0.988728	-2.040306	-1.294585	-0.309857	2.469785	1.586061	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ^0	Σ^1	Σ^2		
		93.66687	11.37945	6.810623	15.84964	168.9558	3330.818		
1s	44.56432	4.674382	0.3258677	0.1433639	5.3619827D-02	3.8831894D-02			
2s	2.025195	0.7128777	1.977253	4.711459	42.94596	673.1221			
2p	0.7749421	0.5298487	0.6050365	2.204381	6.139992	82.95667	1984.496		
6 Carbon Atom		Triplet P State			1s(2)2s(2)2p(2) Configuration			(4s, 2p) Basis	
		$ 2-\nabla/E = 9.2D-07$			$ 1-Norm = 0.0$				
		Total S Cusp: -6.071222			Total P Cusp: -2.274349				
		T	V(1)	V(2)	V(T)	E	C	-Cusp	r max
		37.686514	-88.145619	12.772556	-75.373063	-37.686549	-24.913993		
1s	16.050213	-33.984781	3.305789	-30.678992	-14.628759	-11.322970	6.083464	0.174686	
2s	1.540119	-5.386846	1.571215	-3.814631	-2.274512	-0.703297	5.795374	1.240202	
2p	1.252906	-4.702183	1.509278	-3.192909	-1.940003	-0.430729	2.274349	1.201700	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ^0	Σ^1	Σ^2		
		136.8308	14.69094	7.109677	13.53897	92.47112	1122.493		
1s	65.27954	5.664130	0.2684114	9.7163301D-02	2.4667937D-02	1.2730322D-02			
2s	3.248397	0.8976810	1.590019	2.993874	16.91589	160.6728			
2p	1.641699	0.8874430	0.7836971	1.706409	3.678488	29.29538	400.5611		
6 Carbon Atom		Triplet P State			1s(2)2s(2)2p(2) Configuration			(4s, 3p) Basis	
		$ 2-\nabla/E = 6.2D-08$			$ 1-Norm = 0.0$				
		Total S Cusp: -6.071152			Total P Cusp: -2.540326				
		T	V(1)	V(2)	V(T)	E	C	-Cusp	r max
		37.687358	-88.192880	12.768146	-75.374733	-37.687366	-24.919219		
1s	16.049736	-33.984306	3.305289	-30.679017	-14.629231	-11.323943	6.083358	0.174688	
2s	1.540714	-5.386842	1.570980	-3.815852	-2.275138	-0.704158	5.796262	1.240051	
2p	1.253194	-4.700302	1.507804	-3.192498	-1.939313	-0.421509	2.540326	1.210093	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ^0	Σ^1	Σ^2		
		138.8356	14.69048	7.122237	13.63431	95.77773	1232.190		
1s	65.27781	5.664051	0.2684156	9.7166455D-02	2.4699428D-02	1.2730362D-02			
2s	3.249552	0.8978053	1.579710	2.992608	16.89923	160.4207			
2p	1.670565	0.8904302	0.7833837	1.712993	3.727379	30.96495	455.6617		
6 Carbon Atom		Triplet P State			1s(2)2s(2)2p(2) Configuration			(6s, 4p) Basis	
		$ 2-\nabla/E = 1.7D-07$			$ 1-Norm = 1.1D-16$				
		Total S Cusp: -6.026968			Total P Cusp: -2.956471				
		T	V(1)	V(2)	V(T)	E	C	-Cusp	r max
		37.688405	-88.136933	12.759731	-75.177203	-37.688598	-24.928668		
1s	16.051193	-33.986643	3.304574	-30.682069	-14.630076	-11.325502	6.025222	0.174191	
2s	1.538431	-5.386768	1.568266	-3.812502	-2.273871	-0.705605	6.065351	1.223025	
2p	1.253179	-4.701056	1.507025	-3.194031	-1.940352	-0.433327	2.956471	1.212576	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ^0	Σ^1	Σ^2		
		138.7791	14.68949	7.144196	13.78704	100.0826	1356.536		
1s	65.24100	5.664401	0.2684434	9.7199701D-02	2.4634102D-02	1.2176427D-02			
2s	3.246549	0.8974947	1.589455	3.053259	18.22154	188.9267			
2p	1.690289	0.8920001	0.7835093	1.714199	3.743084	31.74513	489.3290		

Table 12. (Continued)

7 Nitrogen Atom		Quartet S State			1s(2)2s(2)2p(3) Configuration			(4s, 2p) Basis			
		$ 2-\nabla/E = 2.3D-08$ Total S Cusp: -6.877657			$ 1-Norm = 6.9D-17$ Total P Cusp: -2.752317			-			
T	54.397141	V(1) -128.361098	V(2) 19.566818	V(T) -108.794290	Z -54.397140	e -34.830322	-Cusp	r max			
1s	22.141149	-46.570850	4.402969	-42.167860	-20.026732	-15.623763	6.878964	0.148354			
2s	2.243819	-7.550067	2.181450	-5.368617	-3.124798	-0.943348	6.850611	1.024972			
2p	1.875736	-6.706422	2.132660	-4.573762	-2.698027	-0.565367	2.752317	0.972435			
		Σ^3 192.6679	Σ^-2 18.33730	Σ^-1 7.325116	Σ^2 11.91391	Σ^0 59.10916	Σ^0 521.9469	-			
1s	89.60379	6.652979	0.2283131	7.0397797D-02	1.3555142D-02	1.3073483D-02	-				
2s	4.735389	1.078581	1.330363	2.138804	8.897868	63.76832	-				
2p	3.014161	1.329840	0.9580603	1.402601	2.498501	13.76237	131.4614				
7 Nitrogen Atom		Quartet S State			1s(2)2s(2)2p(3) Configuration			(4s, 3p) Basis			
		$ 2-\nabla/E = 3.6D-07$ Total S Cusp: -6.877538			$ 1-Norm = 0.0$ Total P Cusp: -3.069696			-			
T	54.399001	V(1) -128.354253	V(2) 19.556270	V(T) -108.797983	Z -54.398981	e -34.882711	-Cusp	r max			
1s	22.140290	-46.569946	4.401944	-42.168002	-20.027712	-15.625769	6.878882	0.148357			
2s	2.244934	-7.551842	2.180943	-5.370898	-3.125964	-0.945021	6.849708	1.024800			
2p	1.876184	-6.703559	2.130165	-4.573394	-2.697210	-0.567044	3.069696	0.981672			
		Σ^3 192.6790	Σ^-2 18.33632	Σ^-1 7.342938	Σ^2 12.02892	Σ^0 61.97858	Σ^0 590.1746	-			
1s	89.60046	6.652889	0.2263176	7.0400429D-02	1.3555105D-02	1.3058291D-02	-				
2s	4.737294	1.078835	1.330032	2.137748	8.887851	63.65895	-				
2p	3.068702	1.334488	0.9576513	1.408746	2.537580	14.72526	154.2768				
7 Nitrogen Atom		Quartet S State			1s(2)2s(2)2p(3) Configuration			(6s, 4p) Basis			
		$ 2-\nabla/E = 3.2D-08$ Total S Cusp: -7.025662			$ 1-Norm = 9.7D-17$ Total P Cusp: -3.436500			-			
T	54.800897	V(1) -128.351432	V(2) 19.549640	V(T) -108.801793	Z -54.400896	e -34.851256	-Cusp	r max			
1s	22.143163	-46.572650	4.400232	-42.172419	-20.029255	-15.629024	7.023037	0.148606			
2s	2.242675	-7.547200	2.179626	-5.367575	-3.128900	-0.945274	7.079598	1.022675			
2p	1.876407	-6.703910	2.129975	-4.573935	-2.697528	-0.567553	3.436500	0.985131			
		Σ^3 193.2164	Σ^-2 18.33592	Σ^-1 7.349479	Σ^2 12.07655	Σ^0 63.37354	Σ^0 627.0363	-			
1s	89.64875	6.653236	0.2282979	7.0265808D-02	1.2877208D-02	1.46403231D-03	-				
2s	4.755178	1.078171	1.332356	2.150191	9.110311	67.59884	-				
2p	3.096545	1.336182	0.9577015	1.409391	2.585212	15.04239	163.9431				
8 Oxygen Atom		Triplet P State			1s(2)2s(2)2p(4) Configuration			(4s, 2p) Basis			
		$ 2-\nabla/E = 8.5D-08$ Total S Cusp: -7.876184			$ 1-Norm = 0.0$ Total P Cusp: -3.216561			-			
T	74.803513	V(1) -178.094937	V(2) 28.487918	V(T) -149.607019	Z -74.803506	e -46.315589	-Cusp	r max			
1s	29.211825	-61.136280	5.631669	-55.504620	-26.292795	-20.661126	7.876887	0.129286			
2s	3.108121	-10.123097	2.887068	-7.235938	-4.127817	-1.240749	7.863258	0.877582			
2p	2.540905	-8.894086	2.862611	-6.031376	-3.490570	-0.627960	3.216561	0.819586			
		Σ^3 256.5828	Σ^-2 22.26187	Σ^-1 7.576212	Σ^2 10.96340	Σ^0 43.65565	Σ^0 314.9102	-			
1s	118.1036	7.642036	0.1985882	5.3223717D-02	7.6790083D-03	4.9755616D-03	-				
2s	6.568890	1.265376	1.140457	1.575900	4.864186	26.06965	-				
2p	4.833886	1.809843	1.111761	1.224280	1.926288	8.877979	65.69034				
8 Oxygen Atom		Triplet P State			1s(2)2s(2)2p(4) Configuration			(4s, 3p) Basis			
		$ 2-\nabla/E = 2.3D-07$ Total S Cusp: -7.875927			$ 1-Norm = 0.0$ Total P Cusp: -3.655664			-			
T	74.807543	V(1) -178.07704	V(2) 28.462736	V(T) -149.615068	Z -74.807525	e -46.348789	-Cusp	r max			
1s	29.210230	-61.134625	5.629494	-55.505131	-26.294901	-20.665007	7.876656	0.129289			
2s	3.110423	-10.126628	2.886096	-7.240532	-4.130109	-1.240012	7.861701	0.877369			
2p	2.541559	-8.888825	2.857889	-6.010936	-3.489376	-0.631487	3.656664	0.833505			
		Σ^3 256.6094	Σ^-2 22.25973	Σ^-1 7.605164	Σ^2 11.13561	Σ^0 47.27898	Σ^0 387.1456	-			
1s	118.0971	7.541828	0.1985931	5.3226078D-02	7.6795592D-03	4.9635994D-03	-				
2s	6.572726	1.265429	1.140533	1.574643	4.855488	25.99404	-				
2p	4.945938	1.817414	1.111103	1.231724	1.969968	0.388161	93.78690				

Table 12. (Continued)

8	Oxygen Atom	Triplet P State			1s(2)2s(2)2p(4) Configuration			(6s, 4p) Basis	
		12- γ/β_1 = 3.10-08 Total S Cusp: -8.024434			11-Norm: 0.0 Total P Cusp: -3.858964			-Cusp	ϵ max
		T	$\gamma(1)$	$\gamma(2)$	$\gamma(T)$	E	ϵ		
		74.809289	-178.075735	29.457159	-149.619576	-74.809287	-46.352128		
1s	29.213141	-61.137298	5.627829	-55.509470	-26.296329	-20.668500	8.021117	0.129468	
2s	3.108134	-10.122077	2.884918	-7.237159	-4.129025	-1.244107	8.088723	0.876297	
2p	2.541685	-8.089246	2.857916	-6.031330	-3.489645	-0.631729	3.958964	0.835674	
		ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ	ϵ^2	ϵ^3	ϵ^4	
		257.2394	22.25947	7.608392	11.15811	47.89124	401.8153		
1s	118.3892	7.642162	0.1985897	5.3146131D-02	7.3665677D-03	2.0217410D-03			
2s	6.593398	1.265260	1.142006	1.581583	6.951478	27.30140			
2p	4.966836	1.818557	1.111156	1.231800	1.972162	9.493387	86.80211		
9	Fluorine Atom	Doublet P State			1s(2)2s(2)2p(5) Configuration			(4s, 2p) Basis	
		12- γ/β_1 = 6.10-08 Total S Cusp: -8.875667			11-Norm: 0.0 Total P Cusp: -3.686905				
		T	$\gamma(1)$	$\gamma(2)$	$\gamma(T)$	E	ϵ		
		99.400471	-238.703573	39.902638	-198.800935	-99.400465	-59.497827		
1s	37.256477	-77.673059	7.023046	-70.650013	-33.395536	-26.372490	8.875306	0.114563	
2s	4.096893	-13.046382	3.691264	-9.355118	-5.258225	-1.566960	8.882812	0.768438	
2p	3.339546	-11.452938	3.694803	-7.758135	-4.418589	-0.723785	3.686905	0.707603	
		ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ	ϵ^2	ϵ^3	ϵ^4	
		330.2960	26.52262	7.736725	10.01082	32.03673	187.0033		
1s	150.518	8.630380	0.1757380	4.1661350D-02	6.6784499D-03	2.1805296D-03			
2s	8.672250	1.449598	1.000584	1.213836	2.904465	12.15493			
2p	7.334940	2.382390	1.272549	1.076816	1.499964	5.243689	32.53781		
9	Fluorine Atom	Doublet P State			1s(2)2s(2)2p(5) Configuration			(4s, 3p) Basis	
		12- γ/β_1 = 7.20-07 Total S Cusp: -8.875273			11-Norm: 0.0 Total P Cusp: -4.217840				
		T	$\gamma(1)$	$\gamma(2)$	$\gamma(T)$	E	ϵ		
		99.407468	-238.674052	39.859188	-198.814865	-99.407396	-59.548209		
1s	37.252002	-77.670490	7.019566	-70.650924	-33.398922	-26.379356	8.875004	0.114568	
2s	4.100526	-13.052059	3.689885	-9.362174	-5.261688	-1.571762	8.880302	0.768221	
2p	3.340483	-11.445791	3.688057	-7.757734	-4.417251	-0.729194	4.217840	0.723R20	
		ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ	ϵ^2	ϵ^3	ϵ^4	
		330.3367	26.51934	7.772418	10.20625	35.48869	284.2147		
1s	150.5045	8.630054	0.1757430	4.1663363D-02	6.6780465D-03	2.1736120D-03			
2s	8.678214	1.450229	1.000115	1.212601	2.897689	12.10822			
2p	7.526867	2.394259	1.271755	1.084141	1.539544	5.936791	43.99959		
9	Fluorine Atom	Doublet P State			1s(2)2s(2)2p(5) Configuration			(6s, 4p) Basis	
		12- γ/β_1 = 3.45-08 Total S Cusp: -9.023295			11-Norm: 1.80-16 Total P Cusp: -4.241562				
		T	$\gamma(1)$	$\gamma(2)$	$\gamma(T)$	E	ϵ		
		99.409208	-238.667630	39.849218	-198.814412	-99.409204	-59.559987		
1s	37.254864	-77.673059	7.017820	-70.655239	-33.400374	-26.382553	9.019095	0.114700	
2s	4.098514	-13.047925	3.688455	-9.359670	-5.260956	-1.572501	9.101509	0.767700	
2p	3.340490	-11.445132	3.687333	-7.757799	-4.417309	-0.729976	4.241562	0.721435	
		ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ	ϵ^2	ϵ^3	ϵ^4	
		331.0506	26.51863	7.777788	10.23614	36.16641	261.0193		
1s	150.8403	8.630340	0.1757479	4.1612716D-02	6.5175887D-03	9.7756642D-04			
2s	8.700074	1.449769	1.0011989	1.216666	2.942098	12.58807			
2p	7.520728	2.393965	1.271681	1.084923	1.543917	6.056435	47.16824		
10	Neon Atom	Singlet S State			1s(2)2s(2)2p(6) Configuration			(4s, 2p) Basis	
		12- γ/β_1 = 6.10-08 Total S Cusp: -9.875971			11-Norm: 2.2D-16 Total P Cusp: -4.158744				
		T	$\gamma(1)$	$\gamma(2)$	$\gamma(T)$	E	ϵ		
		128.534229	-311.191109	54.122658	-257.068451	-128.534222	-74.411563		
1s	46.269254	-96.181397	9.576570	-87.604827	-41.335573	-32.759002	9.874298	0.102853	
2s	5.210515	-16.321763	4.594395	-11.727368	-6.516453	-7.922458	9.906305	0.684059	
2p	4.262689	-18.364131	4.630121	-9.734010	-5.471562	-3.841480	4.158748	0.622675	
		ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ	ϵ^2	ϵ^3	ϵ^4	
		414.0282	31.11911	7.584745	9.148946	23.78079	113.1103		
1s	186.8393	9.618140	0.1574178	1.3502430D-02	3.0138231D-03	1.0596831D-03			
2s	11.04654	1.632176	0.8010715	0.9656722	1.643770	1.230640			
2p	10.60865	3.043076	1.436413	0.9576277	1.190949	3.346203	16.77448		

Table 12. (Continued)

10	Neon Atom	Singlet S State			1s(2)2s(2)2p(6) Configuration			(4s, 3p) Basis	
		$ 2-V/E = 3.00-08$ Total S Cusp: -9.875525			$ 1-Norm = 8.9D-16$ Total P Cusp: -4.620815				
		T	V(1)	V(2)	V(T)	Z	e	-Cusp	r max
	128.544926	-311.141847	54.051995	-257.089852	-128.544926	-74.492029			
1s	46.265773	-96.177778	8.571645	-87.606132	-41.340360	-32.768715	9.874017	0.102856	
2s	5.215610	-16.329393	4.592160	-11.737232	-6.521622	-1.929461	9.902839	0.683790	
2p	4.263694	-14.354584	4.620731	-9.733854	-5.470160	-0.849430	4.620815	0.637273	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ	Σ^2	Σ^3	Σ^4	
		414.1042	31.11418	7.886849	9.354106	26.90934	157.2475		
1s	186.8266	9.617778	0.1576238	3.35048590-02	3.01438590-03	1.0513401D-03			
2s	11.05876	1.632939	0.8914855	0.9648399	1.845744	6.216905			
2p	10.86220	3.057572	1.435458	0.9647717	1.226236	3.868637	24.13526		
10	Neon Atom	Singlet S State			1s(2)2s(2)2p(6) Configuration			(5s, 3p) Basis	
		$ 2-V/E = 8.3D-09$ Total S Cusp: -10.019158			$ 1-Norm = 1.6D-15$ Total P Cusp: -4.622214				
		T	V(1)	V(2)	V(T)	Z	e	-Cusp	r max
	128.546561	-311.139019	54.045899	-257.093121	-128.546560	-74.500661			
1s	46.268611	-96.180304	8.570054	-87.610250	-41.341639	-32.771585	10.021991	0.102968	
2s	5.213441	-16.325327	4.591118	-11.734209	-6.520769	-1.929651	9.967609	0.683769	
2p	4.263743	-14.354626	4.620592	-9.734034	-5.470291	-0.849698	4.622214	0.637293	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ	Σ^2	Σ^3	Σ^4	
		414.8901	31.11390	7.888188	9.359183	26.95830	157.7070		
1s	187.2072	9.618030	0.1576312	3.34702870-02	2.9243120D-03	5.1490624D-08			
2s	11.06892	1.632533	0.8921185	0.9672702	1.868438	6.423556			
2p	10.86306	3.057641	1.435463	0.9647814	1.226284	3.869263	24.14314		
10	Neon Atom	Singlet S State			1s(2)2s(2)2p(6) Configuration			(5s, 3p) Basis	
		$ 2-V/E = 2.4D-07$ Total S Cusp: -10.019187			$ 1-Norm = 1.8D-15$ Total P Cusp: -4.758186				
		T	V(1)	V(2)	V(T)	Z	e	-Cusp	r max
	128.546899	-311.133177	54.039349	-257.093829	-128.546930	-74.507581			
1s	46.268715	-96.180412	8.569722	-87.610690	-41.341975	-32.772254	10.022027	0.102968	
2s	5.213804	-16.325866	4.590885	-11.735021	-6.521217	-1.930373	9.967528	0.683752	
2p	4.263644	-14.353437	4.619703	-9.733734	-5.470091	-0.850388	4.758186	0.635038	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ	Σ^2	Σ^3	Σ^4	
		414.8942	31.11332	7.891476	9.374312	27.21867	163.1597		
1s	187.2077	9.618041	0.1576310	3.3470199D-02	2.9242884D-03	5.1481620D-04			
2s	11.06559	1.632587	0.8920914	0.9672108	1.868213	6.422492			
2p	10.8634	3.057941	1.435344	0.9653386	1.228825	3.912732	25.05229		
10	Neon Atom	Singlet S State			1s(2)2s(2)2p(6) Configuration			(6s, 3p) Basis	
		$ 2-V/E = 1.8D-08$ Total S Cusp: -10.022245			$ 1-Norm = 1.8D-15$ Total P Cusp: -4.758178				
		T	V(1)	V(2)	V(T)	Z	e	-Cusp	r max
	128.546953	-311.133172	54.039268	-257.093905	-128.546951	-74.507683			
1s	46.268626	-96.180319	8.569729	-87.610590	-41.341964	-32.772235	10.017054	0.102958	
2s	5.21382	-16.325770	4.590768	-11.735002	-6.521160	-1.930392	10.116102	0.683601	
2p	4.263669	-14.353499	4.619712	-9.733787	-5.470117	-0.850405	4.758178	0.635038	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ	Σ^2	Σ^3	Σ^4	
		414.9040	31.11332	7.891826	9.373889	27.21129	163.0664		
1s	187.2029	9.618032	0.1576312	3.3470035D-02	2.9238240D-03	5.1217515D-04			
2s	11.07519	1.632577	0.8920988	0.9671224	1.865915	6.395334			
2p	10.8638	3.057956	1.435350	0.9653276	1.228784	3.912268	25.04578		
10	Neon Atom	Singlet S State			1s(2)2s(2)2p(6) Configuration			(6s, 3p) Basis	
		$ 2-V/E = 3.6D-08$ Total S Cusp: -10.022289			$ 1-Norm = 2.0D-15$ Total P Cusp: -5.102837				
		T	V(1)	V(2)	V(T)	Z	e	-Cusp	r max
	128.547063	-311.133201	54.039071	-257.094131	-128.547068	-74.507997			
1s	46.268658	-96.180575	8.569631	-87.610943	-41.342076	-32.772444	10.017097	0.102958	
2s	5.213770	-16.325661	4.590760	-11.734901	-6.521131	-1.930371	10.116181	0.683593	
2p	4.263631	-14.353455	4.619715	-9.733740	-5.470109	-0.850398	5.102837	0.634126	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ	Σ^2	Σ^3	Σ^4	
		414.9104	31.11332	7.890971	9.370007	27.09784	160.5854		
1s	187.2037	9.618057	0.1576305	3.3469740D-02	2.9237684D-03	5.1219363D-04			
2s	11.07499	1.632566	0.8921100	0.9671597	1.866190	6.397677			
2p	10.90699	3.058849	1.435346	0.9652483	1.228125	3.893268	24.63158		

Table 12. (Continued)

10	Neon Atom	Singlet S State			1s(2)2s(2)2p(6) Configuration			(8s, 6p) Basis	
		$ 2-V/Z = 3.70-08$ Total S Cusp: -10.000639			$ 1-Norm = 1.10-15$ Total P Cusp: -5.071045				
		$\frac{r}{T}$	$\frac{v(1)}{r}$	$\frac{v(2)}{r}$	$\frac{v(T)}{r}$	$\frac{e}{r}$	$\frac{-Cusp}{r}$	$\frac{r_{max}}{r}$	
1s	46.268832	-96.180536	8.569632	-87.610904	-41.342072	-32.772440	10.001778	0.102960	
2s	5.213763	-16.325537	4.590693	-11.734845	-6.521081	-1.930389	9.979973	0.682998	
2p	4.263652	-14.353508	4.619724	-9.733783	-5.470132	-0.850407	5.071045	0.633988	
		$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	
1s	187.1977	9.618054	0.1576305	3.3469633D-02	2.9233003D-03	5.0824314D-04			
2s	11.07099	1.632554	0.8921064	0.9670310	1.863751	6.378413			
2p	10.90657	3.058855	1.435351	0.9652694	1.228412	3.906272	25.02858		
11	Na Sodium Atom	Doublet S State			He(10)3s(1) Configuration			(6s, 2p) Basis	
		$ 2-V/Z = 3.40-06$ Total S Cusp: -10.868433			$ 1-Norm = 2.20-16$ Total P Cusp: -4.748217				
		$\frac{r}{T}$	$\frac{v(1)}{r}$	$\frac{v(2)}{r}$	$\frac{v(T)}{r}$	$\frac{e}{r}$	$\frac{-Cusp}{r}$	$\frac{r_{max}}{r}$	
1s	56.271758	-116.678349	9.969925	-106.709424	-50.436666	-40.466742	10.059188	0.093329	
2s	6.881511	-20.535435	5.451619	-15.083816	-8.242306	-2.790687	10.951904	0.607033	
2p	5.892281	-18.669965	5.633115	-13.036850	-7.148608	-1.511893	4.748217	0.582681	
3s	0.268592	-3.332245	1.841220	-1.891025	-1.622432	-0.181212	10.867003	3.386088	
		$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	
1s	227.0500	10.60712	0.1428382	2.75372020D-02	4.3221791D-03	0.1910096			
2s	18.43891	1.866858	0.7802357	0.7360041	1.177055	14.99565			
2p	16.67923	4.173207	1.697270	0.7950046	0.8078898	1.475900	4.755882		
3s	0.3971652	0.3029313	4.167994	20.25146	714.2986	40304.53			
11	Na Sodium Atom	Doublet S State			He(10)3s(1) Configuration			(6s, 3p) Basis	
		$ 2-V/Z = 1.70-07$ Total S Cusp: -10.863708			$ 1-Norm = 1.10-16$ Total P Cusp: -5.229613				
		$\frac{r}{T}$	$\frac{v(1)}{r}$	$\frac{v(2)}{r}$	$\frac{v(T)}{r}$	$\frac{e}{r}$	$\frac{-Cusp}{r}$	$\frac{r_{max}}{r}$	
1s	56.259397	-116.675938	9.966708	-106.709230	-50.439838	-40.473126	10.858547	0.093332	
2s	6.845126	-20.540606	5.450439	-15.090167	-8.245041	-2.794602	10.984915	0.606477	
2p	5.892416	-18.662445	5.626885	-13.035561	-7.143185	-1.516260	5.229673	0.550317	
3s	0.272305	-3.349349	1.845311	-1.899623	-1.627258	-0.181987	10.877803	3.374850	
		$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	
1s	227.0377	10.60690	0.1428395	2.7537015D-02	4.2964615D-03	0.1889821			
2s	18.44420	1.867328	0.7900909	0.7358063	1.176144	14.87818			
2p	16.95992	4.186273	1.696586	0.7993085	0.8209822	1.598510	5.824981		
3s	0.8032928	0.3046849	4.158359	20.16899	709.7287	39998.97			
11	Na Sodium Atom	Doublet S State			He(10)3s(1) Configuration			(9s, 4p) Basis	
		$ 2-V/Z = 1.50-09$ Total S Cusp: -11.023714			$ 1-Norm = 4.40-16$ Total P Cusp: -5.335814				
		$\frac{r}{T}$	$\frac{v(1)}{r}$	$\frac{v(2)}{r}$	$\frac{v(T)}{r}$	$\frac{e}{r}$	$\frac{-Cusp}{r}$	$\frac{r_{max}}{r}$	
1s	56.275187	-116.681119	9.963793	-106.717326	-50.442179	-40.478386	11.019105	0.093395	
2s	6.882603	-20.540721	5.450575	-15.090185	-8.247582	-2.796967	11.098653	0.608157	
2p	5.892669	-18.662691	5.625969	-13.036722	-7.144053	-1.518085	5.335814	0.549783	
3s	0.267288	-3.315262	1.842932	-1.892330	-1.615082	-0.182110	11.121875	3.393012	
		$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	$\frac{r}{r^2}$	
1s	227.5417	10.60737	0.1428585	2.7480809D-02	1.9684674D-03	3.0958265D-04			
2s	18.45688	1.867338	0.7790703	0.7315237	1.028874	2.493025			
2p	16.98291	4.187200	1.696608	0.7984679	0.8221578	1.626442	6.369628		
3s	0.3984917	0.3013875	4.210100	20.73295	758.8813	43935.13			

Table 12. (Continued)

12	Mg	Magnesium Atom	Singlet S State	Ne(10) 3s(2) Configuration				(6s, 2p) Basis	
				$ 2-V/Z = 1.7D-07$				$ 1-Norm = 8.9D-16$	
			Total S Cusp: -11.870415				Total P Cusp: -5.308386		
			$\frac{T}{T}$	$\frac{V(1)}{V(1)}$	$\frac{V(2)}{V(2)}$	$\frac{V(T)}{V(T)}$	$\frac{E}{E}$	$\frac{e}{e}$	-Cusp
1s	67.268884	-139.170214	11.439632	-127.730581	-60.461697	-49.022065	11.868109	0.085395	
2s	8.743921	-25.288758	6.390880	-18.897918	-10.153997	-3.763157	11.902707	0.544787	
2p	7.751856	-23.827860	6.698887	-16.728573	-8.976716	-2.277829	5.308386	0.479657	
3s	0.534085	-4.795655	2.004971	-2.790684	-2.256599	-0.251628	11.973865	2.603688	
			$\frac{r^{-3}}{r^{-3}}$	$\frac{r^{-2}}{r^{-2}}$	$\frac{r^{-1}}{r^{-1}}$	$\frac{r}{r}$	$\frac{r^2}{r^2}$	$\frac{r^0}{r^0}$	r_{\max}
1s	271.3527	11.59752	0.1305798	2.2986566D-02	2.1825819D-03	4.3186350D-02			
2s	18.34370	2.107396	0.6915158	0.5747958	0.6894789	5.431199			
2p	24.55561	5.456039	1.952288	0.6829220	0.5903631	0.7672681	1.784307		
3s	0.7849288	0.3996379	3.239132	12.27097	264.7087	9210.263			
12	Bg	Magnesium Atom	Singlet S State	Ne(10) 3s(2) Configuration				(6s, 3p) Basis	
				$ 2-V/Z = 2.5D-08$				$ 1-Norm = 8.9D-16$	
			Total S Cusp: -11.870023				Total P Cusp: -5.725956		
			$\frac{T}{T}$	$\frac{V(1)}{V(1)}$	$\frac{V(2)}{V(2)}$	$\frac{V(T)}{V(T)}$	$\frac{E}{E}$	$\frac{e}{e}$	-Cusp
1s	67.267492	-139.168793	11.437245	-127.731588	-60.46056	-49.026811	11.867799	0.085396	
2s	8.746655	-25.292505	6.390031	-18.902474	-10.155819	-3.765787	11.900825	0.544645	
2p	7.751261	-23.420562	6.694041	-16.726522	-8.975261	-2.281221	5.725956	0.483992	
3s	0.537749	-4.805596	2.007831	-2.797768	-2.260017	-0.252185	11.977453	2.598971	
			$\frac{r^{-3}}{r^{-3}}$	$\frac{r^{-2}}{r^{-2}}$	$\frac{r^{-1}}{r^{-1}}$	$\frac{r}{r}$	$\frac{r^2}{r^2}$	$\frac{r^0}{r^0}$	r_{\max}
1s	271.3457	11.59740	0.1305805	2.2986588D-02	2.1787603D-03	4.2950798D-02			
2s	18.34760	2.107709	0.6914469	0.5747211	0.6892943	5.410186			
2p	24.84436	5.467609	1.951714	0.6847995	0.5966252	0.8089734	2.001523		
3s	0.7906824	0.4004663	3.234934	12.24222	263.6873	9170.656			
12	Bg	Magnesium Atom	Singlet S State	Ne(10) 3s(2) Configuration				(9s, 4p) Basis	
				$ 2-V/Z = 1.4D-08$				$ 1-Norm = 1.6D-15$	
			Total S Cusp: -12.024235				Total P Cusp: -5.823350		
			$\frac{T}{T}$	$\frac{V(1)}{V(1)}$	$\frac{V(2)}{V(2)}$	$\frac{V(T)}{V(T)}$	$\frac{E}{E}$	$\frac{e}{e}$	-Cusp
1s	67.274493	-139.175283	11.434611	-127.740572	-60.466179	-49.031568	12.020221	0.085458	
2s	8.743416	-25.293802	6.391341	-18.90261	-10.159045	-3.767708	12.08344	0.546605	
2p	7.751269	-23.420357	6.693433	-16.725924	-8.975658	-2.282221	5.823350	0.484227	
3s	0.535557	-4.792681	2.002036	-2.790643	-2.255086	-0.253048	12.120110	2.588710	
			$\frac{r^{-3}}{r^{-3}}$	$\frac{r^{-2}}{r^{-2}}$	$\frac{r^{-1}}{r^{-1}}$	$\frac{r}{r}$	$\frac{r^2}{r^2}$	$\frac{r^0}{r^0}$	r_{\max}
1s	271.8564	11.59794	0.1305946	2.2956106D-02	1.3712857D-03	1.6743340D-04			
2s	18.38604	2.107817	0.6903396	0.5710982	0.6114708	1.179243			
2p	24.88358	5.468948	1.951696	0.6849901	0.5976918	0.8241269	2.180171		
3s	0.7886473	0.3993901	3.253131	12.42388	275.5763	9992.942			
13	Al	Aluminum Atom	Doublet P State	Ne(10) 3s(2) 3p(1) Configuration				(6s, 4p) Basis	
				$ 2-V/Z = 1.2D-06$				$ 1-Norm = 1.7D-16$	
			Total S Cusp: -12.880181				Total P Cusp: -5.967803		
			$\frac{T}{T}$	$\frac{V(1)}{V(1)}$	$\frac{V(2)}{V(2)}$	$\frac{V(T)}{V(T)}$	$\frac{E}{E}$	$\frac{e}{e}$	-Cusp
1s	79.256515	-163.652759	12.956492	-150.696267	-71.439752	-58.483260	12.881026	0.078694	
2s	10.888507	-30.535866	7.377539	-23.158327	-12.269820	-4.892281	12.863084	0.494426	
2p	9.844927	-28.644699	7.799514	-20.845185	-11.002598	-3.200743	5.961022	0.430385	
3s	0.923110	-6.560466	2.628761	-3.931685	-3.008495	-0.379734	12.966136	2.119387	
3p	0.660019	-5.309665	2.221607	-3.088058	-2.428039	-0.206432	6.140237	2.590261	
			$\frac{r^{-3}}{r^{-3}}$	$\frac{r^{-2}}{r^{-2}}$	$\frac{r^{-1}}{r^{-1}}$	$\frac{r}{r}$	$\frac{r^2}{r^2}$	$\frac{r^0}{r^0}$	r_{\max}
1s	319.6535	12.58867	0.1202515	1.9576381D-02	1.2602477D-03	9.50903670-03			
2s	22.74295	2.388913	0.6208680	0.4610946	0.4195232	1.701163			
2p	38.71642	6.905496	2.203438	0.6009811	0.4552307	0.463036	1.074274		
3s	1.339788	0.5046897	2.599711	7.861769	106.1643	2279.466			
3p	1.256798	0.3516277	3.108469	11.15911	207.0484	5869.661			

Table 12. (Continued)

13 Aluminum Atom		Doublet P State				Ne(10) 3s(2)3p(1) Configuration				(9s, 6p) Basis	
12-V/EI = 3.1D-08 Total S Cusp: -13.024610				1-Norm = 6.4D-16 Total P Cusp: -6.331564							
	T	V(1)	V(2)	V(3)	Z	r	r*	-Cusp	r max		
	241.876633	-578.498493	94.745234	-483.753259	-241.876626	-147.131391					
1s	75.264147	-163.460040	12.947438	-150.712602	-71.448455	-58.501017	13.020480	0.078755			
2s	10.884504	-30.537118	7.370904	-23.166214	-12.281710	-4.410806	13.081059	0.496223			
2p	9.856710	-28.466595	7.796020	-20.870575	-11.014465	-3.218445	6.332328	0.432684			
3s	0.936142	-6.588718	2.629503	-3.559216	-3.023074	-0.393571	13.110699	2.599341			
3p	0.570388	-4.927172	2.073425	-2.853746	-2.283358	-0.209932	6.307795	2.682285			
	T=3	T=2	T=1	Z	T=2	r*	r*				
	730.4027	44.49968	13.72112	33.53146	587.7327	20632.54					
1s		320.1396	12.58923	0.1202586	1.9458840D-02	9.8365515D-04	9.7776372D-05				
2s		22.80173	2.349009	0.6200267	0.4594002	0.3860183	0.5452388				
2p		34.89545	6.9149468	2.205123	0.6005404	0.4553750	0.4638484	0.9277647			
3s			1.360006	0.5068245	2.599198	7.897539	109.1331	2454.423			
3p		1.085576	0.3102157	0.3790132	3.438909	14.05941	365.9054	15717.04			
14 Silicon Atom		Triplet P State				Ne(10) 3s(2)3p(2) Configuration				(6s, 4p) Basis	
12-V/EI = 3.1D-05 Total S Cusp: -14.023738				1-Norm = 1.4D-16 Total P Cusp: -6.522623							
	T	V(1)	V(2)	V(3)	Z	r	r*	-Cusp	r max		
	288.653280	-689.665202	111.967586	-577.697616	-288.844337	-176.876751					
1s	92.247688	-140.13h984	14.548260	-175.540725	-83.343037	-68.794777	14.048654	0.073055			
2s	13.249725	-36.24210	8.433095	-27.816115	-14.566389	-6.133294	13.687902	0.456302			
2p	12.184385	-34.362347	8.972404	-25.389492	-13.205558	-4.233153	6.519317	0.393387			
3s	1.383045	-8.454623	3.275065	-5.181557	-3.798512	-0.523446	13.712242	1.809746			
3p	0.993026	-6.900744	2.810160	-4.090584	-3.097558	-0.287398	6.590906	2.169859			
	T=3	T=2	T=1	Z	T=2	r*	r*				
	856.2189	49.26180	14.19048	30.30826	321.5661	5996.590					
1s		372.4382	13.56136	0.1114296	1.67032450D-02	7.4191666D-04	5.3000829D-04				
2s		27.62406	2.589229	0.5628772	0.3768372	0.2580852	0.2936926				
2p		47.09810	8.512448	0.5359199	0.3596072	0.2848013	0.5570291				
3s		1.986339	0.6040445	2.192486	5.571044	52.46117	778.8074				
3p		2.145427	0.5235366	0.4929103	2.620686	7.975123	107.2086	2217.522			
14 Silicon Atom		Triplet P State				Ne(10) 3s(2)3p(2) Configuration				(6s, 4p) Basis	
12-V/EI = 4.3D-09 Total S Cusp: -14.024903				1-Norm = 7.2D-16 Total P Cusp: -6.839460							
	T	V(1)	V(2)	V(3)	Z	r	r*	-Cusp	r max		
	288.654290	-689.413790	111.705222	-577.708568	-288.854278	-177.149056					
1s	92.244674	-190.135917	14.539421	-175.596496	-83.351822	-68.812401	14.020883	0.073029			
2s	13.262032	-36.265602	8.423477	-27.442125	-14.580093	-6.156616	14.074156	0.454205			
2p	12.197942	-34.3h532	8.967724	-25.421807	-13.223866	-4.256181	6.838538	0.390974			
3s	1.385795	-8.4456689	3.259965	-5.157047	-3.799909	-0.539984	14.134960	1.799791			
3p	0.940819	-6.691111	2.726575	-3.964537	-3.023718	-0.297143	6.836760	2.169502			
	T=3	T=2	T=1	Z	T=2	r*	r*				
	856.2939	49.24384	14.14222	32.28344	411.0197	9909.060					
1s		372.3937	13.58114	0.1114308	1.670923D-02	7.2345547D-04	6.1392436D-05				
2s		27.69567	2.590400	0.5624430	0.3772620	0.2591384	0.2935487				
2p		47.21579	8.5154355	0.5354066	0.3596912	0.2830200	0.3196690				
3s		1.948678	0.6032621	2.207002	5.676555	55.87638	891.1602				
3p		2.051648	0.5010398	0.4775365	2.753514	8.992129	148.5245	4061.817			
15 Phosphorus Atom		Quartet S State				Ne(10) 3s(2)3p(3) Configuration				(6s, 4p) Basis	
12-V/EI = 4.8D-09 Total S Cusp: -15.031754				1-Norm = 1.9D-16 Total P Cusp: -7.078585							
	T	V(1)	V(2)	V(3)	Z	r	r*	-Cusp	r max		
	340.709054	-812.436092	131.018000	-681.418091	-340.709058	-209.491037					
1s	106.217160	-218.604013	16.214214	-202.489795	-96.172635	-76.954417	15.050979	0.068102			
2s	15.869178	-42.471328	9.555804	-32.915524	-17.036346	-7.490542	14.773925	0.414893			
2p	14.762983	-40.564069	10.212821	-30.456246	-15.593265	-5.340444	7.420894	0.359278			
3s	1.878789	-10.402958	3.921065	-6.461793	-3.663034	-0.641938	14.962464	1.600714			
3p	1.400300	-8.688426	3.452966	-5.235460	-3.835160	-0.382194	7.377912	1.962307			
	T=3	T=2	T=1	Z	T=2	r*	r*				
	992.6780	54.16241	14.76800	28.83492	243.4261	3506.784					
1s		428.6697	14.57360	0.1039028	1.4488751D-02	5.5016711D-03	1.4146706D-04				
2s		33.01061	2.81422	0.5155529	0.3156778	0.1794733	0.1683359				
2p		62.46562	10.28912	2.704605	0.4934103	0.2913499	0.1826845	0.2209902			
3s		2.644711	0.6935219	1.924996	0.310738	31.23905	355.0287				
3p		3.3h665L	0.7310563	0.5792284	2.258944	5.935003	59.43056	031.6877			

Table 12. (Continued)

15	Phosphorus Atom	Quartet S State			Ne(10) 3s(2) 3p(3) Configuration			(9s, 6p) Basis	
		$ 2-V/Z = 2.80$ Total S Cusp: -15.025183			$ 1-Norm = 4.70-16$ Total P Cusp: -7.355186				
		T	V(1)	V(2)	V(T)	E	G	-Cusp	r max
		340.718706	-812.220577	130.783174	-681.437403	-340.718697	-209.935522		
1s	106.216330	-218.603129	16.208585	-202.394544	-96.178213	-79.969628	15.021921	0.068078	
2s	15.872784	-42.475508	9.545803	-32.929704	-17.056921	-7.511117	15.065879	0.418698	
2p	14.777952	-40.594224	10.207639	-30.386585	-15.608633	-5.400994	7.354270	0.356586	
3s	1.890428	-10.421425	3.917269	-6.509156	-4.613728	-0.696459	15.161415	1.568636	
3p	1.363970	-8.551703	3.398008	-5.153695	-3.789725	-0.391717	7.371360	1.843921	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ^0	Σ^1	Σ^2		
		992.6338	54.14804	14.97380	30.28339	290.6397	5135.406		
1s	428.6195	14.57354	0.1038048	1.4488361D-02	5.43655380D-04	3.9920737D-05			
2s	33.06095	2.831701	0.5156625	0.3159242	0.1801706	0.1688733			
2p	62.08818	10.28302	2.706282	0.4833996	0.2918019	0.1830307	0.2096196		
3s	2.713847	0.6947617	1.932560	6.346843	32.52840	393.0325			
3p	3.309090	0.7156770	0.5701136	2.323116	6.392689	74.70775	1449.248		
16	Sulfur Atom	Triplet P State			Ne(10) 3s(2) 3p(4) Configuration			(6s, 4p) Basis	
		$ 2-V/Z = 1.80-05$ Total S Cusp: -16.043080			$ 1-Norm = 3.10-16$ Total P Cusp: -8.028432				
		T	V(1)	V(2)	V(T)	E	G	-Cusp	r max
		397.497247	-947.138460	152.149863	-794.988597	-397.491350	-245.341487		
1s	121.178120	-249.060702	17.945215	-231.115487	-109.937366	-91.992151	16.056493	0.063778	
2s	18.725206	-49.178820	10.735147	-38.443673	-19.716467	-8.983320	15.860484	0.388841	
2p	17.569863	-47.242438	11.505842	-35.736596	-18.166733	-6.660891	8.040873	0.330628	
3s	2.459675	-12.557671	4.617226	-7.906464	-5.480969	-0.463743	16.112154	1.434844	
3p	1.838016	-10.522261	4.129908	-6.392353	-4.554337	-0.424429	7.830062	1.648938	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ^0	Σ^1	Σ^2		
		1139.638	59.19615	15.27839	27.81692	194.8681	2315.039		
1s	488.8872	15.56629	9.7148817D-02	1.2685731D-02	4.1785161D-04	4.6824689D-05			
2s	38.87934	3.073676	0.47566676	0.26844448	0.1294624	0.1024200			
2p	80.22668	12.20776	2.952652	0.4410498	0.2418674	0.1252288	0.1306682		
3s	3.519638	0.7868669	1.717474	3.512985	19.42398	172.1253			
3p	4.894278	0.9546963	0.6576413	2.012877	4.744402	38.75224	492.4899		
16	Sulfur Atom	Triplet P State			Ne(10) 3s(2) 3p(4) Configuration			(9s, 6p) Basis	
		$ 2-V/Z = 1.30-08$ Total S Cusp: -16.025568			$ 1-Norm = 4.40-16$ Total P Cusp: -7.891790				
		T	V(1)	V(2)	V(T)	E	G	-Cusp	r max
		397.504772	-946.895811	151.886274	-795.009538	-397.504766	-245.618493		
1s	121.180474	-249.063048	17.939174	-231.123874	-109.943400	-92.004226	16.022246	0.063754	
2s	18.719514	-49.170873	10.736313	-38.442726	-19.727767	-9.004134	16.056033	0.388285	
2p	17.588756	-47.271210	11.500040	-35.771170	-18.182414	-6.682373	7.884593	0.327737	
3s	2.471739	-12.578032	4.613443	-7.965589	-5.492799	-0.479356	16.180206	1.421111	
3p	1.807171	-10.411162	4.083393	-6.327769	-4.520598	-0.437205	8.001266	1.619732	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ^0	Σ^1	Σ^2		
		1139.500	59.16095	15.47466	29.15374	234.4578	3525.857		
1s	488.8215	15.56644	9.7150833D-02	1.2686473D-02	4.1626645D-04	4.6940874D-05			
2s	38.90303	3.073180	0.4757766	0.2685337	0.1293160	0.1021937			
2p	79.71800	12.20184	2.954551	0.4410406	0.2420896	0.1247812	0.1196402		
3s	3.535898	0.7861270	1.720545	3.442352	20.29039	192.6958			
3p	4.843623	0.9420325	0.6506976	2.060369	5.063514	48.21721	784.8857		
17	Cl Chlorine Atom	Doublet P State			Ne(10) 3s(2) 3p(5) Configuration			(6s, 4p) Basis	
		$ 2-V/Z = 5.90-08$ Total S Cusp: -17.051202			$ 1-Norm = 2.20-16$ Total P Cusp: -7.969611				
		T	V(1)	V(2)	V(T)	E	G	-Cusp	r max
		459.458338	-1094.670985	175.754336	-918.916649	-459.458311	-283.703975		
1s	137.128168	-281.506481	19.756769	-261.749713	-124.621545	-108.854776	17.061359	0.059970	
2s	21.890033	-56.380507	11.983947	-44.176560	-22.567557	-10.583610	16.901877	0.362244	
2p	20.601825	-54.368897	12.860345	-41.586462	-20.905918	-8.046472	7.975220	0.121321	
3s	3.082322	-14.120011	5.344440	-4.475571	-6.393248	-1.046808	17.269074	1.305064	
3p	2.361681	-12.616612	4.684391	-7.712222	-5.370541	-0.486150	7.890744	1.459297	
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ^0	Σ^1	Σ^2		
		1297.015	64.39241	15.59193	26.22674	151.0589	1470.815		
1s	553.0698	16.55920	9.1295673D-02	1.1199528D-02	3.2447789D-04	2.1210475D-05			
2s	45.20757	3.315324	0.44161749	0.23111415	0.5477793D-02	6.438343D-02			
2p	100.0214	14.26171	3.198176	0.4666259	0.2053457	0.4152045D-02	0.1720797		
3s	4.406613	0.8717653	1.556608	2.803272	13.06267	94.46842			
3p	6.767910	1.215398	0.7421537	1.794627	3.760688	24.33540	256.1425		

Table 12. (Continued)

17	Cl	Chlorine Atom	Doublet P State	He(10) 3s(2) 3p(5) Configuration				(9s, 6p) Basis							
[2-V/Z] = 2.2D-08 Total S Cusp: -17.025736															
[1-Norm] = 6.4D-16 Total P Cusp: -8.407281															
T	V(1)	V(2)	V(T)	Z	*	-Cusp	r max								
459.481809	-1094.358119	175.394510	-918.963609	-459.481799	-284.087289										
1s	137.136165	-281.514657	19.767277	-261.767380	-124.631215	-104.883939	17.022919	0.059947							
2s	21.796219	-56.344230	11.970486	-48.373748	-22.57527	-10.607080	17.055711	0.361993							
2p	20.632495	-54.425926	12.860811	-41.565115	-20.932620	-8.071809	8.392289	0.303273							
3s	3.108727	-14.873317	5.346088	-9.527233	-6.418506	-1.072422	17.198605	1.290413							
3p	2.320924	-12.467631	6.820390	-7.647241	-5.326316	-0.505927	8.609551	1.445810							
Σ^3		Σ^-		Σ^1		Σ^0		Σ^+							
	1296.953		64.37401	15.81656	27.60339	183.8916	2295.789								
1s	552.9963	16.55969	9.1295556D-02	1.120051D-02	3.2402374D-04	1.8672868D-05									
2s	45.21027	3.314366	0.4417199	0.2311974	9.5374994D-02	6.4544189D-02									
2p	100.3288	14.27680	3.201525	0.4057149	0.2043261	8.8265958D-02	7.2246622D-02								
3s	4.436038	0.8749010	1.555426	2.811850	13.88343	103.9302									
3p	6.779612	1.201482	0.7333900	1.841077	4.053788	31.28075	417.4653								
18	Argon Atom	Singlet S State	He(10) 3s(2) 3p(6) Configuration				(6s, 4p) Basis								
[2-V/Z] = 2.6D-07 Total S Cusp: -18.059978															
[1-Norm] = 1.8D-15 Total P Cusp: -9.072040															
T	V(1)	V(2)	V(T)	Z	*	-Cusp	r max								
526.795197	-1255.336564	201.746305	-1053.590259	-526.795062	-325.048757										
1s	150.075376	-315.989802	21.638592	-294.311209	-140.235834	-118.597241	18.068470	0.056590							
2s	25.115479	-64.012129	13.297782	-50.714387	-25.598668	-12.301086	17.947435	0.339140							
2p	23.906761	-62.056027	14.299552	-47.756075	-23.849298	-9.549324	9.101151	0.284339							
3s	3.781833	-17.273924	6.716670	-11.157054	-7.375221	-1.258351	18.383319	1.197016							
3p	2.901523	-14.754782	5.640017	-9.114765	-6.213242	-0.573225	8.673567	1.330655							
Σ^3		Σ^-		Σ^1		Σ^0		Σ^+							
	1465.340		69.74092	15.89295	24.95780	121.1281	982.2783								
1s	621.2432	17.55277	8.61022650D-02	9.9588716D-03	2.5590122D-04	1.3186154D-05									
2s	51.98545	3.556229	0.8122139	0.2012333	7.2111647D-02	4.2058598D-02									
2p	128.9925	16.52729	3.447557	0.3753963	0.1742068	6.3394738D-02	4.3646714D-02								
3s	5.399718	0.9596624	1.421750	2.3747840	9.081646	58.05251									
3p	8.999443	1.4866603	0.8197101	1.633406	3.135895	17.08662	145.6379								
18	Argon Atom	Singlet S State	He(10) 3s(2) 3p(6) Configuration				(9s, 6p) Basis								
[2-V/Z] = 1.5D-08 Total S Cusp: -18.025687															
[1-Norm] = 2.0D-15 Total P Cusp: -8.917114															
T	V(1)	V(2)	V(T)	Z	*	-Cusp	r max								
526.817028	-1255.071022	201.436974	-1053.634048	-526.817020	-325.380046										
1s	154.083517	-315.958046	21.632488	-294.325558	-140.242041	-118.609553	18.023428	0.056568							
2s	25.102045	-63.994895	13.285738	-50.709156	-25.607112	-12.321373	18.035259	0.339038							
2p	23.937266	-62.099199	14.295614	-47.803585	-23.866320	-9.570706	8.894105	0.282175							
3s	3.802612	-17.315905	6.118402	-11.197508	-7.398892	-1.276890	18.218028	1.183748							
3p	2.869514	-14.656356	5.598339	-9.058017	-6.188502	-0.590163	9.203995	1.306747							
Σ^3		Σ^-		Σ^1		Σ^0		Σ^+							
	1865.074		69.72617	16.06267	25.98869	144.0840	1491.566								
1s	621.1442	17.55322	8.6103706D-02	9.9597063D-03	2.5592678D-04	1.3197406D-05									
2s	51.98098	3.556272	0.8122867	0.2012310	7.1969587D-02	4.2232589D-02									
2p	128.2993	16.52428	3.489956	0.3753310	0.1743402	6.3361754D-02	4.1101867D-02								
3s	5.416362	0.9619947	1.421962	2.349268	9.382530	60.09784									
3p	8.994831	1.474265	0.8142420	1.661664	3.303622	20.79905	228.5665								
19	K	Potassium Atom	Doublet S State	Ar(18) 4s(1) Configuration				(8s, 4p) Basis							
[2-V/Z] = 1.5D-07 Total S Cusp: -18.061670															
[1-Norm] = 3.7D-16 Total P Cusp: -9.394533															
T	V(1)	V(2)	V(T)	Z	*	-Cusp	r max								
599.147398	-1423.023512	224.728627	-1198.294884	-599.147486	-374.418859										
1s	172.000026	-352.371938	23.422968	-328.948971	-156.948945	-133.525977	18.857434	0.053538							
2s	28.703337	-72.200410	14.506669	-57.693742	-28.990405	-18.483736	18.917861	0.317802							
2p	27.447801	-70.211665	15.625277	-54.586388	-27.138587	-11.513311	9.421151	0.26829							
3s	5.795166	-20.388210	6.923867	-13.860742	-8.665577	-1.782109	18.831746	1.091686							
3p	3.869057	-17.908555	6.545547	-11.363008	-7.493951	-0.948403	9.092230	1.179154							
4s	0.249189	-4.389073	1.997477	-2.391597	-2.142407	-0.144930	18.400386	4.221807							
Σ^3		Σ^-		Σ^1		Σ^0		Σ^+							
	1642.119		74.89597	19.59614	54.59116	2411.787	271608.1								
1s	692.2692	18.54589	8.1452881D-02	8.9219516D-03	1.1610335D-03	0.1970928									
2s	59.23132	3.800022	0.3865665	0.1775398	0.1434715	18.16242									
2p	152.1247	18.92639	3.6945351	0.3494951	0.1507307	8.6955673D-02	2.6056755D-02								
3s	6.777832	1.072853	1.277864	1.883080	5.953920	57.41210									
3p	12.91777	1.956643	0.9425555	1.422749	2.366988	59.527785									
4s	0.2770037	0.2310039	5.470902	35.34577	2342.141	271101.2									

Table 12. (Continued)

19	Potassium Atom	Doublet S State			Ar(18)4s(1) Configuration			(12s, 6p) Basis		
		T	V(1)	V(2)	V(T)	Z	e	-Cusp	r max	
{2-V/Z} = 1.5D-09 Total S Cusp: -19.005836										
1s	599.164566	-1422.963291	224.634158	-1198.329133	-599.164567	-374.530409				
2s	28.663569	-72.156261	18.501443	-57.654819	-28.991250	-18.489808	18.986372	0.318827		
2p	27.470103	-70.243382	15.627067	-54.616316	-27.146212	-11.519185	9.404983	0.263866		
3s	6.814097	-20.822256	6.929775	-13.492481	-8.678385	-1.748610	18.955879	1.082359		
3p	3.848413	-17.841901	6.521618	-11.320283	-7.475865	-0.958267	9.567913	1.164861		
4s	0.264439	-4.495093	2.041598	-2.453895	-2.189056	-0.147457	18.990606	4.342923		
		Σ^2	Σ^-2	Σ^1	Σ^0	Σ^2	Σ^0	Σ^0		
		1644.179	74.89280	19.44982	51.21887	1706.546	128758.2			
1s	693.2549	18.54735	8.1466519D-02	8.9134911D-03	2.0840315D-04	6.5846087D-04				
2s	59.25327	3.797698	0.3863912	0.1765980	5.5428552D-02	7.0861324D-02				
2p	151.7358	18.92578	3.697020	0.3894274	0.1508014	4.7060072D-02	2.5790760D-02			
3s	6.817566	1.074856	1.277067	1.883531	5.849312	28.89977				
3p	12.97055	1.946853	0.9390478	1.436681	2.439839	10.73114	78.45815			
4s	0.2951501	0.2365839	5.243322	31.53695	1630.067	128229.4				
{2-V/Z} = 3.2D-06 Total S Cusp: -20.058110										
20	Calcium Atom	Singlet S State			Ar(18)4s(2) Configuration			(8s, 4p) Basis		
		T	V(1)	V(2)	V(T)	Z	e	-Cusp	r max	
1s	676.745597	-1603.449527	249.960495	-1353.489032	-676.743635	-426.782940				
2s	190.964375	-390.831778	25.258368	-365.573410	-174.609035	-149.350668	20.059035	0.050850		
2p	32.477135	-80.822914	15.768362	-65.058553	-32.577418	-16.809056	19.963541	0.300790		
3s	31.233096	-78.861836	17.006788	-61.855088	-30.622042	-13.615294	9.795464	0.248032		
3p	5.920201	-23.687301	7.770768	-15.916533	-9.996332	-2.225564	20.685430	1.007590		
4s	0.499391	-21.226203	7.482475	-13.743728	-8.806209	-1.323738	9.525485	1.063786		
		-6.118655	2.715083	-3.403572	-2.904181	-0.189097	21.173675	3.432077		
		Σ^2	Σ^-2	Σ^1	Σ^0	Σ^2	Σ^0	Σ^0		
		1834.692	80.17248	20.92369	53.24249	1182.162	52280.65			
1s	769.5065	19.54159	7.7300669D-02	8.0226649D-03	1.6627389D-04	4.5566642D-05				
2s	67.03826	4.041186	0.3635003	0.1562012	4.3366228D-02	4.1131850D-02				
2p	182.9927	21.48758	3.943092	0.3269165	0.1316932	3.5615776D-02	1.5917545D-02			
3s	8.390152	1.184365	1.163773	1.556820	5.143376	38.45585				
3p	17.63857	2.467560	1.061310	1.266561	1.869690	5.853062	27.98163			
4s	0.5456185	0.3059327	0.076839	18.89645	569.2280	26017.83				
{2-V/Z} = 2.9D-09 Total S Cusp: -20.006697										
20	Calcium Atom	Singlet S State			Ar(18)4s(2) Configuration			(12s, 6p) Basis		
		T	V(1)	V(2)	V(T)	Z	e	-Cusp	r max	
1s	676.757143	-1603.203436	249.689148	-1353.514288	-676.757145	-427.067997				
2s	190.970478	-390.838377	25.252024	-365.586353	-174.615475	-149.363451	20.066679	0.050829		
2p	32.47251	-80.817365	15.761178	-65.056187	-32.583536	-16.822358	19.987064	0.300799		
3s	31.245500	-78.879380	17.002489	-61.876892	-30.631392	-13.628903	9.866255	0.247691		
3p	5.956582	-23.778973	7.786706	-15.988267	-10.031725	-2.245019	19.956239	0.996868		
4s	0.9177906	-21.177420	7.459652	-13.717768	-8.799862	-1.380210	10.132201	1.056101		
		-6.000601	2.658248	-3.342358	-2.854074	-0.195830	19.983525	3.472454		
		Σ^2	Σ^-2	Σ^1	Σ^0	Σ^2	Σ^0	Σ^0		
		1834.803	80.16017	21.24388	56.53172	1424.831	72163.01			
1s	769.3793	19.54192	7.7300663D-02	8.0231921D-03	1.67752110D-04	2.7704533D-04				
2s	67.03438	0.040868	0.3635034	0.1561733	4.3118709D-02	3.5827453D-02				
2p	182.8650	21.48875	3.943969	0.3268738	0.1317160	3.5671116D-02	1.6884110D-02			
3s	8.395973	1.188749	1.159437	1.546094	3.853961	14.80655				
3p	17.76096	2.463060	1.058871	1.275403	1.918740	6.548427	38.82472			
4s	0.5365927	0.3000301	4.214870	20.41620	688.7660	35951.34				

Table 12. (Continued)

21	Scandium Atom	Doublet D State			Ar(18) 3d(1)4s(2) Configuration			(8s, 4p, 3d) Basis		
		$ 2-\nabla/E = 2.6D-06$			$ 1-\text{Norm} = 5.1D-16$					
		Total S Cusp:	-20.857402		Total P Cusp:	-10.272456		Total D Cusp:	-5.306532	
		T	V(1)	V(2)	V(T)	E	e	-Cusp	r max	
		759.719897	-1799.890619	280.452769	-1519.837849	-759.717952	-879.265183			
1s	210.883798	-631.249679	27.236122	-804.013557	-193.129759	-165.893637	20.860621	0.048376		
2s	36.573014	-90.021351	17.185011	-72.836380	-36.263327	-19.078316	20.825511	0.283468		
2p	35.257052	-87.996840	18.536180	-69.460660	-34.203608	-15.667428	10.302110	0.233661		
3s	6.955790	-26.892793	8.686318	-18.206476	-11.250486	-2.568169	20.827413	0.936178		
3p	5.864466	-24.287287	8.405696	-15.841552	-9.977106	-1.571011	9.996762	0.983238		
3d	3.070433	-16.773086	6.680494	-10.092592	-7.021659	-0.381165	5.306532	1.135380		
4s	0.547188	-6.662680	2.953059	-3.709621	-3.162432	-0.209373	20.561045	3.188911		
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ	Σ^2	Σ^3	Σ^4		
		2031.695	85.70908	21.58488	55.02059	1402.313	83036.62			
1s	848.8337	20.53570	7.3522371D-02	7.26072290D-03	3.4148462D-04	2.5843605D-02				
2s	75.18829	4.286731	0.383638	0.1401039	8.9704988D-02	6.862830				
2p	218.2738	24.21907	4.190326	0.3071233	0.1160741	2.7549940D-02	1.1460331D-02			
3s	9.775644	1.280609	1.079590	1.339012	2.911522	13.93964				
3p	22.26633	2.916774	1.154631	1.167379	1.568391	4.216427	17.05545			
3d	1.417980	0.8845160	0.7987184	1.670181	3.579294	29.71851	437.6016			
4s	0.5999457	0.3172705	4.037266	19.12088	670.5636	4.117.65				
24	Cr	Chromium Atom	Septuplet S State			Ar(18) 3d(5)4s(1) Configuration			(8s, 4p, 3d) Basis	
			$ 2-\nabla/E = 1.3D-04$			$ 1-\text{Norm} = 3.1D-16$				
			Total S Cusp:	-23.846223		Total P Cusp:	-11.386543		Total D Cusp:	-5.372573
			T	V(1)	V(2)	V(T)	E	e	-Cusp	
		1042.200231	-2477.183827	392.689718	-2084.534107	-1042.333876	-649.684162			
1s	276.721267	-564.580581	33.613758	-530.966823	-254.245556	-220.631798	23.855166	0.042261		
2s	50.157527	-120.408048	21.884877	-48.523171	-48.365644	-26.480767	23.763208	0.244156		
2p	48.764256	-118.317995	23.572929	-94.785066	-45.980810	-22.407881	11.407520	0.197802		
3s	10.265393	-37.110257	11.683171	-25.427086	-15.161693	-3.478522	23.733232	0.784702		
3p	8.897261	-34.010940	11.442246	-22.566894	-13.671433	-2.229186	11.213731	0.01633		
4s	0.788475	-9.021788	3.987062	-5.034726	-4.286251	-0.259190	23.598269	2.703288		
3d	4.230855	-21.998130	8.881598	-13.156532	-8.925677	-0.088080	5.372573	0.875784		
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ	Σ^2	Σ^3	Σ^4		
		2692.943	103.2160	21.08757	39.69751	521.5379	17334.67			
1s	1112.611	23.52179	6.4136037D-02	5.5195990D-03	1.03747370D-04	1.9163262D-03				
2s	102.7659	5.017002	0.2941976	0.1025979	4.0968607D-02	1.699993				
2p	349.0249	33.35784	4.929916	0.2602934	8.3216350D-02	1.4359949D-02	6.2704936D-03			
3s	14.38553	1.546261	0.9023238	0.9368879	1.8417133	5.300880				
3p	39.45412	6.379534	1.817122	0.9600201	1.080563	1.996269	5.759512			
4s	0.8729656	0.3759078	3.426731	13.75825	339.5470	14003.82				
3d	2.399488	1.224611	0.9165888	1.563529	3.373315	33.39230	656.4505			
25	Eu	Banganese Atom	Hextuplet S State			Ar(18) 3d(5)4s(2) Configuration			(8s, 4p, 3d) Basis	
			$ 2-\nabla/E = 5.2D-06$			$ 1-\text{Norm} = 2.9D-16$				
			Total S Cusp:	-24.851969		Total P Cusp:	-12.387922		Total D Cusp:	-6.809069
			T	V(1)	V(2)	V(T)	E	e	-Cusp	
		1149.840774	-2735.392604	435.717012	-2299.675592	-1149.830818	-714.117806			
1s	300.691919	-613.048283	35.917895	-577.130388	-276.439869	-240.521574	24.864128	0.040547		
2s	55.111.596	-131.454047	23.621476	-107.832571	-52.719975	-29.098499	24.730534	0.233393		
2p	53.710589	-129.369271	25.427186	-103.982085	-50.231536	-24.804350	12.443990	0.189973		
3s	11.335881	-40.593789	12.726325	-27.867464	-16.531583	-3.805257	24.759092	0.746862		
3p	9.910890	-37.386345	12.508208	-24.882137	-14.971646	-2.467383	11.923368	0.766112		
3d	6.471806	-29.194272	11.046674	-18.187597	-11.675791	-0.629117	6.809069	0.785875		
4s	0.738358	-9.347655	8.181940	-5.165715	-4.427356	-0.245416	24.770624	2.735041		
		Σ^{-3}	Σ^{-2}	Σ^{-1}	Σ	Σ^2	Σ^3	Σ^4		
		2935.815	109.4157	21.75565	42.98042	559.6554	22943.22			
1s	1208.890	24.52193	6.1517490D-02	5.0770389D-03	8.7021337D-05	1.2899081D-03				
2s	112.8108	5.258162	0.2809156	9.33515919D-02	3.21868950D-02	1.048830				
2p	404.5828	36.74304	5.174771	0.2475816	7.5165558D-02	1.18429564D-02	3.0597276D-03			
3s	15.89049	1.623752	0.8604677	0.8523308	1.175345	2.596810				
3p	46.37952	4.879824	1.495454	0.9080023	0.9626566	1.551620	3.815395			
3d	4.139456	1.850276	1.167771	1.124870	1.612762	6.068127	42.06938			
4s	0.8224293	0.3739062	3.396000	13.39391	308.7606	11351.33				

Table 12. (Continued)

29	Cu	Copper Atom	Doublet S State			Ar(1s)3d(10)4s(1) Configuration			(8s, 4p, 3d) Basis		
			$ 2-\nabla/E = 3.2D-05$			$ \Gamma-\text{Norm} = 3.7D-16$					
			Total S Cusp:	-28.845242		Total P Cusp:	-14.523154		Total D Cusp:	-7.380582	
			T	V(1)	V(2)	V(T)	E	e	-Cusp	r max	
			1638.828038	-3929.582035	651.873869	-3277.708166	-1638.880128	-987.006258			
1s	406.498381	-826.849685	85.795819	-781.054266	-374.555884	-328.760465	28.855091	0.034897			
2s	77.386193	-180.618298	31.223997	-149.394301	-72.008108	-40.784112	28.817461	0.198608			
2p	75.933332	-178.485789	33.484671	-145.001118	-69.067786	-35.583115	18.620179	0.160147			
3s	16.362138	-56.365832	17.517309	-38.888523	-22.486385	-4.969076	28.335242	0.629255			
3p	14.650832	-52.608429	17.336998	-35.271431	-20.620599	-3.283601	13.750842	0.633161			
3d	9.416209	-40.438163	15.283501	-25.154662	-15.738453	-0.654952	7.380582	0.613264			
4s	0.667546	-10.967471	5.035396	-5.932075	-5.260529	-0.229132	28.711186	2.708303			
			Γ^{-3}	Γ^{-2}	Γ^{-1}	Γ	Γ^2	Γ^4	Γ^6		
			4010.136	135.5028	20.86858	31.08151	331.2857	10126.20			
1s		1633.208	28.51206	5.28714890-02	3.7475632D-03	4.9958974D-05	7.6482960D-04				
2s		158.1413	6.228217	0.2376165	6.6687280D-02	1.1634136D-02	0.21338300				
2p	674.0800	51.80923	6.154682	0.2075905	5.2734597D-02	5.6052223D-03	1.05155110-03				
3s		22.86378	1.943649	0.7236100	0.6047464	0.6573753	4.361818				
3p	81.30670	7.182696	1.814084	0.7530005	0.6641529	0.7455596	1.285138				
3d	7.407473	2.699573	1.394419	0.9757738	1.261087	4.258261	28.91314				
4s		0.7620723	0.3781887	3.319098	12.81896	282.8580	9820.198				
30	Zn	Zinc Atom	Singlet S State			Ar(1s)3d(10)4s(2) Configuration			(8s, 4p, 3d) Basis		
			$ 2-\nabla/E = 7.3D-07$			$ \Gamma-\text{Norm} = 1.3D-15$					
			Total S Cusp:	-29.840743		Total P Cusp:	-15.005522		Total D Cusp:	-7.964649	
			T	V(1)	V(2)	V(T)	E	e	-Cusp	r max	
			1777.781456	-4262.488097	706.926488	-3555.561609	-1777.780153	-1070.853666			
1s	435.436477	-885.286297	48.285468	-837.000828	-401.564351	-353.278883	29.854941	0.033723			
2s	83.529370	-194.102265	33.118382	-160.983883	-77.458513	-44.336131	29.755241	0.191366			
2p	82.075166	-191.973619	35.499130	-156.874488	-78.399323	-38.90192	15.10229	0.154031			
3s	18.000183	-61.109701	18.750788	-62.358913	-28.358730	-5.607942	29.396585	0.602359			
3p	16.225721	-57.267441	18.615725	-38.651716	-22.425996	-3.910271	18.277231	0.604647			
3d	11.206623	-45.973960	17.005031	-28.968929	-17.762307	-0.757275	7.964649	0.578057			
4s	0.988525	-13.152804	5.938883	-7.213920	-6.228995	-0.286111	29.566787	2.319211			
			Γ^{-3}	Γ^{-2}	Γ^{-1}	Γ	Γ^2	Γ^4	Γ^6		
			4307.857	142.0829	21.90471	34.46551	363.1574	9257.971			
1s		1749.255	29.50958	5.1076689D-02	3.4967342D-03	3.7364013D-05	2.7634844D-06				
2s		170.5289	6.470076	0.2288782	6.1893763D-02	1.0658119D-02	0.1875011				
2p	755.6069	55.96243	6.399121	0.1995597	4.8718219D-02	4.7789466D-03	8.2512028D-04				
3s		25.14047	2.036990	0.6910245	0.5505609	0.5152293	1.798298				
3p	93.63238	7.933745	1.908915	0.7152592	0.5983550	0.6011601	0.9227161				
3d	9.371680	3.200879	1.532465	0.8677248	0.9755015	2.384128	11.47688				
4s		1.113416	0.43084269	2.898295	9.798074	167.3183	4566.851				
31	Gallium Atom	Doublet P State	Ar(1s)3d(10)4s(2)4p(1) Configuration			(8s, 6p, 3d) Basis					
			$ 2-\nabla/E = 2.9D-07$			$ \Gamma-\text{Norm} = 5.7D-16$					
			Total S Cusp:	-30.838202		Total P Cusp:	-15.512165		Total D Cusp:	-8.534074	
			T	V(1)	V(2)	V(T)	E	e	-Cusp	r max	
			1923.159351	-4609.929631	763.611492	-3846.318140	-1923.158789	-1159.547297			
1s	465.376472	-945.724774	50.797657	-854.927116	-429.550645	-378.752987	30.857671	0.032628			
2s	89.891136	-208.051325	35.028857	-173.022668	-83.131332	-48.102476	30.681135	0.184642			
2p	88.452157	-205.946139	37.531811	-168.418328	-79.62170	-42.430359	15.642225	0.148842			
3s	19.780366	-66.139890	20.019152	-46.120738	-26.339771	-6.320619	30.527714	0.576570			
3p	17.838526	-61.977449	19.866464	-42.11042	-28.272516	-8.806110	18.627223	0.578335			
3d	13.121513	-51.679221	18.716876	-32.962345	-19.840832	-1.123956	8.534074	0.543662			
4s	1.410158	-15.634041	6.920155	-8.713886	-7.303728	-0.383573	30.575530	2.043272			
4p	1.282556	-14.495839	6.521780	-7.974059	-6.691403	-0.169623	12.577706	2.258240			
			Γ^{-3}	Γ^{-2}	Γ^{-1}	Γ	Γ^2	Γ^4	Γ^6		
			4615.539	148.7074	22.63959	35.00407	296.0049	5367.904			
1s		1869.352	30.50725	4.9400014D-02	3.2703353D-03	2.9232596D-05	6.8124800D-05				
2s		183.3388	6.711333	0.2207802	5.7615115D-02	9.6656769D-03	0.1380431				
2p	843.7378	60.27617	6.643824	0.1921233	4.5143350D-02	4.1335681D-03	1.3555675D-03				
3s		27.62608	2.133565	0.6605362	0.5020527	0.4106859	0.6322315				
3p	106.8172	8.707907	1.999273	0.6454078	0.5519309	0.5438276	1.290495				
3d	11.61397	3.733341	1.667072	0.7842662	0.7839374	1.461103	5.246643				
4s		1.568341	0.5043239	2.535109	7.462589	95.66381	1953.470				
4p	6.605828	0.5295687	0.9676077	2.600087	7.566797	85.93771	1399.205				

Table 12. (Continued)

31	Gallium Atom	Doublet P State	Ar(18)3d(10)4s(2)4p(1) Configuration				(1s, 9p, 8d) Basis	
			$ Z-V/E = 8.8D-09$				$\{1\text{-Norm}\} = 1.0D-15$	
		Total S Cusp: -31.011532	Total P Cusp: -15.478789				Total D Cusp: -9.619867	
		T	$V(1)$	$V(2)$	$V(T)$	E	e	-Cusp
		1923.259894	-4608.214406	761.694601	-3846.519805	-1923.259911	-1161.565310	r max
1s	465.454933	-945.801504	50.764831	-895.036672	-429.581740	-378.816909	31.011960	0.032622
2s	89.806530	-207.970577	34.998598	-172.971979	-63.165448	-48.166850	31.020491	0.185281
2p	68.516154	-206.023408	37.507438	-168.515970	-79.999816	-42.492377	15.419652	0.148005
3s	19.811756	-66.189639	19.992406	-46.197234	-26.385477	-6.393072	30.930846	0.573203
3p	17.872358	-62.080865	19.863894	-42.216951	-24.344593	-4.480699	15.858684	0.573885
3d	13.100111	-51.610357	18.659230	-32.951127	-19.851016	-1.191786	9.618867	0.530280
4s	1.500056	-15.977620	7.027073	-8.950547	-7.450491	-0.423418	30.928395	1.998749
4p	0.781154	-11.606636	5.308492	-6.298184	-5.516989	-0.208497	16.377728	2.667877
		E^{-3}	E^{-2}	E^{-1}	E	E^2	E^3	E^4
		4620.458	148.6521	23.35959	40.52678	520.4434	15833.89	
1s		1871.200	30.50973	4.9407745D-02	3.2708234D-03	2.7343020D-05	3.4216600D-06	
2s		183.7540	6.708728	0.2206332	5.735264D-02	5.7098992D-03	1.4979742D-03	
2p	841.6717	60.27602	6.645916	0.1920862	4.5149669D-02	4.1045338D-03	6.5685300D-04	
3s		27.75816	2.135150	0.6604676	0.5026458	0.40771123	0.5022711	
3p	108.24C2	8.732772	2.002608	0.6806698	0.5505713	0.5295840	0.8388584	
3d	11.64535	3.729706	1.664850	0.7879965	0.7965059	1.565849	6.269046	
4s		1.669477	0.5154071	2.489596	7.209955	90.31731	1831.680	
4p	2.928782	0.3455349	0.3744076	3.382482	13.44034	320.1228	12101.82	
32	Germanium Atom	Triplet P State	Ar(18)3d(10)4s(2)4p(2) Configuration				(8s, 6p, 3d) Basis	
			$ Z-V/E = 1.9D-07$				$\{1\text{-Norm}\} = 4.4D-16$	
		Total S Cusp: -31.841506	Total P Cusp: -16.045836				Total D Cusp: -9.087922	
		T	$V(1)$	$V(2)$	$V(T)$	E	e	-Cusp
		2075.253267	-4972.463907	821.957728	-4150.506179	-2075.252893	-1253.295165	r max
1s	496.3198E0	-1008.166501	53.335883	-954.830618	-458.510758	-405.174875	31.864702	0.031594
2s	96.4780E5	-222.471224	36.959602	-185.511622	-89.307527	-52.077924	31.625564	0.178445
2p	95.0553E3	-220.391955	39.584757	-180.807197	-85.751808	-46.167047	16.198896	0.143269
3s	21.691675	-71.429340	21.313049	-50.116291	-28.424616	-7.111567	31.678075	0.552551
3p	19.686764	-67.179905	21.208131	-45.971774	-26.285010	-5.076879	15.037788	0.553730
3d	15.104E74	-57.453752	20.3994962	-37.058790	-21.954316	-1.559354	9.087922	0.512229
4s	1.895019	-18.218283	7.909125	-10.309158	-8.414139	-0.505014	31.692432	1.848081
4p	1.497155	-15.962268	7.107728	-8.854540	-7.357385	-0.249657	13.793105	2.124681
		E^{-3}	E^{-2}	E^{-1}	E	E^2	E^3	E^4
		4935.177	155.3895	23.45031	36.24701	272.6996	4018.070	
1s		1993.536	31.50520	4.7830547D-02	3.0653797D-03	2.4648047D-05	1.8552520D-05	
2s		196.6360	6.952226	0.2132193	5.3730553D-02	8.1069871D-03	8.4588182D-02	
2p	938.6977	64.74869	6.887249	0.1852268	4.1941521D-02	3.5765826D-03	1.382509D-03	
3s		30.29828	2.232167	0.6322092	0.4598929	0.3388629	0.3781392	
3p	122.1507	9.583728	2.099372	0.6527825	0.4997597	0.4881747	1.161848	
3d	14.08631	4.283204	1.795430	0.7191022	0.6509390	0.9651535	2.691355	
4s		2.088233	0.5693213	2.253902	5.860166	57.47645	883.1138	
4p	5.786543	0.6203882	0.4988209	2.468853	6.866852	72.34534	1108.512	
33	As	Arsenic Atom	Quartet S State	Ar(18)3d(10)4s(2)4p(3) Configuration				(8s, 6p, 3d) Basis
			$ Z-V/E = 7.5D-06$				$\{1\text{-Norm}\} = 4.4D-16$	
		Total S Cusp: -33.090210	Total P Cusp: -16.5040729				Total D Cusp: -9.608061	
		T	$V(1)$	$V(2)$	$V(T)$	E	e	-Cusp
		2234.144774	-5351.466305	883.193542	-4468.272763	-2234.127990	-1350.938447	r max
1s	528.302270	-1072.645469	55.915793	-1016.729677	-488.427406	-432.511614	33.081410	0.030636
2s	103.222152	-237.318708	38.932958	-198.385750	-95.163598	-56.230643	32.942782	0.173276
2p	101.913630	-235.345988	41.680040	-193.665944	-91.752253	-50.072213	16.697737	0.138384
3s	23.612101	-76.774195	22.602238	-54.171957	-30.530756	-7.928519	34.481850	0.534509
3p	21.678134	-72.680183	22.610037	-50.070146	-28.391962	-5.781925	15.503502	0.530682
3d	17.176552	-63.362406	22.080735	-41.281671	-24.105110	-2.024375	9.608061	0.483491
4s	2.517701	-21.283764	9.068981	-12.215783	-9.697082	-0.628101	34.970686	1.679831
4p	1.820421	-17.880326	7.868599	-10.011727	-8.191306	-0.322707	14.720147	1.966482
		E^{-3}	E^{-2}	E^{-1}	E	E^2	E^3	E^4
		5271.647	162.1656	24.03072	36.23038	237.8253	2932.028	
1s		2123.725	32.50441	4.6363104D-02	2.8792335D-03	2.1435895D-05	5.1130367D-06	
2s		210.6663	7.191476	0.2059752	4.9966001D-02	4.3420348D-03	1.0250415D-03	
2p	1040.207	69.38514	7.131696	0.1787963	3.9065838D-02	3.0975289D-03	1.0721716D-03	
3s		33.24419	2.326491	0.6079387	0.4248933	0.3026213	0.6469314	
3p	139.2760	10.52364	2.202430	0.6221588	0.4531636	0.3655831	0.8735797	
3d	16.81881	4.856033	1.920073	0.6659728	0.5530090	0.6733317	1.508935	
4s		2.755687	0.6449625	1.976750	4.439418	31.09627	326.9538	
4p	7.4486580	0.7505057	0.5418280	2.297073	5.970866	55.35779	752.1624	

Table 12. (Continued)

34	Selenius Atom	Triplet P State	$\text{Ar}(18)3d(10)4s(2)4p(8)$ Configuration						(8s, 6p, 3d) Basis									
$ 2-\nabla/Z = 5.3D-06$																		
Total S Cusp: -3s.101477 Total P Cusp: -16.115989 Total D Cusp: -10.095927																		
$ \Gamma-\text{Norm} = 5.3D-16$																		
T	V(1)	V(2)	V(T)	E	* ^o	-Cusp	r max											
2399.781802	-5744.349683	944.773411	-4799.576272	-2399.794471	-1455.021060													
1s	561.229087	-1139.070621	58.510285	-1080.560376	-519.331289	-860.821048	38.087427	0.029727										
2s	110.316223	-252.757468	40.904786	-211.852703	-101.536480	-60.631738	34.019353	0.167737										
2p	109.057847	-250.841662	43.780535	-207.061127	-98.003280	-54.222745	16.088103	0.132640										
3s	25.778568	-82.539326	23.938026	-58.601300	-32.822732	-8.884707	35.408507	0.518674										
3p	23.843792	-78.583646	24.058816	-54.528830	-30.681038	-6.622222	16.283541	0.505646										
3d	19.312643	-69.376239	23.724530	-45.649709	-26.337066	-2.612536	10.095927	0.457873										
4s	3.113566	-23.938481	10.006065	-13.932415	-10.818849	-0.812784	35.525969	1.558283										
4p	2.092662	-19.360923	8.483460	-10.917463	-8.824801	-0.381341	16.261689	1.864238										
Γ^{-3}	Γ^{-2}	Γ^{-1}	Γ	Γ^2	Γ^*	Γ^*	Γ^*											
5613.051	168.9515	25.08175	39.02913	270.9303	3555.684													
1s	2255.870	33.50208	4.6977522D-02	2.7094182D-03	1.9145538D-05	6.2789429D-06												
2s	225.4070	7.434083	0.1993420	4.6796652D-02	3.7834157D-03	4.9243232D-04												
2p	1135.940	74.03579	7.377696	0.1728677	3.6643515D-02	4.3699630D-03	3.7826689D-02											
3s	36.19050	2.827627	0.5831493	0.3904775	0.2506487	0.4178966												
3p	158.7526	11.53205	2.311284	0.5928390	0.4108895	0.2893522	0.4136633											
3d	19.78967	5.485359	2.040419	0.6218229	0.4782565	0.4884403	0.8975558											
4s	3.391776	0.7040730	1.828013	3.798630	22.73316	203.8438												
4p	9.278179	0.8661295	0.5694389	2.240180	5.771036	58.57713	783.8590											
35	Bromine Atom	Doublet P State	$\text{Ar}(18)3d(10)4s(2)4p(5)$ Configuration						(8s, 6p, 3d) Basis									
$ 2-\nabla/Z = 4.1D-06$																		
Total S Cusp: -3s.111234 Total P Cusp: -16.620335 Total D Cusp: -10.520769																		
T	V(1)	V(2)	V(T)	E	* ^o	-Cusp	r max											
2572.361427	-6155.320454	1010.587159	-5144.733295	-2572.371869	-1561.784710													
1s	595.150342	-1207.490242	61.160812	-1146.329430	-551.179088	-490.018276	35.092488	0.028870										
2s	117.647729	-268.683902	42.934890	-225.794812	-108.101703	-65.167213	35.089325	0.162545										
2p	116.400933	-266.781698	45.934148	-220.847550	-104.486647	-58.512499	16.591201	0.128452										
3s	28.035687	-88.546859	25.339963	-63.206896	-35.171209	-9.831246	36.321569	0.496078										
3p	26.072086	-84.567557	25.525505	-59.02053	-32.969967	-7.484462	16.787386	0.485993										
3d	21.563101	-75.593907	25.421114	-50.172793	-28.609692	-3.188578	10.520769	0.436600										
4s	3.696669	-26.563597	10.938732	-15.601865	-11.905196	-0.970464	36.298923	1.462328										
4p	2.566333	-21.754131	9.375622	-12.378508	-9.812175	-0.436553	16.800817	1.711967										
Γ^{-3}	Γ^{-2}	Γ^{-1}	Γ	Γ^2	Γ^*	Γ^*	Γ^*											
5966.555	175.8663	25.60869	38.84096	243.1521	2837.863													
1s	2391.987	38.49972	4.3672246D-02	2.5541791D-03	1.718563D-05	6.5183215D-06												
2s	280.3221	7.676683	0.1931167	4.3917112D-02	3.3380980D-03	4.6039635D-08												
2p	1251.017	78.99024	7.622334	0.1672397	3.8273009D-02	3.6207250D-03	2.6337716D-02											
3s	39.29559	2.529910	0.5601253	0.3598871	0.2096484	0.2798345												
3p	179.3562	12.57857	2.416216	0.5667917	0.3756505	0.2409429	0.3264742											
3d	23.06984	6.064745	2.159826	0.5837029	0.4187238	0.3656907	0.5647981											
4s	4.010799	0.7581885	1.707330	3.311691	17.19113	133.0639												
4p	11.96234	1.052587	0.6215466	2.071797	8.951617	40.64808	512.6819											
36	Krypton Atom	Singlet S State	$\text{Ar}(18)3d(10)4s(2)4p(6)$ Configuration						(8s, 6p, 3d) Basis									
$ 2-\nabla/Z = 4.9D-08$																		
Total S Cusp: -3s.871318 Total P Cusp: -17.127311 Total D Cusp: -10.905569																		
T	V(1)	V(2)	V(T)	E	* ^o	-Cusp	r max											
2751.989705	-6583.273757	1079.294211	-5503.979546	-2751.989841	-1672.695630													
1s	630.059206	-1277.898942	63.856288	-1214.042658	-583.983452	-520.127168	35.896646	0.028089										
2s	125.197293	-285.067106	44.995849	-240.065257	-114.867964	-69.872114	35.591191	0.157514										
2p	123.989407	-283.218087	48.126172	-235.091915	-111.102509	-62.976337	17.097850	0.128521										
3s	30.499081	-94.953121	26.814367	-68.138753	-37.639673	-10.825305	35.997292	0.874985										
3p	28.437768	-90.823180	27.039109	-63.780471	-35.346303	-8.307195	17.289320	0.867649										
3d	23.901073	-81.977076	27.136750	-54.840326	-30.939253	-3.802502	10.905569	0.513816										
4s	4.265358	-29.010629	11.804773	-17.205856	-12.940498	-1.135725	35.981787	1.386601										
4p	3.062341	-24.234632	10.332080	-13.902552	-10.840212	-0.508132	17.324608	1.585128										
Γ^{-3}	Γ^{-2}	Γ^{-1}	Γ	Γ^2	Γ^*	Γ^*	Γ^*											
6325.922	182.8687	26.04892	38.32276	215.1377	2220.567													
1s	2530.292	35.49719	4.2437801D-02	2.4119878D-03	1.4998238D-05	2.5488623D-06												
2s	254.9010	7.918364	0.1873816	4.1404022D-02	3.6664257D-03	9.5430563D-03												
2p	1373.778	88.11003	7.867169	0.1619606	3.2122040D-02	3.0320564D-03	1.8240084D-02											
3s	42.50410	2.637587	0.5379437	0.3320105	0.1738961	0.1486396												
3p	201.8965	13.68465	2.522866	0.5428357	0.3440575	0.2000359	0.2380125											
3d	26.64155	6.707319	2.277141	0.5506901	0.3707214	0.2810035	0.3716588											
4s	8.609365	0.8058508	1.619383	2.985747	14.04131	98.62253												
4p	14.85370	1.248731	0.6731842	1.923157	4.272553	30.44525	336.2919											

Table 12. (Continued)

37	Rubidium Atom	Doublet S State		Kr(36)5s(1) Configuration				(10s, 6p, 3d) Basis		
		T	V(1)	V(2)	V(T)	E	r	-Cusp	r max	
12-V/E1 = 2.7D-08										
		Total S Cusp:	-36.842977	Total P Cusp:	-17.702223	Total D Cusp:	-11.27893			
		T	V(1)	V(2)	V(T)	E	r	-Cusp	r max	
2938.301818	-7018.504513	1141.900797	-5876.603717	-2938.301898	-1796.401102					
1s	665.93906%	-1350.277837	66.456431	-1283.821400	-617.482331	-551.425899	36.870510	0.027288		
2s	133.05967%	-302.023139	46.968260	-255.054979	-121.995200	-75.026940	36.595286	0.152839		
2p	131.82251%	-300.199278	50.221802	-249.927476	-118.104963	-67.883161	17.688251	0.120887		
3s	32.97628%	-101.408714	28.157570	-73.251146	-40.274861	-12.117291	36.688941	0.460326		
3p	30.90108%	-97.278545	28.452867	-68.825678	-37.924590	-9.471723	17.770659	0.450257		
4s	26.36351%	-88.598244	28.758209	-59.84036	-33.476525	-4.718317	11.278893	0.395002		
4p	5.162567	-32.398903	12.862786	-19.536117	-14.373549	-1.510763	36.573943	1.291043		
5s	3.95680%	-27.932256	11.588485	-17.383771	-12.386964	-0.798478	17.860359	1.449234		
	0.309065	-8.144418	3.849689	-4.294728	-3.985664	-0.135974	36.134463	4.744102		
		Σ^3	Σ^2	Σ^1	Σ	Σ^2	Σ^3	Σ^4		
		6700.778	189.6893	29.58581	65.73524	1940.075	140621.3			
1s	2673.734	36.49400	4.1271847D-02	2.2810922D-03	1.4985310D-05	2.1132519D-04				
2s	270.735	8.162788	0.1818727	3.9163125D-02	2.0153467D-02	2.108257				
2p	1506.486	89.42294	8.112143	0.1569953	3.0142644D-02	2.2777746D-03	6.7663341D-03			
3s	45.82125	2.740776	0.5180992	0.3076300	0.1687669	0.1939562				
3p	225.9216	14.83031	2.629150	0.5210451	0.3166211	0.1679523	0.1718192			
3d	30.56929	7.382878	2.398547	0.5211529	0.3303909	0.2191110	0.2501626			
4s	5.537150	0.8756460	1.500693	2.563320	10.41842	79.57948				
4p	20.04602	1.583013	0.7589259	1.719664	3.391623	18.58161	153.9641			
5s	0.2785287	0.2201194	5.504187	34.17622	1804.198	139530.2				
12-V/E1 = 2.7D-08										
		Total S Cusp:	-37.856226	Total P Cusp:	-18.270574	Total D Cusp:	-11.640973			
38	Strontium Atom	Singlet S State		Kr(36)5s(2) Configuration				(10s, 6p, 3d) Basis		
		T	V(1)	V(2)	V(T)	E	r	-Cusp	r max	
3131.498055	-7470.077830	1207.082566	-6262.995263	-3131.497208	-1924.414641					
1s	702.874055	-1824.711065	69.08013	-1355.623052	-652.748997	-583.660984	37.884290	0.026563		
2s	161.075040	-319.382008	46.968483	-270.413524	-129.338484	-80.370000	37.580505	0.148635		
2p	139.903012	-317.579159	52.349888	-265.229271	-125.326249	-72.976361	18.267876	0.117456		
3s	35.600161	-108.136479	29.537615	-78.598865	-42.998704	-13.461089	37.844593	0.844504		
3p	33.472932	-103.958480	29.900026	-74.058854	-40.585521	-10.685495	18.273616	0.834527		
3d	28.931818	-95.422803	30.404535	-65.018269	-36.086451	-5.681916	11.640973	0.377965		
4s	6.167774	-36.007343	13.977545	-22.029798	-15.862023	-1.884478	37.611770	1.215796		
4p	4.901927	-31.604107	12.807664	-18.796462	-13.894535	-1.086889	18.391509	1.343254		
5s	0.539265	-10.262766	4.774275	-5.488891	-4.949226	-0.174951	37.355211	3.916535		
		Σ^3	Σ^2	Σ^1	Σ	Σ^2	Σ^3	Σ^4		
		7089.155	196.5810	31.60875	73.76282	1801.518	91797.24			
1s	2822.038	37.49240	4.0170815D-02	2.1607886D-03	1.3149832D-05	1.1187503D-04				
2s	286.9867	8.408790	0.1766583	3.6908668D-02	1.2872706D-02	0.9187178				
2p	1646.954	98.89518	8.357346	0.1523211	2.8347773D-02	1.8823562D-03	2.8723160D-03			
3s	49.46915	2.845697	0.4997220	0.2861028	0.1297777	0.2932085				
3p	252.124	16.02993	2.735789	0.5080601	0.2921365	0.1615029	0.1216356			
3d	34.88346	8.006582	2.511126	0.4948542	0.2966204	0.1738257	0.1733564			
4s	6.585319	0.9875617	1.392184	2.196455	7.068908	45.68278				
4p	25.83474	1.937031	0.8316870	1.566532	2.600674	12.37075	80.94248			
5s	0.4790403	0.2700728	4.562390	23.51321	854.7361	45607.86				
12-V/E1 = 2.7D-08										
		Total S Cusp:	-38.853042	Total P Cusp:	-18.792647	Total D Cusp:	-11.226519			
39	Yttrium Atom	Doublet D State		Kr(36)4d(1)5s(2) Configuration				(10s, 6p, 4d) Basis		
		T	V(1)	V(2)	V(T)	E	r	-Cusp	r max	
3331.590973	-7943.273709	1280.112053	-6663.161656	-3331.580683	-2051.468630					
1s	740.785201	-1501.126944	71.824494	-1429.300845	-688.511244	-616.686744	38.882021	0.025876		
2s	169.361809	-337.266909	51.075261	-286.171644	-136.027840	-85.752579	38.569718	0.148579		
2p	148.222510	-335.497949	54.583945	-280.914054	-132.691514	-78.107569	18.793252	0.114174		
3s	38.122010	-115.073266	31.025095	-84.048172	-45.726171	-14.701076	38.846024	0.430418		
3p	36.168612	-110.683890	31.454608	-79.424282	-43.255671	-11.796063	18.775942	0.419805		
3d	31.574634	-102.391112	32.136991	-70.254121	-38.679517	-6.542527	11.212759	0.362122		
4s	7.032020	-39.218444	15.033968	-28.190476	-17.148977	-2.114429	38.634333	1.156786		
4p	5.682873	-38.720508	13.892147	-20.82361	-15.145888	-1.523340	18.909647	1.266151		
4d	3.181457	-25.290066	10.942550	-14.347516	-11.166059	-0.222509	11.422475	1.666517		
5s	0.597574	-11.049505	5.133874	-5.915631	-5.318057	-0.184183	38.393007	3.718601		
		Σ^3	Σ^2	Σ^1	Σ	Σ^2	Σ^3	Σ^4		
		7486.915	203.6737	31.97852	71.06464	1547.892	72750.99			
1s	2973.995	38.49038	3.9125627D-02	2.0496173D-03	1.1373978D-05	5.7508879D-05				
2s	303.7366	8.607689	0.177354	3.4871557D-02	1.1285549D-02	0.7579914				
2p	1794.594	100.5105	8.602513	0.1479212	2.6719550D-02	1.5891093D-03	1.8085284D-03			
3s	53.20027	2.950597	0.495221	0.2665950	0.1121926	0.2298801				
3p	280.47799	17.28556	2.943177	0.4819873	0.2703676	0.1205111	9.3971169D-02			
3d	39.36729	8.808899	2.625413	0.4714920	0.2682117	0.1415778	0.1342032			
4s	7.491479	1.005499	1.317195	1.964843	5.934043	30.61385				
4p	31.01708	2.230411	0.8902694	1.469139	2.461482	9.507728	54.18575			
4d	2.394567	0.7579129	0.6484632	1.927639	4.226438	27.79321	263.2688			
5s	0.5317771	0.2833206	0.4360259	21.53804	724.3946	36048.74				

Table 12. (Continued)

42	Holmium Atom	Heptuplet S State	Kr(36)4d(5)5s(1) Configuration	(10s, 6p, 4d) Basis		
			$ 2-V/E = 7.43D-08$			$ 1-Norm = 4.3D-16$
	Total S Cusp:	-41.845040	Total P Cusp:	-20.292499	Total D Cusp:	-11.953260
	T	V(1)	V(2)	V(T)	E	e
	3975.349065	-9476.117307	1525.419472	-7950.697836	-3975.348771	-2449.929299
	T-Cusp				-Cusp	r max
1s	860.535180	-1742.367420	80.370853	-1661.996567	-801.461388	-721.090535
2s	175.688055	-393.882721	57.726773	-336.155948	-160.467883	-102.741110
2p	174.649108	-392.225673	61.620031	-330.605642	-155.956534	-94.336503
3s	47.067595	-137.120414	35.790045	-101.330370	-54.262775	-18.472730
3p	44.845147	-132.897555	36.438947	-96.458608	-51.613061	-15.174114
3d	40.160374	-124.571464	37.621174	-86.905290	-46.789916	-9.168742
4s	9.680411	-48.915576	18.295598	-30.619982	-20.939571	-2.643977
4p	8.037519	-43.959210	17.153125	-26.806085	-18.768171	-1.615046
4d	4.385685	-31.698464	13.515833	-18.182631	-13.796646	-0.280813
5s	0.677477	-12.843458	5.989429	-6.850429	-6.176552	-0.187124
	Γ^{-3}	Γ^{-2}	Γ^{-1}	Γ^0	Γ^1	Γ^2
	8786.300	225.6218	31.20179	53.01123	669.5959	24149.22
1s	3453.883	41.48494	3.62922170-02	1.76305840-03	7.97069290-06	5.24201760-06
2s	356.9041	9.378160	0.1548695	2.96711370-02	7.73866810-03	0.4372656
2p	2288.105	118.3165	9.338706	0.1361162	2.25996340-02	7.51553590-04
3s	65.17271	3.264772	0.4374959	0.2188631	7.43048120-02	7.92824790-02
3p	377.8622	21.31465	3.164228	0.4334710	0.2182830	7.78886930-02
3d	55.3770	11.15378	2.965987	0.4142402	0.2054597	8.26976180-02
4s	10.28281	1.164657	1.148717	1.494203	3.402828	11.65861
4p	88.20444	3.113016	1.066648	1.262898	1.821801	5.225615
4d	3.733759	1.060937	0.7547253	1.693553	3.319632	18.18821
5s	0.6075518	0.3057966	0.304764	18.49339	539.0507	23244.72
43	Technetium Atom	Hextuplet S State	Kr(36)4d(5)5s(2) Configuration	(10s, 6p, 4d) Basis		
			$ 2-V/E = 2.2D-06$			$ 1-Norm = 3.3D-16$
	Total S Cusp:	-42.845283	Total P Cusp:	-20.843312	Total D Cusp:	-12.332300
	T	V(1)	V(2)	V(T)	E	e
	4203.738576	-10009.583676	1602.097159	-8407.486517	-4203.747941	-2601.650782
	T-Cusp				-Cusp	r max
1s	902.445371	-1826.776339	83.134384	-1743.641956	-641.196585	-758.062201
2s	188.951776	-413.745463	59.889547	-353.895916	-168.948180	-109.094594
2p	183.953401	-412.132257	63.872520	-348.259738	-164.305937	-100.433417
3s	50.220004	-144.940918	37.287681	-107.653237	-57.433233	-20.145551
3p	47.956583	-140.661881	38.006370	-102.675510	-58.718927	-16.712557
3d	43.252690	-132.426229	39.359710	-93.066518	-49.818362	-10.454618
4s	10.901974	-52.891848	19.430766	-33.661082	-22.559108	-1.722242
4p	9.228308	-48.008595	18.381326	-29.627268	-20.398960	-2.017634
5s	0.878908	-14.528134	6.713950	-7.818188	-6.935276	-0.221326
4d	5.116691	-34.923918	14.861220	-20.102698	-14.986007	-0.164787
	Γ^{-3}	Γ^{-2}	Γ^{-1}	Γ^0	Γ^1	Γ^2
	9190.744	232.7870	33.10947	61.61075	875.8915	29761.79
1s	3621.880	42.48317	3.5437003D-02	1.68081670-03	7.2083551D-06	2.1493551D-06
2s	375.6163	9.621998	0.1548814	2.8186502D-02	6.5521157D-03	0.3329369
2p	2472.571	124.6010	9.588471	0.1325808	2.1830171D-02	9.6193331D-04
3s	69.45207	3.370779	0.4242050	0.2057032	6.5639594D-02	7.5162560D-02
3p	414.4963	22.75947	3.271672	0.4193084	0.20401007	6.7599145D-02
3d	61.55513	11.99851	3.079680	0.3980335	0.1892811	6.9400419D-02
4s	11.55387	1.230063	1.091165	1.346023	2.732053	8.038571
4p	57.05189	3.550071	1.116479	1.186975	1.605164	3.997710
5s	0.785897	0.3378636	3.690472	15.54349	387.8500	14577.97
4d	4.565869	1.207876	0.6121841	1.580866	2.896720	13.89826
	Γ^{-3}	Γ^{-2}	Γ^{-1}	Γ^0	Γ^1	Γ^2
	9190.744	232.7870	33.10947	61.61075	875.8915	29761.79
1s	3621.880	42.48317	3.5437003D-02	1.68081670-03	7.2083551D-06	2.1493551D-06
2s	375.6163	9.621998	0.1548814	2.8186502D-02	6.5521157D-03	0.3329369
2p	2472.571	124.6010	9.588471	0.1325808	2.1830171D-02	9.6193331D-04
3s	69.45207	3.370779	0.4242050	0.2057032	6.5639594D-02	7.5162560D-02
3p	414.4963	22.75947	3.271672	0.4193084	0.20401007	6.7599145D-02
3d	61.55513	11.99851	3.079680	0.3980335	0.1892811	6.9400419D-02
4s	11.55387	1.230063	1.091165	1.346023	2.732053	8.038571
4p	57.05189	3.550071	1.116479	1.186975	1.605164	3.997710
5s	0.785897	0.3378636	3.690472	15.54349	387.8500	14577.97
	Γ^{-3}	Γ^{-2}	Γ^{-1}	Γ^0	Γ^1	Γ^2
	9190.744	232.7870	33.10947	61.61075	875.8915	29761.79
46	Palladium Atom	Singlet S State	Kr(36)4d(10)5s(0) Configuration	(8s, 6p, 4d) Basis		
			$ 2-V/E = 6.7D-08$			$ 1-Norm = 1.1D-15$
	Total S Cusp:	-45.920017	Total P Cusp:	-22.263162	Total D Cusp:	-12.949547
	T	V(1)	V(2)	V(T)	E	e
	4937.518831	-11778.695258	1903.657925	-9875.037334	-4937.518502	-3033.860577
	T-Cusp				-Cusp	r max
1s	1034.274286	-2092.105949	92.345182	-1994.706767	-965.481481	-873.136299
2s	210.091206	-476.186378	67.158125	-409.030163	-194.938958	-127.780743
2p	213.31152	-474.796556	71.570641	-403.225915	-189.137362	-118.343121
3s	60.12703	-169.455571	42.653061	-126.802511	-66.675848	-44.022377
3p	57.820971	-165.189056	43.595075	-121.598470	-63.777499	-40.186924
3d	55.024322	-157.020273	45.414689	-111.606044	-56.576722	-33.162033
4s	13.651098	-62.961585	22.966329	-39.995255	-26.344158	-3.378628
4p	11.611052	-57.326248	21.789876	-35.536372	-23.921320	-2.131444
4d	6.643923	-42.319386	17.755890	-24.563496	-17.919572	-0.163682
	Γ^{-3}	Γ^{-2}	Γ^{-1}	Γ^0	Γ^1	Γ^2
	10586.69	256.0586	30.30270	36.68374	117.5331	680.8592
1s	4151.395	45.48056	3.3098373D-02	1.4659166D-03	5.5036289D-06	2.7938977D-07
2s	414.9696	10.35192	0.1435882	2.4243323D-02	1.03979910D-03	2.3209700D-04
2p	3076.296	144.3366	10.123064	1.8933516D-02	7.1394563D-04	3.1556838D-04
3s	83.16084	3.63H3817	0.3893041	0.1731195	4.5874719D-02	1.8333353D-02
3p	537.3307	27.37579	3.591066	0.3824437	0.1697083	4.6753006D-02
3d	82.15737	14.645983	3.413494	0.3575762	0.1528165	4.8812584D-02
4s	14.45152	14.451502	1.368730	0.9856997	1.098103	1.807016
4p	77.63423	4.4415188	1.246273	1.071324	1.313594	2.730791
4d	6.601469	1.562165	0.9199866	1.416053	2.355128	9.666755

Table 12. (Continued)

47	Silver Atom	Doublet S State			Kr(36) 4d(10) 5s(1) Configuration			(10s, 6p, 4d) Basis											
$ 2-V/E = 6.50-08$																			
Total S Cusp: -46.830944 Total P Cusp: -22.799722 Total D Cusp: -13.512573																			
11-Normal = 6.60-16																			
5197.390089 -12389.767452	1994.987613 -10394.779839	-5197.389750	-3202.402138	-Cusp	r max														
1s 1080.090617 -2184.417744	95.377542 -2089.106202	-1009.015585	-913.704044	46.867269	0.021443														
2s 224.427569 -498.117836	69.471100 -428.646736	-204.219166	-134.748066	46.485662	0.118680														
2p 223.583247 -496.669304	70.015994 -422.653311	-199.070064	-125.054072	22.809065	0.093188														
3s 63.634531 -178.067867	44.322323 -133.745544	-70.111013	-25.788689	46.728801	0.343513														
3p 61.300185 -173.765059	45.324861 -128.440199	-67.140014	-21.815153	22.739779	0.330044														
3d 56.531312 -165.710539	47.319400 -118.391139	-61.859827	-14.540427	13.500420	0.270810														
4s 14.810641 -66.804220	24.069952 -82.734268	-27.923627	-3.353675	47.202386	0.855749														
4p 12.804464 -61.371863	23.074187 -38.375676	-25.533212	-2.539025	22.884238	0.897095														
4d 7.930369 -47.148053	19.400528 -27.747525	-19.817155	-0.416627	13.627473	1.054830														
5s 0.719183 -15.528838	7.308252 -8.220586	-7.501403	-0.193151	46.235548	3.197225														
Σ^3	Σ^2	Σ^1	Σ	Σ^2	Σ^1														
11070.29	263.6121	32.23486	47.30994	460.9189	13841.06														
1s 4333.581	46.47697	3.23824820D-02	1.4031591D-03	5.0606210D-06	3.4912408D-06														
2s 455.2454	10.59825	0.1403559	2.3253183D-02	4.5109343D-03	0.2021049														
2p 3300.051	151.2656	10.56743	0.1201100	1.7569375D-02	6.3719762D-04	1.9331583D-04													
3s 87.76290	3.788678	0.3787390	0.1637898	4.1180803D-02	3.0938784D-02														
3p 583.6610	28.92696	3.697129	0.3715559	0.1601115	4.1418172D-02	1.8895871D-02													
3d 90.38325	15.61298	3.525756	0.3455951	0.1420954	3.9840563D-02	4.1270674D-02													
4s 15.69315	1.421366	0.9530421	1.028940	1.639418	5.714109														
4p 87.85513	4.876830	1.305784	1.023561	1.196289	2.237261	6.191643													
4d 8.226564	1.848255	1.003150	1.298473	1.972072	6.661739	34.82580													
5s 0.6528436	0.3304008	3.693769	15.48967	376.8570	13443.24														
48	Cadmium Atom	Singlet S State			Kr(36) 4d(10) 5s(2) Configuration			(10s, 6p, 4d) Basis											
$ 2-V/E = 9.90-08$																			
Total S Cusp: -47.833861 Total P Cusp: -23.334410 Total D Cusp: -16.076621																			
11-Normal = 2.90-15																			
5464.89190 -13019.060468	2089.277204 -10929.783264	-5464.891360	-3375.614156	-Cusp	r max														
1s 1126.99737	-2278.823261	98.302879	-2100.520382	-1053.523011	-955.220131	47.870353	0.020993												
2s 234.92786	-520.471706	71.809639	-448.662067	-213.734206	-141.924567	47.473769	0.116051												
2p 234.12649	-519.068334	76.485781	-442.582553	-208.456056	-131.970275	23.349225	0.091110												
3s 67.25474	-186.899821	46.022932	-140.876590	-73.621886	-27.599014	47.802507	0.334788												
3p 64.86877	-182.539183	47.091845	-135.447338	-70.578567	-23.496722	23.246233	0.321572												
3d 60.06533	-174.548844	49.261454	-125.287390	-65.222060	-15.960606	16.114823	0.270853												
4s 16.13662	-71.044994	25.278293	-45.766701	-29.630074	-4.351781	88.207539	0.827727												
4p 14.08679	-65.605778	24.278255	-41.327525	-27.240731	-2.962478	23.425054	0.864131												
4d 9.29792	-51.940735	20.980774	-30.959661	-21.662032	-0.681257	15.722783	0.997759												
5s 1.066868	-18.203069	8.446177	-9.756892	-8.690024	-0.243847	47.694292	2.745393												
Σ^3	Σ^2	Σ^1	Σ	Σ^2	Σ^1														
11571.09	271.2304	33.61069	52.61010	545.7437	14530.37														
1s 4521.618	47.47548	3.1699843D-02	1.3445425D-03	4.6626191D-06	3.6635039D-06														
2s 476.4367	10.84316	0.1372020	2.2210932D-02	3.70520579D-03	0.1415142														
2p 3534.67	158.3721	10.81392	0.1173382	1.6761672D-02	5.7304807D-04	1.2645648D-04													
3s 97.7544	3.853738	0.3688582	0.1553439	3.6757423D-02	1.2416239D-02														
3p 632.169	30.57774	3.802900	0.3612870	0.1513265	3.6845322D-02	1.5202274D-02													
3d 100.012	16.51109	3.636434	0.3337232	0.11718573	3.2122128D-02	1.4015693D-02													
4s 17.07037	1.480104	0.9167312	0.9505559	1.379901	3.988621														
4p 99.16927	5.344356	1.366787	0.9791269	1.092682	1.86613	4.559609													
4d 10.14873	2.153498	1.082099	1.206839	1.695967	4.788403	20.26816													
5s 0.9599650	0.3792306	3.274759	12.25417	241.6968	7145.909														

Table 12. (Continued)

49	Indium Atom	Doublet P State		Kr(36)4d(10)5s(2)5p(1) Configuration				(10s, 8p, 4d) Basis	
		T	V(1)	V(2)	V(T)	E	e	-Cusp	r max
12-V/EI = 3.8D-06									
Total S Cusp: -48.849443 Total P Cusp: -24.306223 Total D Cusp: -14.626701									
1s	178.932908	-2375.257578	101.313377	-2273.944197	-1099.011288	-997.697911	48.883135	0.020561	
2s	245.579431	-543.221557	76.172903	-469.048654	-223.469223	-149.296319	48.485554	0.113620	
2p	244.891621	-541.937446	78.983567	-462.953878	-218.062257	-139.078690	24.424922	0.089380	
3s	71.017967	-195.993946	47.731198	-148.262742	-77.244776	-29.513578	48.998271	0.326289	
3p	68.510248	-191.474752	48.852146	-142.622606	-74.112358	-25.260212	23.641705	0.315031	
3d	63.829587	-183.697133	51.198973	-132.502160	-68.672573	-17.477600	14.624565	0.257432	
4s	17.534979	-75.433876	26.522591	-48.911283	-31.376313	-4.853722	49.132662	0.800206	
4p	15.408365	-69.855782	25.528761	-44.327021	-28.918676	-3.389914	24.526527	0.831516	
4d	10.634037	-56.739482	22.575130	-38.164352	-23.530313	-0.955183	14.643613	0.936332	
5s	1.507860	-21.206450	9.685433	-11.521018	-10.013158	-0.327725	48.698334	2.438389	
5p	1.289799	-19.408734	8.981438	-10.427296	-9.137496	-0.156058	27.024406	2.705078	
		Σ^3	Σ^-	Σ^1	Ξ	Ξ^2	Ξ^0	Ξ^*	
		12083.77	278.8900	34.56254	54.14581	475.6452	9245.647		
1s	4713.978	48.47864	3.10454980-02	1.2895183D-03	4.2227092D-06	5.9331293D-07			
2s	498.0452	11.08615	0.1341954	2.1227848D-02	2.5098145D-03	6.6360459D-02			
2p	3790.203	165.7114	11.05995	0.1146920	1.6050412D-02	5.0777847D-04	1.1630329D-04		
3s	97.97703	3.999876	0.3598489	0.1475851	3.4206230D-02	5.3820111D-02			
3p	684.7567	32.29710	3.907648	0.3515611	0.1432752	3.7369489D-02	0.1439160		
3d	108.1702	17.61303	3.748921	0.3238332	0.1241729	2.8708124D-02	1.5094479D-02		
4s	18.51370	1.539467	0.8830671	0.8811240	1.163882	2.335166			
4p	111.9611	5.833721	1.825628	0.9425680	1.014429	1.615220	4.194784		
4d	11.91688	2.404624	1.157949	1.126262	1.475398	3.630085	13.56570		
5s	1.339556	0.4327847	2.885091	9.462659	141.3669	3134.108			
5p	6.646047	0.4408247	0.3960966	3.022978	10.08015	143.9997	2810.679		
50	Tin Atom	Triplet P State		Kr(36)4d(10)5s(2)5p(2) Configuration				(10s, 8p, 4d) Basis	
		T	V(1)	V(2)	V(T)	E	e	-Cusp	r max
12-V/EI = 7.2D-08									
Total S Cusp: -49.867669 Total P Cusp: -24.218148 Total D Cusp: -15.249763									
1s	1223.865745	-2473.689077	104.334900	-2369.354177	-1145.488433	-1041.153533	49.898378	0.020146	
2s	256.508451	-566.503294	76.537919	-489.965375	-233.456884	-156.918964	49.517393	0.111287	
2p	255.877110	-565.273202	81.483179	-483.790023	-227.912912	-146.429733	24.188399	0.087118	
3s	74.850968	-205.257684	49.425680	-155.822008	-80.971096	-31.535016	50.122542	0.318445	
3p	72.411962	-200.871912	50.656768	-150.215143	-77.803182	-27.146413	24.343140	0.305721	
3d	67.611012	-192.976398	53.132506	-139.843892	-72.232880	-19.100374	15.256885	0.251450	
4s	19.018287	-79.987779	27.762881	-52.224898	-33.206611	-5.443731	50.019988	0.774190	
4p	16.844827	-74.412514	26.832248	-47.508266	-30.735839	-3.503591	24.298196	0.805965	
4d	12.078667	-61.627893	24.119634	-37.508258	-25.429591	-1.309957	15.197156	0.889664	
5s	2.035281	-24.336401	10.923363	-13.413037	-11.377757	-0.4504393	49.626786	2.208178	
5p	1.261608	-19.451269	8.969166	-10.492103	-9.220494	-0.251328	24.550235	2.726084	
		Σ^3	Σ^-	Σ^1	Ξ	Ξ^2	Ξ^0	Ξ^*	
		12606.52	286.5568	36.09347	60.34118	610.0074	13123.59		
1s	4910.380	49.47378	3.0417737D-02	1.2378137D-02	3.8867613D-06	1.6265140D-07			
2s	520.2467	11.33007	0.1313074	2.0304187D-02	1.69670815-03	2.9843933D-02			
2p	4026.101	172.9427	11.30546	0.1121857	1.5334349D-02	1.0279680D-03	2.2435486D-02		
3s	103.2673	4.105154	0.3505505	0.1403475	3.1597509D-02	6.1371957D-02			
3p	739.1425	34.06812	4.017438	0.3421207	0.1356947	3.2029548D-02	0.1209539		
3d	117.7738	18.64758	3.859528	0.3140392	0.1165693	2.4882881D-02	1.0127520D-02		
4s	20.03953	1.599756	0.8515141	0.8185689	0.9949393	1.708773			
4p	124.0978	6.341313	1.488250	0.9035704	0.9306572	1.385043	2.996629		
4d	14.02699	2.761220	1.232558	1.058833	1.300931	2.787076	8.968993		
5s	1.789038	0.4867280	2.580432	7.534605	87.87416	1501.391			
5p	6.663796	0.4395036	0.3890254	3.164523	11.32297	197.9072	5004.290		

Table 12. (Continued)

51	Antimony Atom	Quartet S State	Kr(36)4d(10)5s(2)5p(3) Configuration					(10s, 8p, 4d) Basis	
			$12^{-1}/E^1 = 1.5D-08$					-Norm = 7.8D-16	
		Total S Cusp: -50.880008	Total P Cusp: -25.305444					Total D Cusp: -15.788276	
T		V(1)	V(2)	V(T)	Z	e	-Cusp	r	max
6313.332786	-15011.601264	2384.935598	-12626.665666	-6313.332880	-3928.397282				
1s	1273.778666	-2574.100728	107.405528	-2466.695200	-1192.916534	-1085.511006	50.908750	0.019748	
2s	267.713938	-590.313827	78.956366	-511.356861	-243.642923	-164.685957	50.584193	0.109027	
2p	267.163798	-589.167365	84.033813	-505.133551	-237.969763	-153.935950	25.440632	0.085569	
3s	78.777253	-214.727903	51.201564	-163.526439	-84.749186	-33.547722	51.176536	0.311100	
3p	76.204562	-210.145737	52.463621	-157.682116	-81.477554	-29.013933	24.566565	0.299821	
3d	71.530032	-202.518131	55.137402	-147.380728	-75.850697	-20.173294	15.805170	0.245285	
4s	20.561721	-84.658605	29.062910	-55.595696	-15.033974	-5.971065	50.953466	0.750228	
4p	18.380255	-79.124405	28.198642	-50.929764	-32.549508	-8.354867	25.589483	0.775518	
4d	13.549797	-66.505862	25.676524	-40.829338	-27.279542	-1.603018	15.670740	0.849101	
5s	2.466214	-26.879718	11.938083	-14.941636	-12.475421	-0.537339	50.543663	2.064445	
5p	1.815762	-23.124910	10.504660	-12.620250	-10.804488	-0.299829	26.768877	2.386306	
E ⁻³	E ⁻²	E ⁻¹	E	E ²	E ³	E ⁴	E ⁵		
	13142.54	294.3451	36.47246	57.89871	409.7276	6768.879			
1s	5110.606	50.47256	2.98147060-02	1.18914680-03	3.61351570-06	8.04649120-07			
2s	542.9598	11.57478	0.1285379	1.94443380-02	1.28367660-03	1.62063400-02			
2p	4312.545	180.7017	11.55230	0.1097507	1.46490540-02	4.23046150-04	5.00604420-05		
3s	108.6490	4.210351	0.3420471	0.1336039	2.86440350-02	4.91601790-02			
3p	795.0930	35.84485	4.120505	0.3335975	0.1289884	3.0948490-02	0.1249221		
3d	127.8500	19.71390	3.970946	0.3047979	0.1096794	2.18420810-02	7.5657870-03		
4s	21.63188	1.659973	0.8223158	0.7628539	0.8593016	1.359040			
4p	140.0238	6.909270	1.551459	0.8687605	0.8593683	1.422285	2.607945		
4d	16.26275	3.082478	1.304037	1.001840	1.162833	2.206269	6.236178		
5s	2.153824	0.5270533	2.390495	6.447054	63.48289	906.3937			
5p	10.06143	0.6142000	0.4530296	2.702336	8.142419	97.22000	1624.803		
52	Tellurium Atom	Triplet P State	Kr(36)4d(10)5s(2)5p(4) Configuration					(10s, 8p, 4d) Basis	
			$12^{-1}/E^1 = 2.9D-08$					-Norm = 6.9D-16	
		Total S Cusp: -51.891125	Total P Cusp: -25.806013					Total D Cusp: -16.143942	
T		V(1)	V(2)	V(T)	Z	e	-Cusp	r	max
6611.644581	-15711.457670	2488.168315	-13223.289354	-6611.644774	-4123.674658				
1s	1324.691940	-2676.512817	110.486326	-2566.026490	-1241.334550	-1130.842228	51.918063	0.019366	
2s	279.157774	-614.611934	81.381701	-533.230233	-254.072459	-172.690757	51.571773	0.106852	
2p	278.668775	-613.526709	86.592286	-526.934423	-248.265648	-161.673362	25.984438	0.083798	
3s	82.810130	-224.417206	52.964811	-171.452395	-88.642265	-35.677458	52.209533	0.304118	
3p	80.214048	-219.815449	54.299245	-165.516204	-85.301715	-31.002469	25.049984	0.292722	
3d	75.545254	-212.258816	57.137348	-155.121468	-79.576212	-22.438864	16.166561	0.238861	
4s	22.181366	-89.477710	30.364666	-59.113044	-36.931678	-6.567012	51.891275	0.727767	
4p	19.964387	-83.944373	29.553220	-54.391153	-34.426772	-4.873552	26.130693	0.750271	
4d	15.078519	-71.472230	27.215579	-48.256651	-29.178132	-1.962553	15.995734	0.812699	
5s	2.973895	-29.609483	12.989704	-16.619779	-13.645885	-0.656180	51.511471	1.933947	
5p	2.172687	-25.292431	11.390931	-13.894400	-11.720713	-0.321682	26.985486	2.235432	
E ⁻³	E ⁻²	E ⁻¹	E	E ²	E ³	E ⁴	E ⁵		
	13688.80	302.1434	37.29529	59.02791	433.3072	5992.606			
1s		5374.814	51.47140	2.92349990-02	1.14328650-03	3.36954230-06	1.30082900-06		
2s		566.1515	71.81946	0.1258555	1.66405680-02	1.0081890-03	9.09831860-03		
2p	4590.843	188.4430	11.79859	0.1074352	1.40343870-02	3.87661130-04	3.20913780-05		
3s		114.1664	4.315716	0.3339354	0.1273181	2.58284560-02	3.56863380-02		
3p	856.6373	37.69196	4.227220	0.3252583	0.1225994	2.79353900-02	0.1052201		
3d	138.2603	20.80096	4.081900	0.2961755	0.1034817	1.93640270-02	6.08746480-03		
4s		23.30267	1.720725	0.7949349	0.7124766	0.7462680	1.100132		
4p	155.6184	7.480862	1.614315	0.8364065	0.7959937	0.9784043	2.164350		
4d	18.65520	3.414602	1.374466	0.9514221	1.047469	1.777260	4.465295		
5s		2.583273	0.5698131	2.222716	5.562829	46.79775	566.5121		
5p	12.49257	0.7295124	0.4863929	2.547825	7.269430	78.53971	1199.740		

Table 12. (Continued)

S3	Iodine Atom	Doublet P State	Kr(36) 4d(10) 5s(2) 5p(5) Configuration	(10s, 8p, 4d) Basis				
		$ 2-\Delta/E = 3.60-08$	$ 1-\Delta_{\text{Total}} = 4.00-15$					
		Total S Cusp: -52.89952	Total P Cusp: -26.307352	Total D Cusp: -16.458023				
		τ^1	τ^2	τ^3				
1s	1376.600020	-2780.919768	113.596693	-2667.323076	-1290.723055	-1177.126363	52.925342	0.018998
2s	290.849160	-639.407632	83.833589	-555.574083	-264.724883	-180.891298	52.595298	0.104754
2p	290.420069	-638.382724	89.178495	-549.204229	-258.784180	-169.605685	26.443990	0.082097
3s	86.951242	-234.328392	54.754613	-179.573780	-92.622538	-37.667925	53.228060	0.297443
3p	84.338245	-229.715771	56.163005	-173.552766	-89.214521	-31.051516	25.557078	0.285917
3d	79.664850	-222.209986	59.161160	-163.048789	-83.383939	-24.222779	16.488883	0.232657
4s	23.862470	-98.419264	31.690533	-62.728732	-38.866255	-7.175722	52.842231	0.706752
4p	21.608659	-88.861561	30.934073	-57.947888	-36.338829	-5.404756	26.677232	0.726754
4d	16.659710	-76.533556	28.764596	-87.758961	-31.099253	-2.334655	16.291549	0.779833
5s	3.490236	-32.281996	14.006575	-18.275421	-14.785186	-0.778611	52.496448	1.026346
5p	2.579557	-27.634816	12.345159	-15.286957	-12.710120	-0.364960	27.301252	2.095317
		τ^{-3}	τ^{-2}	τ^{-1}	τ^0	τ^1	τ^2	τ^3
1s	5522.962	52.47018	2.8677322D-02	1.1000235D-03	3.1395143D-06	1.4579989D-06		
2s	589.8246	12.046429	0.1233409	1.7887645D-02	8.2968549D-03	5.0791033D-03		
2p	4880.829	196.3882	12.04496	0.1052146	1.3457302D-02	3.5562606D-03	2.0790013D-05	
3s	119.8228	4.421290	0.3261845	0.1214521	2.3113693D-02	2.5816485D-02		
3p	917.32C7	39.59033	4.334260	0.3172946	0.1166390	2.4909968D-02	8.0803600D-02	
3d	149.1856	21.91538	4.192641	0.2880559	9.7821083D-02	1.7251822D-02	5.0248842D-03	
4s	25.03740	1.781496	0.7693916	0.6670842	0.6518082	0.8986870		
4p	172.25.6	8.073042	1.677011	0.8065249	0.7397307	0.8837124	1.788838	
4d	21.21455	3.756878	1.443841	0.9063849	0.9496068	1.451074	3.265269	
5s	3.019102	0.6090943	2.096823	4.894833	35.96624	378.0292		
5p	15.320C7	0.8590596	0.5214116	2.398629	6.462041	62.61307	663.9243	
S4	Xenon Atom	Singlet S State	Kr(36) 4d(10) 5s(2) 5p(6) Configuration	(10s, 8p, 4d) Basis				
		$ 2-\Delta/E = 5.40-08$	$ 1-\Delta_{\text{Total}} = 2.90-15$					
		Total S Cusp: -53.907013	Total P Cusp: -26.809882	Total D Cusp: -16.766265				
		τ^1	τ^2	τ^3				
1s	7232.015954	-17167.726327	2703.694851	-14464.031476	-7232.015543	-4529.320692	-	
2s	1429.504240	-2887.322875	116.736633	-2770.596242	-1341.082022	-1224.345389	53.931333	0.018648
2p	302.787101	-664.69996	86.312609	-578.347387	-275.600286	-189.287678	53.615373	0.102733
3s	302.417556	-663.715380	91.792356	-571.942995	-269.525399	-177.733013	26.942441	0.080468
3p	91.200409	-244.460082	56.570944	-187.889858	-96.689450	-40.118506	54.239118	0.291048
3d	88.57450	-239.843621	58.054377	-181.789245	-93.214674	-35.160298	26.075308	0.279402
4s	83.888649	-232.372370	61.209474	-171.162896	-87.274227	-26.064753	16.796779	0.226758
4p	25.60377	-99.481508	33.040219	-66.441289	-40.837522	-7.797303	53.801353	0.686999
4d	23.346811	-94.004286	32.354006	-61.649879	-38.303068	-5.948662	27.224614	0.704625
5s	18.294866	-81.666777	30.326269	-51.340508	-33.045642	-2.719373	16.586563	0.749968
5p	4.017736	-34.930001	15.003988	-14.426017	-15.908261	-0.904276	53.489167	1.734857
		τ^{-3}	τ^{-2}	τ^{-1}	τ^0	τ^1	τ^2	τ^3
1s	5735.072	53.46494	2.8140471D-02	1.0591572D-03	2.927331D-06	1.4405342D-06		
2s	613.9807	12.30526	0.1208971	1.7160579D-02	7.0416139D-04	3.4773101D-03		
2p	5182.852	204.4177	12.29140	0.1030830	1.2514879D-02	3.2709323D-04	1.4710095D-05	
3s	125.6209	4.527052	0.3197748	0.1159748	2.11018010-02	1.8998171D-02		
3p	98J.1848	41.53918	4.441549	0.3096922	0.1110861	2.2201533D-02	6.0341744D-02	
3d	160.6755	23.05781	4.303192	0.2803963	9.2625161D-02	1.5428212D-02	4.2016786D-03	
4s	26.83420	1.842250	0.7455014	0.6260427	0.5722670	0.7394241		
4p	140.2233	8.696958	1.740820	0.7791067	0.6878184	0.7232783	1.388394	
4d	23.95115	4.109759	1.512348	0.8658209	0.8656181	1.198231	2.429318	
5s	3.463575	0.6468519	1.971864	4.367658	28.86969	264.1665		
5p	18.18821	0.9866219	0.5556937	2.262708	5.759499	19.98952	620.9646	

Table 12. (Continued)

54	Xenon Atom	Singlet S State		Kr(36) 4d(10) 5s(2) 5p(6) Configuration				(1s, 12p, 6d) Basis	
		T	V(1)	V(2)	V(T)	Z	e	-Cusp	r max
12-V/E = 1.9D-08 Total S Cusp: -54.023967 Total P Cusp: -26.991783 Total D Cusp: -17.666220									
1s	1429.523327	-2887.381449	116.710535	-2770.630913	-1301.107587	-1224.397051	54.022672	0.018646	
2s	302.811553	-664.735717	86.292373	-578.443344	-275.631790	-189.339417	54.037031	0.102750	
2p	302.832166	-663.751459	91.768783	-571.962676	-269.550509	-177.781726	27.000756	0.080441	
3s	91.202532	-244.472102	56.547309	-187.924793	-96.722262	-80.174953	54.011162	0.291274	
3p	88.577644	-220.022070	58.054752	-181.947317	-93.275668	-35.220916	26.934383	0.278367	
3d	83.926152	-232.433768	61.194743	-171.239024	-87.312872	-26.118129	17.603052	0.226693	
4s	25.610114	-99.497511	33.015968	-66.481543	-40.871430	-7.855462	54.047022	0.687211	
4p	23.345257	-94.041002	32.344110	-61.696892	-38.351635	-6.007525	27.062529	0.705090	
5s	18.220537	-81.483840	30.242918	-51.240526	-33.019988	-2.777074	16.007484	0.748209	
5p	4.055584	-34.980662	14.990633	-19.990029	-15.938445	-0.943813	54.177234	1.712948	
5d	2.928249	-29.540233	13.077584	-16.462648	-13.538399	-0.456815	27.113556	1.940238	
E^3 E^2 E^1 E E^2 E^1 E^0 E^0									
	14819.99	317.8747	39.05485	62.60167	468.1836	6951.993			
1s	5736.284	53.46929	2.8141027D-02	1.0591620D-03	2.8421744D-06	8.0250815D-08			
2s	614.9363	12.30992	0.1208726	1.7159530D-02	0.0022632D-04	2.2634830D-05			
2p	5182.4465	204.4189	12.29169	0.1030820	1.2915085D-02	3.2722555D-04	2.1652060D-05		
3s	125.5044	4.527261	0.3186975	0.1158712	2.0302282D-02	5.3215347D-03			
3p	989.6887	41.61152	4.488483	0.1094287	0.1108205	1.9888242D-02	8.3235281D-03		
3d	160.8111	23.06614	8.304329	0.2803358	9.2632849D-02	1.5664221D-02	4.1897971D-03		
4s	26.85896	1.842547	0.7452695	0.6255767	0.5706585	0.7185660			
4p	189.5944	8.690340	1.741500	0.7770238	0.6855305	0.7074986	1.048983		
5s	4.100333	1.508953	0.870037	0.8792026	1.286055	2.952122			
5p	3.505438	0.6477900	1.980961	4.438842	30.42177	309.2911			
5d	17.81369	0.9700469	0.5470413	2.337728	6.271783	64.79633	1049.347		
55	Cesium Atom	Doublet S State		Xe(54) 6s(1) Configuration				(12s, 8p, 6d) Basis	
12-V/E = 3.8D-08 Total S Cusp: -54.687153 Total P Cusp: -27.302629 Total D Cusp: -17.078395									
T	V(1)	V(2)	V(T)	Z	e	-Cusp	r max		
7553.835737	-17912.289798	2804.618608	-15107.671189	-7553.835452	-4749.216844				
1s	1483.384797	-2995.702799	119.795317	-2875.907482	-1392.522685	-1272.727369	54.915839	0.018303	
2s	314.987977	-690.501085	88.706819	-601.794267	-286.806290	-198.094471	54.545954	0.100743	
2p	310.669273	-689.593377	94.323517	-595.269860	-280.600585	-186.277068	27.396740	0.078878	
3s	95.577092	-256.839918	58.305575	-196.535339	-100.957248	-42.651673	55.191742	0.264792	
3p	92.985995	-250.239165	59.870352	-190.368813	-97.422818	-37.552465	26.744038	0.273014	
3d	88.21854	-242.749745	63.172435	-179.577309	-91.358763	-28.186328	17.111993	0.221157	
4s	27.412211	-104.681302	34.308355	-70.372947	-42.960737	-8.652382	55.006846	0.669281	
4p	25.115644	-99.194153	33.677159	-65.516994	-40.001351	-6.724192	27.783363	0.684731	
5s	20.005774	-86.953584	31.806241	-55.147343	-35.141566	-3.335327	16.889939	0.722598	
5p	4.804732	-38.326497	16.162525	-22.163973	-17.359240	-1.196716	55.094586	1.624876	
5d	3.753859	-33.632956	14.618110	-15.018857	-15.255992	-0.650678	28.028101	1.822045	
6s	0.350291	-11.195407	5.363830	-5.831577	-5.481286	-0.117457	56.694145	5.238582	
E^3 E^2 E^1 E E^2 E^1 E^0 E^0									
	15394.74	325.6780	42.22756	88.35216	2190.687	138279.2			
1s	5050.637	50.46732	2.7622842D-02	1.0205208D-03	2.9776149D-06	3.6842482D-05			
2s	638.3913	12.55457	0.1185629	1.6562885D-02	4.4343330D-03	0.4257765			
2p	5495.909	212.6461	12.53806	0.1010350	1.2406458D-02	3.0166546D-03	1.5507698D-05		
3s	131.5656	4.633453	0.3116729	0.1105254	2.7606326D-02	0.9806526			
3p	1053.442	43.55251	4.549803	0.3023411	0.10501818	1.8722272D-02	2.2617597D-02		
3d	172.7363	24.22855	4.413632	0.2731405	8.7800141D-02	1.3094505D-02	3.5442089D-03		
4s	28.72550	1.903296	0.7228326	0.5882168	0.50232051	0.6329075			
4p	209.0621	9.331543	1.803530	0.7519475	0.6416747	0.6223063	1.005405		
4d	26.90491	4.677807	1.580974	0.6285070	0.7917077	0.9956295	1.822733		
5s	4.132173	0.6968454	1.8464615	3.825795	21.93090	185.4061			
5p	23.53771	1.222089	0.6115083	2.072124	4.822281	34.66409	353.2185		
6s	0.2611857	0.2035529	5.795790	36.97867	1923.827	135760.6			

Table 12. (Continued)

56	Barium Atom	Singlet S State			$\text{Xe}(54)6s(2)$ Configuration			(12s, 8p, 4d) Basis		
		$ 2-\gamma/E = 6.90-08$			$ \text{t-Norm} = 1.30-15$					
		Total S Cusp: -55.895976	Total P Cusp: -27.798069	Total D Cusp: -17.391112						
		T	$\nabla(1)$	$\nabla(2)$	$\nabla(3)$	E	e	-Cusp	r max	
		7883.462430	-18675.051731	2908.126314	-15766.925417	-7893.462981	-4975.336667			
1s	1538.290234	-3106.107152	122.875986	-2982.231166	-1444.940932	-1322.064946	55.923269	0.017974		
2s	327.419137	-716.788002	91.123758	-625.664285	-298.245108	-207.121350	55.567217	0.098975		
2p	327.165322	-715.965847	96.876371	-619.069476	-291.904154	-195.027783	27.863551	0.077358		
3s	100.049945	-265.422977	60.059315	-205.363661	-105.313717	-45.254401	56.217406	0.278897		
3p	97.418951	-260.840539	61.070478	-199.136061	-101.717072	-40.012594	27.372671	0.266930		
3d	92.657372	-253.345884	65.156110	-186.189774	-95.532402	-30.376292	17.427034	0.215824		
4s	29.295115	-110.023630	35.600011	-74.423619	-45.128505	-9.528493	55.958184	0.651348		
4p	26.948072	-104.511740	35.021522	-69.490218	-42.541516	-7.519994	28.299090	0.665747		
5d	21.781234	-92.359119	33.303230	-59.055885	-37.274651	-3.971018	17.198639	0.697373		
5s	5.678035	-41.915343	17.374870	-24.504073	-18.862438	-1.487568	56.098856	1.501730		
5p	4.553618	-37.189023	15.877897	-21.311526	-16.757909	-0.680412	28.393979	1.704955		
6s	0.545826	-13.282301	6.292898	-6.989403	-6.443577	-0.150680	56.982551	0.535627		
		τ^{-3}	τ^{-2}	τ^{-1}	τ^0	τ^1	τ^2	τ^3	τ^4	
		15987.37	333.49831	44.78245	102.6897	2585.524	138996.3			
7s		6170.790	55.46620	2.71244040D-02	9.83976120D-04	2.95747420D-06	4.7394140D-05			
2s		663.5846	12.79979	0.1162995	1.5925063D-02	3.1594953D-03	0.2538587			
2p	5821.785	221.0406	12.78475	9.9066056D-02	1.1923733D-02	2.7854534D-04	1.3974710D-05			
3s		137.6709	4.739696	0.308803	0.1061266	2.4626035D-02	0.7019920			
3p	1126.706	45.61125	4.657867	0.2953694	0.1009430	1.6481780D-02	1.0057371D-02			
3d	185.3985	25.42685	4.524036	0.2562558	8.3418953D-02	1.2460254D-02	3.0110529D-03			
4s		30.66437	1.966708	0.7015686	0.5539333	0.4845590	0.4945289			
4p	228.9787	9.987177	1.866281	0.7275283	0.6001738	0.5395308	0.7563918			
4d	30.06581	4.858679	1.6495270	0.7949371	0.7269542	0.8339989	1.3849283			
5s		4.860340	0.7484883	1.724076	3.329719	16.27375	115.7739			
5p	29.31815	1.465302	0.6640897	1.920830	4.138241	25.27607	216.7136			
6s		0.4087664	0.2371839	5.085877	28.73246	1194.287	68721.58			
57	Lanthanum Atom	Doublet D State			$\text{Xe}(54)54(1)6s(2)$ Configuration			(12s, 8p, 6d) Basis		
		T	$\nabla(1)$	$\nabla(2)$	$\nabla(3)$	E	e	-Cusp	r max	
		Total S Cusp: -56.897470	Total P Cusp: -28.297596	Total D Cusp: -17.795249						
		τ^{-3}	τ^{-2}	τ^{-1}	τ^0	τ^1	τ^2	τ^3	τ^4	
		8220.9753.2	-19462.631474	3020.681732	-16441.949742	-8220.974810	-5200.292678			
1s	1594.189127	-3218.505135	126.038943	-3092.466192	-1498.277056	-1372.238113	56.924625	0.017657		
2s	340.0993.9	-743.573069	93.622058	-649.951011	-309.851682	-216.229623	56.567405	0.097063		
2p	339.9052.9	-742.792255	99.510940	-643.281315	-303.376036	-203.865096	28.351313	0.075902		
3s	104.6283.8	-276.223861	61.895183	-214.328678	-109.700309	-47.805126	57.231994	0.273215		
3p	101.9840.0	-271.636130	63.616270	-208.019859	-106.035829	-42.419559	27.923293	0.261195		
3d	97.2053.2	-264.156799	67.219446	-196.935353	-99.730411	-32.514565	17.896582	0.210886		
4s	31.2304.7	-115.478476	36.971210	-78.503755	-47.273309	-10.302188	56.957150	0.634623		
4p	26.8519.1	-105.962337	36.446316	-73.514021	-46.662070	-8.213754	28.814905	0.647462		
4d	23.5933.1	-97.798457	34.848984	-62.949478	-39.356162	-6.507178	17.581140	0.673938		
5s	6.4075.9	-44.978092	18.65030	-26.523062	-20.115872	-1.660442	57.090721	1.476500		
5p	5.2121.4	-40.172847	16.575637	-23.197210	-17.985076	-1.009635	28.836460	1.620532		
5d	2.9371.5	-29.784294	13.303522	-16.408772	-13.543627	-0.240105	17.171658	2.107526		
6s	0.6107.6	-18.191571	6.711129	-7.480441	-6.869666	-0.158536	57.964185	0.319678		
		τ^{-3}	τ^{-2}	τ^{-1}	τ^0	τ^1	τ^2	τ^3	τ^4	
		16590.38	341.4497	45.30923	99.92461	2223.998	108489.6			
1s		6396.785	56.46500	2.6643457D-02	9.4934722D-04	2.7223485D-06	3.8509583D-05			
2s		689.1294	13.04514	0.1141235	1.5330561D-02	2.7009091D-03	0.1997536			
2p	6160.939	229.6025	13.03184	9.7171762D-02	1.469967D-02	2.5759443D-04	1.1513976D-05			
3s		143.9131	4.846023	0.2933758	0.1016328	2.2152058D-02	0.5801360			
3p	1202.532	47.70911	4.765546	0.2887487	0.64580553D-02	1.4858223D-02	6.7197423D-03			
3d	198.8135	26.66169	4.634365	0.2547153	7.9319555D-02	1.12409329D-02	2.6112441D-03			
4s		32.66159	2.025875	0.6815864	0.5226587	0.3946503	0.4139858			
4p	250.1339	10.66658	1.9291764	0.7047462	0.5628732	0.4722933	0.6073191			
4d	33.43230	5.248170	1.715762	0.7637816	0.6752372	0.7407060	1.430746			
5s		5.471101	0.7890893	1.641718	3.017357	13.2958	81.17081			
5p	34.36513	1.665560	0.7047868	1.818236	3.707427	20.22236	154.3163			
5d	2.847959	0.5897884	0.5225315	2.370234	6.275181	57.68013	727.3465			
6s		0.4531288	0.2489749	4.857858	26.25934	1003.655	53323.82			

Table 12. (Continued)

6s	Gadolinium Ion (+3)	Octuplet S State	Xe(54) 6s(0) 4f(7) Configuration	{10s, 8p, 4d, 3f} Basis					
				12-V/EI = 3.50-05					
Total S Cusp:	-64.92829		Total P Cusp:	-31.370896		Total D Cusp:	-20.333262	Total F Cusp:	-11.941710
T	V(1)	V(2)	V(T)	E	e	-Cusp	r max		
10818.8620t2	-25623.976782	3985.869866	-21638.106916	-10819.244854	-6833.374989				
1s	2013.506745	-4061.315548	149.790249	-3911.525299	-1898.018555	-1748.228306	64.175927	0.015723	
2s	436.2967c5	-945.560134	112.866751	-832.693383	-396.396679	-283.529928	64.436200	0.085902	
2p	436.393525	-945.072929	119.687281	-825.385648	-388.992123	-269.304843	31.387856	0.066953	
3s	139.5910t8	-357.871755	76.643346	-281.228410	-141.637352	-64.994007	64.147144	0.239173	
3p	136.9122f2	-353.276316	78.879256	-274.398361	-137.486108	-58.607853	31.275811	0.226690	
3d	131.997049	-345.825569	83.480322	-262.345247	-130.48198	-46.967876	20.400677	0.181507	
4s	42.997779	-151.589083	47.069936	-104.179108	-61.481328	-14.411393	62.765890	0.548087	
4p	40.207861	-145.433886	46.693295	-98.740591	-58.532730	-11.839345	31.455710	0.552814	
4d	34.0286f5	-131.807844	45.283153	-86.524691	-52.500086	-7.216933	20.008591	0.562115	
5s	8.3960t7	-57.873683	23.300621	-34.573062	-26.176985	-2.876364	62.958286	1.283941	
5p	6.8729L7	-51.854133	21.455460	-30.398673	-23.525726	-2.070266	31.487359	1.400329	
4f	22.107042	-103.639563	39.801080	-63.838483	-47.731841	-1.930361	11.941710	0.572956	
T ⁻²	T ⁻¹	T ⁻¹	T	T ²	T ⁴	T ⁴			
21121.63	400.3746	33.94083	35.81981	99.56165	515.4729				
1s	8078.213	63.45806	2.3701081D-02	7.5108427D-04	2.2266587D-06	6.3585997D-06			
2s	884.7760	14.77438	0.1008285	1.1932830D-02	2.6063195D-04	1.6962206D-04			
2p	8907.517	294.2919	14.76676	8.5639601D-02	8.8999930D-03	1.6944002D-04	1.6896640D-04		
3s	151.3566	5.591766	0.2595316	7.6787545D-02	8.8529198D-03	1.5033052D-03			
3p	1821.5t7	63.65530	5.519947	0.2497218	7.2156664D-02	8.4968990D-03	5.2044934D-03		
3d	311.67t2	36.11120	5.493525	0.2219158	5.7706296D-02	5.8832545D-03	9.7393118D-04		
4s	44.83286	2.367954	0.5867731	0.3876568	0.2186396	0.1859852			
4p	392.91t6	18.74117	2.272404	0.6008767	0.4097162	0.2498510	0.2135234		
4d	56.01774	7.514419	2.059498	0.6386783	0.4710427	0.3520874	0.3829200		
5s	7.165784	0.9042763	1.429642	2.286619	7.625947	35.47778			
5p	51.08755	2.191189	0.8102208	1.582756	2.815595	11.77814	69.46503		
4f	8.954592	3.344071	1.619368	0.7742811	0.7380877	1.150808	3.171367		
86	Radium Atom	Singlet S State	Xe(54) 4f(14) 5d(10) 6s(2) 6p(6) Config.	{12s, 10p, 6d, 3f} Basis					
				12-V/EI = 4.40-08					
Total S Cusp:	-85.734328		Total P Cusp:	-42.300270		Total D Cusp:	-27.299712	Total F Cusp:	-18.904578
T	V(1)	V(2)	V(T)	E	e	-Cusp	r max		
21866.627446	-51979.840664	A246.586729	-43733.253935	-21866.626489	-13620.039761				
1s	3650.316415	-7348.115260	233.765125	-7114.350135	-3464.033730	-3230.269605	85.954552	0.011679	
2s	816.992212	-1738.485914	182.311482	-1556.17433	-739.182141	-554.870659	85.614759	0.063283	
2p	818.5219t1	-1739.464982	192.157638	-1587.311844	-728.789903	-536.632265	42.289574	0.048998	
3s	280.921431	-681.077366	130.882715	-550.194644	-269.273243	-138.395028	86.474009	0.171642	
3p	278.264311	-676.521202	134.807436	-541.713766	-263.449045	-124.641968	42.392215	0.161209	
3d	273.978731	-670.128336	182.740897	-527.387838	-253.409048	-110.668550	27.316855	0.126050	
4s	97.375738	-304.108520	86.419041	-217.686479	-120.306681	-33.688640	86.045663	0.377679	
4p	93.705032	-296.713938	86.776014	-209.937924	-116.232902	-29.456888	42.118118	0.376699	
4d	86.350517	-282.270664	87.311023	-194.959641	-108.608653	-21.297630	27.229259	0.365182	
4f	73.357416	-257.781515	87.173503	-170.68012	-97.250598	-10.077095	18.904578	0.318004	
5s	26.657935	-132.734467	49.601698	-83.132769	-56.474833	-6.873135	86.340491	0.804013	
5p	23.862533	-124.506819	47.25275	-76.781544	-52.918959	-5.193685	42.240826	0.837935	
5d	17.805332	-105.805718	42.851899	-62.953819	-45.148837	-2.2965328	27.355766	0.933889	
6s	4.422310	-50.509278	22.614667	-27.894612	-23.472301	-0.857635	86.975567	1.908538	
6p	3.128053	-42.588338	19.522975	-23.053633	-19.737311	-0.816335	42.055759	2.165037	
T ⁻²	T ⁻¹	T ⁻¹	T	T ²	T ⁴	T ⁴			
39021.77	604.4168	53.43232	79.69693	587.1252	8488.617				
1s	14630.91	65.44320	1.75917682D-02	4.1348254D-04	4.5757442D-07	5.0215742D-07			
2s	1649.565	20.21495	7.38021573D-02	6.3856738D-03	1.1523511D-04	8.8284237D-04			
2p	22680.45	550.4802	20.22639	6.2351197D-02	4.7061273D-03	7.0518743D-05	8.4430568D-04		
3s	383.3463	7.919508	0.1446296	3.8856450D-02	2.5164176D-03	5.3346585D-03			
3p	5064.098	128.10438	7.866526	0.1756236	3.5585269D-02	2.1199775D-03	4.6614250D-03		
3d	911.3519	74.50338	7.792190	0.1529047	2.7259783D-02	1.3264165D-03	2.8144675D-04		
4s	101.0031	3.536099	0.3583669	0.1788923	4.472007D-02	1.6658100D-02			
4p	1254.777	33.68033	3.450162	0.3998719	0.1811371	4.9320114D-02	6.4577807D-02		
4d	204.7819	18.64457	3.282217	0.4005858	0.1844623	5.2682498D-02	2.2852923D-02		
4f	49.26306	10.91193	2.997459	0.3982733	0.1866329	6.2945596D-02	3.4545456D-02		
5s	22.48467	1.543424	0.8619231	0.8238358	0.9688189	1.532637			
5p	248.5696	7.270370	1.467756	0.9112325	0.9305557	1.250720	2.317918		
5d	31.30099	3.375901	1.230299	1.052835	1.253813	2.372411	6.386622		
6s	3.206199	0.5873172	2.135709	5.086897	37.49135	381.9976			
6p	25.71874	0.8803495	0.4952132	2.526460	7.206486	79.52510	1273.767		

BASES FOR MOLECULE CALCULATIONS

General Considerations

Although exponential-type primitive functions are best for expanding atomic orbital components of molecular wavefunctions, the calculation of three- and four-centered integrals is a major bottleneck to their use. An alternative has been widely applied and accepted, namely the Gaussian type primitive functions defined by Eq. 19, with $p=2$. This function has been gaining popularity ever since it was introduced in 1950 by Boys (60). Reviews enumerating the molecules which have been treated using Gaussians have been written by Allen and Karo (61), Allen (62), and Clark and Stewart (63).

All integrals which form matrix elements of the molecular Hamiltonian operator are readily evaluated in closed form, the multi-centered integrals in particular being expressed in terms of the well-known incomplete gamma function. A defect of the Gaussian primitive is that it does not have the proper short- or long-range behavior required to satisfy the demonstrable boundary conditions of molecular or atomic wavefunctions (42,43). As a consequence, large numbers of Gaussian primitives centered at each atom nucleus with widely ranging exponential parameters must be used in order to simulate the proper behavior, especially the short-range one.

The use of Gaussians therefore implies relatively large bases for good calculations on polyatomic molecules. Now, the number of two-electron integrals generated by an n function basis is $[n(n+1)/2] \cdot [n(n+1)/2 + 1]/2$. SCF matrix elements formed from these and other integrals require at least $n(n+1)/2$ words of core. These figures rapidly approach certain practical limits. For example, in a (9s,5p) basis for diatomic nitrogen there are forty-eight primitive Gaussians ($n=48$). This implies of the order of $7 \cdot 10^5$ integrals and symmetric matrices which require 1176 words. In order to reduce this number so that even larger molecules may be considered, the idea of independent variation of a basis of Gaussian primitives must be abandoned and instead certain linear combinations of Gaussian primitives taken as basis. These "contracted" or "combined" Gaussians play a role similar to that played by single exponentials. Integrals over Gaussian primitives need not be stored since only those over combined functions are ever used. This saves data handling time. Moreover, in the new basis the matrices are smaller and require less core storage during SCF processing.

Contracted Gaussian function bases representing atomic orbitals were suggested in 1963 by Reeves and Harrison (18). Subsequently, Taketa et al. (64) in 1966 discussed procedures for efficiently calculating integrals for combined functions. The functions have variously been called combined

Gaussian-type orbitals (CGTO's), contracted Gaussian functions, and combined Gaussian functions (CGF's). Since the individual Gaussian primitives are not separate variational functions, the coefficients in the linear combination which define the combined function $G_{lm}(r)$ are required prior to the molecular calculation. Various authors differ in the method of pre-forming the contracted basis functions. References (37, 44, 65-85) contain various Gaussian atomic orbitals and recommended contractions for molecular calculations. The following deals with a new method for generating the contracted or combined Gaussian functions.

Even-Tempered Gaussian Representations

of SCF Atomic Orbitals

A logical procedure to construct combined Gaussian functions is first to seek such functions which represent good approximations of accurate SCF atomic orbitals. These also form a good zeroth order approximation for a molecular basis where those atoms so represented are involved. Accurate atomic SCF orbitals such as those described earlier in this work are available at this time for nearly every atom up to krypton and for many beyond. They are generally given in terms of a basis of exponential functions. In 1965, Reeves and Fletcher (86) described Gaussian approximations to single exponentials as linear combinations of Gaussians found by an

optimized least-squares fitting. Subsequently O-ohata et al. (37) presented various length expansions of this type showing that "basic" Gaussians (i.e. composed of a solid harmonic and a pure Gaussian without additional powers of r) are sufficient to represent more general types of exponential-type functions. Shavitt (87) generalized the Gaussian fitting expressions to include arbitrary smooth functions of r while also including an arbitrary weighting function $W(r)$.

In this work, even-tempered Gaussian primitives are used, and the weighted least-squares deviation

$$(25) \quad D_{lm}(\alpha, \beta) = \int dV [\phi_{nlm}(r) - G_{lm}(r)]^2 W(r)$$

is minimized with respect to the orbital exponent parameters α and β of Eq. 22 and the coefficients in the even-tempered expansions $G_{lm}(r)$ of Eq 20. The accurate SCF atomic orbital is denoted by $\phi_{nlm}(r)$. Since dV is the ordinary three-dimensional volume element, the angular parts of $\phi_{nlm}(r)$ and $G_{lm}(r)$ integrate to unity leaving the radial integral with volume element $r^2 dr$. The weighting function $W(r)$ is discussed below.

Rather than using specific Gaussian primitives for fitting each atomic orbital, the same set of Gaussian primitives is used to fit all atomic SCF orbitals of the same symmetry in an atom, a symmetry being defined by a value of the quantum number l . In order to obtain optimal values for α and β ,

which serve equally well for all SCF atomic orbitals of one symmetry, the sum of deviations for those orbitals,

$$(26) \quad D_1(\alpha, \beta) = \sum_n w(n) N(n) D_{nl}(\alpha, \beta) ,$$

is minimized. Here the $N(n)$ are the SCF orbital occupation numbers and the $w(n)$ are additional adjustable weighting factors. For the expansions included in this work, the atomic orbital weights were taken to decrease from core level to valence level by orders of magnitude. For example, the s orbitals of sodium were weighted by $(wN)=2.0, 0.2$, and 0.01 in the sequence 1s, 2s, and 3s. This is a matter of judgement and experience, but is felt to be logically consistent since first of all, the inner orbitals are more important to the energy. Energy quantities for atoms regularly vary in orders of magnitude in the same way the weights have been chosen. For examples, see Table 12. Secondly, since core orbitals describing inner shells are essentially constant in different molecular environments, it is anticipated that little adjustment of the atomic core orbitals is needed for molecules. Therefore it is consistent to represent these more invariant orbitals as well as possible and the more deformable valence atomic orbitals less well. The addition of correction terms may then be confined to only those corrections needed for the valence shells.

We return to the question of the weighting function $W(r)$ in Eq. 25. First, the possibilities r^{-2} , r^{-1} , and 1 were investigated. For several typical atoms, the corresponding Gaussian representations were calculated for all SCF atomic orbitals. Then the total atomic energies were calculated. It was found that the representations obtained using $W(r)=r^{-1}$ gave energies closest to the actual SCF energies. This is reasonable, because of the major role played by r^{-1} in the large nuclear attraction energy term of the atomic Hamiltonian. This weighting function was generally adopted.

Because of various quantities involved in the analytic expressions for integrals, it was thought that a great deal of time might be saved during integral computation if the exponents for Gaussian primitives of all symmetries of an atom were drawn from the same even-tempered set. To obtain the optimal basis set, one (α, β) pair was determined by minimizing a total atomic least-squares deviation. This total deviation was again taken as a weighted sum of individual values similar to Eq. 26

$$(27) \quad D(\alpha, \beta) = \sum_{nl} w(n) N(n, l) D_{nl}(\alpha, \beta) .$$

Again, the $N(n, l)$ are occupation numbers and $w(n)$ adds additional weight factors. As indicated in the equation, the additional factors $w(n)$ are assumed to depend only upon the

principal quantum number n . These are taken to be 1, 0.1, 0.01, etc. for $n=1, 2, 3, \dots$. For example, combined weights and occupation numbers (w_N) for atomic orbitals 1s, 2s, and 2p of the nitrogen atom were taken to be 2.0, 0.2, and 0.3 respectively. This weighting produced fittings which appear satisfactory and logically consistent with the considerations applied in the case where the symmetries were left unmixed. This limitation to one pair, (α, β) , for all symmetries is of course a substantial restriction on the primitive basis as compared to allowing separate pairs for different symmetries. The effect of the restriction was measured by comparing the optimal least-squares deviations obtained for the individual orbitals by the two methods. It was found that the unified (α, β) approach leads to worse least-squares deviations but the deterioration is not substantial.

Considering the 1s, 2s, and 3s orbitals of argon, the 2s orbital is more expanded radially than the 1s orbital, likewise the 3s is more expanded than the 2s. The optimal Gaussian s-basis will contain primitives required to represent all three atomic orbitals. Some primitives with large exponents are important for the 1s, others with smaller exponents for the 2s, and still smaller for the 3s. However, to some degree all Gaussians contribute to all the SCF atomic orbitals. This is mainly a consequence of the orthogonality requirement. The SCF orbital orthogonality is similar to that of

orbitals obtained by Schmidt orthogonalizing Slater-type 1s, 2s, and 3s orbitals. That is, outer SCF atomic orbitals have considerable contributions of a similar character from inner SCF atomic orbitals and the nodal behavior is also mimicked. In view of this analogy, it is to be expected that one might express the orthogonal atomic SCF orbitals in terms of an equal number of non-orthogonal atomic SCF orbitals, which like the Slater-type atomic orbitals, would have substantial values over a smaller range of the radial variable. The resulting distribution of charge would be considerably more localized radially. For this reason, each non-orthogonal orbital would require a substantial contribution from a limited number of the Gaussian primitive set only and such "de-orthogonalized" SCF atomic orbitals would therefore offer certain advantages. It must be noted, however, that in contrast to specific orthogonalization procedures, the process of deorthogonalization is not unique in any sense. Nevertheless, a method was devised to form non-orthogonal SCF atomic orbitals which have just the desired qualities. It is described in Appendix B.

In this manner, it became feasible to restrict the number of Gaussian primitives contributing to each non-orthogonal SCF atomic orbital to less than the total number of primitives. This shortening of the expansions saves valuable time in summations defining contracted Gaussian inte-

grals in terms of primitives. In the present work, all SCF atomic orbitals are non-orthogonal SCF atomic orbitals in Gaussian expansion. Examples are displayed in Tables 13-18, 20, and 22. The columns labelled with the unprimed ordinary atomic orbital designations, 1s, 2s, 2p, etc. are the Gaussian representations of non-orthogonal atomic orbitals. Note that the zetas for the p Gaussians are a subset of the s Gaussian zetas. Representations of SCF atomic orbitals restricted to include only a part of the Gaussian primitive functions as just discussed are found only in Tables 17, 18, 20, and 22.

Atomic Orbital Bases for Molecular Calculations

Certainly SCF atomic orbitals are the most important atomic orbitals for molecular calculations. They may be termed a minimal basis set of SCF atomic orbitals or a zeroth order basis. In particular, one finds that localized core or inner shell functions occupied by core electrons in a molecule are very nearly the same as compared to the atom inner shells. However, the physical environment of an atom in a molecule is substantially different from that of an isolated atom and the valence shells, which extend over the entire molecule, have substantial distortions arising from the extra-atomic molecular forces. In order to take these effects into

account, additional contracted basis functions are provided. The distortions can be divided into two categories: scaling and polarization.

Scaling and pseudoscaling

Scaling of a basis function implies the inclusion of a scale parameter t ($t>0$) which multiplies the radial coordinate r wherever it appears. Each orbital is also multiplied by $t^{3/2}$ to keep it normalized to unity. This parameter has the effect of "contracting" ($t>1$) or "dilating" ($t<1$) the original function ($t=1$) while retaining its shape. Dilation creates a more diffuse charge while contraction causes the charge to concentrate about the nucleus. Infinite contraction ($t\rightarrow\infty$) would simulate a point charge at the nucleus, while infinite dilation ($t\rightarrow 0$) would approach a distribution uniformly spread over all space. The scale parameters of the contracted atomic orbitals represent valuable additional parameters for the molecular variational process and the best molecular energy is obtained by minimizing the energy with respect to the scale parameters (49). This is a procedure which implies recalculating the energy for many different t values. Explicit scaling such as this involves the repetition of almost all steps in the molecular calculation because the primitive basis actually changes with t . This is clearly to be avoided if possible. An alternative which avoids such

non-linear parameter optimization consists in adding to the SCF minimal basis set additional atomic orbitals, judiciously chosen and of such nature that, in the enlarged linear space, reasonably good approximations to the scaled SCF orbitals may be found. If done effectively, this use of the enlarged basis set in a molecular calculation obviates the need for introducing explicit scale parameters. The determination of the contribution of each basis function to the molecular wavefunction is then reduced to that of finding coefficients for linear expansions.

The procedure for selecting additional effective variational functions (88) is as follows. First, a scaling range (t_i, t_f) is established for an SCF atomic orbital. For twenty different values of the scale parameter t distributed over the scaling range, Gaussian representations of the scaled SCF atomic orbitals are found in terms of the same Gaussian primitives determined for the unscaled ($t=1$) SCF atomic orbital. Finally, a set of orthogonal basis functions is determined in the space spanned by the resulting twenty expansions and its members are ranked in order of importance. The important ones are selected as additional contracted Gaussian functions. Good judgement and accumulated experience defines which atomic orbitals are to be used to form the extra functions. Since this procedure derives from the explicit scaling of SCF atomic orbitals, the functions created by it ought

to be useful for the simulation of scaling through linear variation. Since the additional functions are found independently for each SCF atomic orbital, substantial overlap between those arising from separate SCF atomic orbitals may exist. Checks for such linear dependencies must be made and dependent functions discarded. This function so determined will be referred to as a "pseudo-scaled" basis.

The determination of the transformation is carried out as follows. Extending Eq. 20 one may write the twenty scaled Gaussian linear combinations

$$(28) \quad G_{jlm}(\underline{r}) = G_{lm}(t_j \underline{r}) = \sum_{k=1}^M X(klm) c_{jk}$$

where $j=1, 2, \dots, 20$, with $M \leq 20$, and with the t_j 's equally distributed over the chosen scaling range. The Gaussian primitives $X(klm)$ depend on the same exponential parameters α and β which were determined for the original unscaled SCF atomic orbital. The new set of contracted Gaussian functions $G_{ilm}'(\underline{r})$ are formed according to the relations

$$(29) \quad G_{ilm}'(\underline{r}) = \sum_j G_{jlm}(\underline{r}) T_{ji}$$

where

$$(30) \quad \underline{T} = \underline{U} S_D^{-1/2} .$$

The matrix \underline{U} is that orthogonal matrix ($\underline{U}^+ = \underline{U}^{-1}$) which diagonalizes \underline{S} , the real symmetric matrix of overlap integrals between the scaled functions $G_{jlm}(r)$. The diagonal matrix is \underline{S}_D . This transformation is called canonical orthonormalization (89) and has the important feature of allowing easy discrimination between the new functions with respect to importance. Each new function is associated with one of the eigenvalues of \underline{S} . Because of linear dependence, ($20-M$) of the eigenvalues will be exactly zero. The M functions G_{ilm}' corresponding to non-zero eigenvalues span the entire space and are mutually orthogonal. Moreover they may be ordered according to the magnitude of the eigenvalues, the largest being the most important, etc., with respect to expanding the scaled SCF atomic orbital in the given scaling range. In the present work, the first two or three of these transformed basis orbitals have been used as extra basis functions. It may be observed that the first, most important function is very nearly the same as the original unscaled ($t=1$) Gaussian contraction. Somewhat better results were obtained by replacing it with the original unscaled SCF atomic orbital although this introduced a slight non-orthogonality between this SCF atomic orbital and the second and third of the G' functions. A newer version of this method, not used here, first projects the unscaled SCF atomic orbitals out of the space of all scaled SCF atomic orbitals. The canonical or-

thonormalization is then used to obtain the extra functions G' from the remaining space which is orthogonal to the original unscaled SCF atomic orbital. The columns in Tables 13-18, 20, and 22, headed with primed atomic orbital designations (e.g. $2s'$, $2p''$) are those obtained by the above procedure.

An extensive set of programs was developed to automatically form scaled atomic orbital contracted Gaussian basis functions as described (90). The input consists of a set of accurate atomic orbitals for some atom expressed as linear combinations of exponential functions, weight parameters $w(n)N(n,l)$ for the least-squares step, the numbers of Gaussian primitives to be used to expand each symmetry along with designated subsets for each non-orthogonal atomic orbital if desired, scaling ranges for each individual atomic orbital, and the number of scaled functions to be produced for each atomic orbital. Output consists of data defining the contracted Gaussian functions which may be used for molecular calculations.

Polarization

Polarization functions are those atomic orbitals used in a molecular calculation which belong to a symmetry not occupied in the SCF approximation to the ground states of constituent atoms. Examples are p functions in hydrogen atom

and d functions in sulfur. In the atomic SCF wavefunctions they are excluded by symmetry constraints. However, the molecular symmetry orbitals are mixtures of atomic orbitals with much less restriction as to the symmetry of included atomic functions and hence, polarization functions play an important role in molecular orbitals. Early calculations showed that the incorporation of a p_z function into the basis for a hydrogen molecule has a significant effect on the result (91). Mulliken distinguished between two classes of polarization effect (55): valence polarization, representing a redistribution of electrons among the valence orbitals, and Coulomb polarization, being a deformation of a valence orbital due to the field of neighboring nuclei. While atomic SCF orbitals are a good zeroth order expansion basis for a molecule, the addition of appropriate polarization functions is necessary to produce more realistic charge distributions. The selection of these functions is not trivial and often much work in terms of expensive calculations must be expended. Each function which is added must be optimal for a molecular environment in order that the number of such functions is kept small. This is because of the fact that functions of higher symmetry usually require more calculation time than those of lower symmetry.

The radial extent of useful polarization functions must be approximately that of the outermost valence atomic orbi-

tals (55). Roos and Siegbahn (75) applied this criterion to recommend single δ -Gaussian polarization functions for all atoms from boron through fluorine and aluminum through chlorine on the basis of their optimal results for oxygen and sulfur. They also recommended single and double p-Gaussians for polarization of hydrogen.

In the work described here which involves the atoms hydrogen, lithium, and sodium, only p-type polarization functions are included. It was not possible to incorporate d functions because of program restrictions. It can however be assumed that they are less important than the p functions. For these atoms, the highest occupied orbital in the neutral atom 2S ground state is an ns orbital, $n=1,2,3$ respectively. In order to obtain p functions with an appropriate radial extent, the 2P states of lithium and sodium, formed by moving the lone electron from the ns to the np orbital, were computed using the atomic SCF programs described in Appendix A. In this way an SCF p-atomic orbital was found which accurately describes the lone np electron and which is concentrated near the nucleus to about the same extent as a single ns electron. The radii at which these lithium 2s and 2p orbital distributions attain their respective maxima are 3.10 a.u. and 3.82 a.u.; for sodium they are 3.39 a.u. and 4.90 a.u.

For hydrogen a different tack was necessary. The 2p orbital of the 2P state has a radial distribution maximum at

4.0 a.u., far beyond that of the 1s function, which is 1.0 a.u. It is desirable to determine "atomic orbitals" in the form of a 1s and a 2p Slater-type orbital for use in the fitting process. In order to do this, first of all, the exponent of a 1s exponential-type orbital was optimized with respect to the performance of its contracted even-tempered Gaussian pseudo-scaled (6s)/[3s] basis in an SCF calculation on the hydrogen diatomic molecule ($r=1.4$ a.u.). The optimal energy was obtained for an exponent of 1.1. Secondly, a single 2p exponential was expanded in a Gaussian pseudo-scaled (4p)/[3p] p basis where the Gaussian exponents of the p primitives are the same as those of the s Gaussians obtained above, in accord with an earlier restriction. By a series of hydrogen molecule calculations with different fitted bases obtained from Slater-type p-orbitals with different exponents, the single p-exponent was fixed optimally at 1.30. Through trial of various weights, (w_N), for these fittings it was found that unweighted p-orbitals produced the best energies. Hence molecule adapted single exponential atomic orbitals were determined for hydrogen to be used in generating even-tempered Gaussian pseudo-scaled bases of any desired expansion length. The distribution of the hydrogen s atomic orbital so found attains a radial maximum at 0.91 a.u. while the p orbital distribution has a maximum at 1.54 a.u.

Choice of Two Bases

With the methods described, two contracted even-tempered pseudo-scaled Gaussian bases were formed representing the SCF atomic orbitals and polarization atomic orbitals for each of the three atoms. Weights, $w(n)N(n,l)$, of Eq. 27 for polarization orbitals were taken to be zero on the basis of some earlier exploratory calculations on the hydrogen molecule so that the expansions of SCF atomic orbitals which are important to the energy would not be biased by the less important polarization atomic orbitals. With the help of the procedure for adding functions to describe orbital scaling, this minimal, polarized basis was augmented with several contracted functions. The resulting pseudo-scaled bases, referred to as A(smaller) and B(larger), which are displayed in Tables 13-18 were then used for all molecular calculations described in the following section.

Although we have been forced to use least-squares approximations to SCF atomic orbitals, it is generally known that comparable Gaussian expansions, which are optimal for the atom in terms of the energy, are best as a zeroth order molecular basis. An improvement to the method described here then would be to determine even-tempered Gaussian SCF atomic orbitals directly as was done with exponentials in the preceding section using the energy criterion. These could be converted to non-orthogonal atomic orbitals and furthermore,

a scaling procedure much like that used here could be constructed. Alternatively, the deorthogonalized virtual orbitals of an atomic SCF calculation might be used as added basis functions. It is hoped that these possibilities will be investigated in future work.

Before going on to the molecular calculations, we now display some results which serve to give some idea about the quality of the bases used here.

Comparison of Segmented Contracted Bases with Pseudo-Scaled Contracted Bases

Contracted Gaussian basis sets for molecular calculations are derived from a primary set of atomic orbitals. This is done because it has become apparent that the space spanned by the atomic orbitals as such forms a subspace of Hilbert space which is extremely important for the expansion of molecular orbitals. Many investigators in this field use SCF atomic orbitals directly. Here use is made of weighted least-squares approximations to SCF atomic orbitals as described. The present method of deriving a contracted pseudo-scaled basis from the primary atomic orbitals is substantially different from that used by others. Since new, it is therefore of interest to compare it with the conventional technique.

In the present method, the SCF atomic orbitals are used

for some of the contracted basis functions. Additional linearly independent functions are formed such that the entire set spans a space designed to contain the space spanned by the scaled primary SCF atomic orbitals. In contrast, in other methods, the primary SCF atomic orbitals are not part of the molecular basis. Instead, the primary atomic orbitals are cut into segments, each expanded in terms of a subset of the primitive expansion basis. From these are selected those segments to be used as contracted functions. An important criterion used is that no one Gaussian primitive will contribute to more than one contracted basis function. The coefficients in these atomic orbital segments are ordinarily the self-consistent field coefficients defining the SCF atomic orbitals. In the common case in which several atomic orbitals of the same symmetry depend on the same group of Gaussian primitives, those specific linear coefficients are taken which correspond to the orbital to which each Gaussian primitive is the principal contributor. For example, the Gaussian primitives with large exponents are more important to inner shell atomic orbitals while those with small exponents are important for the outer or valence shells. This contraction scheme will be referred to as the segmented-basis scheme.

Recently Dunning has proposed segmented bases of this type for the first-row atoms (81) where the segmentation scheme is optimal. These contractions were optimized in the

framework of a molecular calculation in order to remove any bias in favor of atoms. Several choices are given, depending on the number of contracted basis functions desired. The Gaussian primitives used are the energy optimized (9s,5p) results of Huzinaga (44).

A direct comparison is difficult since it is not possible using the present programs to form a pseudo-scaled basis with the same Gaussian primitives. Thus the comparison made here is to form a pseudo-scaled even-tempered basis as prescribed above, and compare it with a segmented even-tempered basis made using the same (9s,5p) Gaussian primitive set. The segmented basis contracted functions parallel those of Dunning (81) since the same numbers of Gaussian primitives, (9s,5p), were used and the zeta values are similar.

The segmented-basis and scaled-basis contractions are presented in Tables 19-22 for oxygen and nitrogen. For the segmented bases, each column of coefficients headed by the notation [ns] or [mp] represents a group of contracted functions. The horizontal lines separate different contracted functions, one from another (note that the s functions are rearranged from their normal increasing zeta order so that the contracted functions could be displayed in the simple fashion used). In the case of the scaled bases, each column adjacent to the column of zetas represents one contracted function. The 1s, 2s, and 2p contracted functions are the

best weighted least-squares approximations to the SCF atomic orbitals used for fitting. The $2s'$, $2s''$, $2p'$, and $2p''$ are the extra contracted functions added to include "pseudo-scaling" in the basis. No $1s'$ or $1s''$ functions are displayed since these were found to be relatively ineffective with respect to improvement of molecular energies. This is in accord with Dunning's observation that segments representing diffuse distributions are more important to the molecular energy. Since no primed functions of the $1s$ variety were used it is easy to see how a collection of scaled-contracted functions is selected. The $1s$, $2s$, and $2p$ SCF type functions serve as a core which is the zeroth-order or unscaled approximation. To these are added primed functions either $2s$ or $2p$ as desired. Presumably, if $1s$ primed functions were included, they would follow the $2s''$ function in effectiveness with respect to molecular calculations. Calculations on H_2O and N_2 with Dunning's geometries were carried out. For H_2O , the same segmented-basis contracted functions were used for hydrogen in both cases since only the effect of the contracted oxygen bases is of interest.

The results of calculations with these bases on H_2O are presented in Table 23. We see that the pseudo-scaled basis far surpasses the segmented-basis in terms of the energy obtainable with the smallest number of contracted functions. The [2s, 2p] pseudo-scaled basis gives a lower energy than the

[4s,3p] segmented basis. The difference is five contracted basis functions. Also it may be noted that the virial coefficient V/E attains a better value for the pseudo-scaled basis. This is a partial confirmation of the goal that the pseudo-scaled basis incorporates a large share of the variability obtainable by scaling. Moreover, this large reduction of the basis size saves substantial computation time.

Table 24 displays the corresponding results for the N₂ calculations. Here the advantage of the scaled-basis scheme is less distinct but an energy advantage over the segmented-basis is still held. There is also a better virial coefficient as before.

The two examples given do not prove conclusively that the pseudo-scaled technique is better than the segmented-basis technique in all cases, however, the comparison presented here is consistent within itself and does display a pseudo-scaled basis advantage. An interesting comparison would be one in which Dunning's results (81) are compared directly to pseudo-scaled basis results where the pseudo-scaled basis employs the same (9s,5p) Gaussian primitives as those used by Dunning. It is anticipated that the proposed comparison will show the same trend of results as those in Tables 23 and 24.

Table 13. Even-tempered Gaussian basis for hydrogen

		Alpha = 0.0427223860	Beta = 3.9731435139	
k	Zeta(s)	1s	1s'	1s''
1	0.169742	0.534455	-1.208282	-0.826963
2	0.674410	0.468828	0.980996	1.774329
3	2.679528	0.101798	0.229149	-1.360457
4	10.646148	0.033332	0.108512	-0.136441
k	Zeta(p)	2p	2p'	
1	0.169742	0.596790	-1.000601	
2	0.674410	0.495517	0.899730	
3	2.679528	0.070379	0.234315	

Table 14. Even-tempered Gaussian A basis for lithium

		Alpha = 0.0134244004		Beta = 4.4087312064		
k	Zeta(s)	1s	1s'	2s	2s'	2s''
1	0.059185	0.003252	-0.130685	-1.061541	0.976542	-0.322977
2	0.260929	-0.066512	0.700168	0.069603	-1.767186	1.069250
3	1.150365	-0.591262	0.423309	0.069026	0.553693	-1.956915
4	5.071651	-0.401180	-0.723107	0.006087	-0.089143	0.843025
5	22.359547	-0.083839	-0.221324	0.000385	0.036194	-0.196413
6	98.577233	-0.022567	-0.065388	0.000021	-0.007027	0.043458

k	Zeta(p)	2p	2p'
1	0.059185	0.914106	-0.763537
2	0.260929	0.133451	1.205215
3	1.150365	0.041092	-0.070168
4	5.071651	0.002641	0.056667

Table 15. Even-tempered Gaussian A basis for sodium

		Alpha = 0.0271184334				Beta = 3.9423341969		
k	Zeta(s)	1s	2s	2s'	3s	3s'	3s''	
1	0.106910	-0.007788	-0.033856	0.054357	1.360960	-0.804941	-0.262917	
2	0.421475	0.019317	0.361320	-1.134960	-0.664814	2.042962	0.997811	
3	1.661594	-0.032587	0.754129	0.618581	0.149688	-1.036179	-2.186997	
4	6.550559	0.224629	-0.011300	0.705214	-0.065941	0.289267	1.211562	
5	25.824492	0.601177	-0.091333	-0.219804	0.024256	-0.106019	-0.372887	
6	101.808777	0.276783	-0.002165	-0.007127	-0.008173	0.037920	0.127342	
7	401.364224	0.053918	-0.001910	-0.007504	0.002575	-0.011837	-0.041100	
8	1582.311905	0.016922	0.000284	0.001206	-0.000547	0.002519	0.008750	
		2p	2p'	3p	3p'			
1	0.106910	0.008281	0.078311	1.155731	-0.533538			
2	0.421475	-0.284352	0.779946	-0.342481	1.448923			
3	1.661594	-0.596840	-0.087016	0.101145	-0.470394			
4	6.550559	-0.301361	-0.538045	-0.033729	0.134937			
5	25.824492	-0.071911	-0.218352	0.007116	-0.034088			

Table 16. Even-tempered Gaussian B basis for hydrogen

		Alpha = 0.0418240224	Beta = 3.1025201259	
k	Zeta (s)	1s	1s'	1s''
1	0.129760	0.344044	-1.203521	-1.419455
2	0.402583	0.504461	0.570174	2.530538
3	1.249021	0.208773	0.428332	-1.410156
4	3.875112	0.066852	0.199827	-0.072639
5	12.022612	0.012219	0.027673	-0.184594
6	37.300396	0.007000	0.021674	-0.023197

k	Zeta (p)	2p	2p'
1	0.129760	0.351883	-1.054385
2	0.402583	0.603455	0.523789
3	1.249021	0.166022	0.447151
4	3.875112	0.042314	0.112437

Table 17. Even-tempered Gaussian B basis for lithium

		Alpha = 0.0115700412		Beta = 3.4255209394		
k	Zeta(s)	1s	1s'	2s	2s'	2s''
1	0.039633	0.0	0.0	-0.800126	1.380438	-0.809067
2	0.135765	0.010834	0.003525	-0.326396	-1.795111	2.087963
3	0.465066	-0.210444	0.907008	0.138628	0.217677	-2.480206
4	1.593094	-0.537969	-0.007375	0.028699	0.031214	0.751112
5	5.457177	-0.299765	-0.587888	0.006591	0.031603	-0.053095
6	18.693674	-0.096477	-0.235659	0.0	0.0	0.0
7	64.035572	-0.019614	-0.060476	0.0	0.0	0.0
8	219.355193	-0.007962	-0.022461	0.0	0.0	0.0
		2p	2p'			
1	0.039633	0.755860	-1.051079			
2	0.135765	0.289905	1.203019			
3	0.465066	0.077801	0.016285			
4	1.593094	0.012141	0.093219			
5	5.457177	0.005053	0.000343			

Table 18. Even-tempered Gaussian B basis for sodium

		Alpha = 0.0163978687			Beta = 3.3577611334		
k	Zeta(s)	1s	2s	2s'	3s	3s'	3s''
1	0.055060	0.0	0.0	0.0	1.194917	-1.277291	0.583843
2	0.184879	0.0	0.001993	-0.221654	-0.258506	2.306001	-1.824016
3	0.620779	0.0	0.515018	-0.992937	-0.021215	-0.872782	2.766856
4	2.084427	-0.007211	0.601007	0.898107	-0.049076	0.184419	-1.275815
5	6.999006	0.199525	-0.049888	0.522637	0.011826	-0.092733	0.306452
6	23.500992	0.549964	-0.078733	-0.174691	-0.001896	0.020937	-0.073667
7	78.910717	0.296943	-0.011472	-0.038524	0.0	0.0	0.0
8	264.963339	0.087799	0.0	0.0	0.0	0.0	0.0
9	889.683602	0.018048	0.0	0.0	0.0	0.0	0.0
10	2987.345019	0.007414	0.0	0.0	0.0	0.0	0.0

k	Zeta(p)	2p	2p'	3p	3p'
1	0.055060	0.0	0.0	1.105064	-0.837234
2	0.184879	-0.036207	0.311914	-0.195974	1.608357
3	0.620779	-0.375665	0.644501	0.079142	-0.440708
4	2.084427	-0.509383	-0.234273	-0.032909	0.135953
5	6.999006	-0.257626	-0.491214	0.006020	-0.037185
6	23.500992	-0.060573	-0.175109	0.0	0.0
7	78.910717	-0.012261	-0.041587	0.0	0.0

Table 19. Even-tempered segmented bases for oxygen

k	Zeta (s)	[3s]	[4s]	[5s]
1	0.237088	0.389215	1.0	1.0
2	0.764797	0.651556	1.0	1.0
4	7.958322	1.0	1.0	1.0
3	2.467084	0.069826	0.069826	0.069826
5	25.671964	0.413528	0.413528	0.413528
6	82.812644	0.141225	0.141225	0.141225
7	267.137104	0.041909	0.041909	0.041909
8	861.731117	0.007347	0.007347	0.007347
9	2779.773039	0.003797	0.003797	0.003797
<hr/>				
k	Zeta (p)	[2p]	[3p]	
1	0.237088	1.0	1.0	
2	0.764797	0.473480	1.0	
3	2.467084	0.303268	0.303268	
4	7.958322	0.079705	0.079705	
5	25.671964	0.022109	0.022109	

Table 20. Even-tempered pseudo-scaled bases for oxygen

k	Zeta (s)	1s	2s	2s'	2s''
1	0.237088	0.0	0.389215	-1.145496	1.325788
2	0.764797	0.0	0.651556	0.601045	-2.378377
3	2.467084	0.069826	0.075350	0.767300	1.569574
4	7.958322	0.476983	-0.108228	-0.105985	0.244298
5	25.671964	0.413528	-0.018300	-0.074484	-0.165110
6	82.812644	0.141225	-0.003141	-0.008068	-0.027911
7	267.137104	0.041909	0.0	0.0	0.0
8	861.731117	0.007347	0.0	0.0	0.0
9	2779.773039	0.003797	0.0	0.0	0.0

k	Zeta (p)	2p	2p'	2p''
1	0.237088	0.369977	-0.921457	0.924088
2	0.764797	0.473480	0.274249	-1.375734
3	2.467084	0.303268	0.442420	0.481172
4	7.958322	0.079705	0.248480	0.407096
5	25.671964	0.022109	0.064976	0.212321

Table 21. Even-tempered segmented bases for nitrogen

k	Zeta (s)	[3s]	[4s]	[5s]
1	0.188809	0.438292	1.0	1.0
2	0.605375	0.623792	1.0	1.0
4	6.223404	1.0	1.0	1.0
3	1.941004	0.078531	0.078531	0.078531
5	19.953986	0.402541	0.402541	0.402541
6	63.978097	0.138554	0.138554	0.138554
7	205.131796	0.041123	0.041123	0.041123
8	657.710300	0.007311	0.007311	0.007311
9	2108.804429	0.003801	0.003801	0.003801
k	Zeta (p)	[2p]	[3p]	
1	0.188809	1.0	1.0	
2	0.605375	0.489950	1.0	
3	1.941004	0.281715	0.281715	
4	6.223404	0.070625	0.070625	
5	19.953986	0.020051	0.020051	

Table 22. Even-tempered pseudo-scaled bases for nitrogen

k	Zeta (s)	1s	2s	2s'	2s''
1	0.188809	0.0	0.438292	-1.204585	1.302996
2	0.605375	0.0	0.623792	0.764094	-2.447176
3	1.941004	0.078531	0.043756	0.677153	1.748323
4	6.223404	0.483364	-0.102515	-0.116432	0.120923
5	19.953986	0.402541	-0.016065	-0.067527	-0.138741
6	63.978097	0.138554	-0.002964	-0.007436	-0.029749
7	205.131796	0.041123	0.0	0.0	0.0
8	657.710300	0.007311	0.0	0.0	0.0
9	2108.804429	0.003801	0.0	0.0	0.0

k	Zeta (p)	2p	2p'	2p''
1	0.188809	0.374662	-0.952647	0.933998
2	0.605375	0.489950	0.335456	-1.474053
3	1.941004	0.281715	0.445845	0.650616
4	6.223404	0.070625	0.215303	0.344239
5	19.953986	0.020051	0.057069	0.183967

Table 23. Molecular comparison of contraction schemes: H₂O

Contraction	Segmented Energy	Basis V/E	Scaled Energy	Basis V/E
[2s,2p 2s]			-75.986463	2.00298
[3s,2p 2s]	-75.981672	1.99726	-75.987852	2.00043
[4s,2p 2s]	-75.984105	1.99802	-75.990762	2.00053
[4s,3p 2s]	-75.986285	1.99735	-75.995725	2.00016
[5s,3p 2s]	-75.990754	1.99764		
(9s,5p 4s)	-75.999510		-75.999510	

Table 24. Molecular comparison of contraction schemes: N₂

Contraction	Segmented Energy	Basis V/E	Scaled Energy	Basis V/E
[2s,2p]			-108.783000	2.00220
[3s,2p]	-108.784867	1.99659	-108.833053	2.00339
[4s,2p]	-108.861102	1.99720	-108.861188	2.00023
[4s,3p]	-108.866344	1.99752	-108.871894	1.99901
[5s,3p]	-108.870924	1.99777		
(9s,5p)	-108.877096		-108.877096	

MOLECULE CALCULATIONS

Triatomic Alkali Ions

The molecule H_3^+ has been known to mass spectroscopists since Thomson reported it in 1912 (92). It is of theoretical interest because it is the simplest prototype molecule for a three-center two-electron bond and possesses a high stability. It is as stable with respect to dissociation into H_2 and H^+ as H_2 itself is with respect to dissociation into neutral hydrogen atoms. A careful theoretical treatment of the dissociation of H_3^+ and H_2 was given by Schwartz and Schaad (93) in 1967.

Alkali-metal atoms are similar to hydrogen and, although their valence electron is shielded from the nucleus by an inner core of electrons, it is natural to inquire about the possible stability of the triatomic alkali ions. This inquiry is the object of the present calculations. Considered are the homonuclear triatomics H_3^+ , Li_3^+ , and Na_3^+ and the heteronuclear triatomics composed of at most two kinds of atom, i.e. LiH_2^+ , Li_2H^+ , NaH_2^+ , $NaLi_2^+$, Na_2H^+ , and Na_2Li^+ . Also treated here are the atomic ions Li^+ and Na^+ and the neutral diatomics H_2 , LiH , Li_2 , NaH , $NaLi$, and Na_2 . Some of these have been calculated previously. A tabulation of SCF calculations along with some more approximate results is presented in Table 25 with those obtained here. These results

will be discussed later.

Because of the kind of atoms involved in the molecular calculations described here, the ground state configurations are rather straightforwardly assigned. All inner shell molecular orbitals resemble either the atomic inner shell orbitals or some symmetric linear combinations of them which are also easily identified. In diatomics, two electrons remain for the valence shell. They occupy a bonding orbital in order for the molecule to be stable. Although this could be an E_{1u} (or π_u) symmetry orbital, the A_{1g} (or σ_g) bonding orbital is generally lower and provides better bonding, with the electrons distributed along the molecular bond axis. With very accurate open-shell SCF calculations on NaLi, Bertoncini et al. (94) found the : Σ^+ state to be the most stable out of a set of possible states including the : Π and therefore predicted it to be the ground state. Analogously, in the positive triatomic ions there are also two electrons left for the valence shell. For these, the valence or highest occupied orbital is again most naturally one belonging to the totally symmetric irreducible representation for the molecular point group in question. Since the ions are electron deficient, with two electrons forming a three-center bond, the electrons most naturally would want to be in the plane of the atoms. Other bonding configurations would be analogous to the II states of a diatomic, and would have a node in the

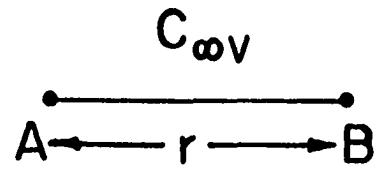
atomic plane. Some experimentation with the alternate configurations was tried but all the molecules were finally predicted to have a ground state characterized by a totally symmetric valence orbital.

Equilibrium Nuclear Configurations

In order to establish the stability of the ions, it is necessary to first determine their ground state equilibrium structure. Using the smaller A bases, the geometry parameters were systematically varied and an energy computed for each new geometry to determine a minimum. Further variations were carried out using the larger B bases. These minima are predicted to be the ground state nuclear configurations. Table 26 contains all of the geometries so established with both A and B bases. All geometry parameters are in reference to Figure 1.

Homonuclear triatomic ions

H_3^+ has been found previously to have the conformation of an equilateral triangle (93). The same is true for Li_3^+ but from rather approximate calculations (95,96). We first of all optimized the geometry of Li_3^+ under an equilateral constraint. Secondly, small deviations from the equilibrium equilateral structure were made in order to determine whether some distorted equilateral triangle has a lower energy. In this way, Li_3^+ was shown to be equilateral in our calcula-



z

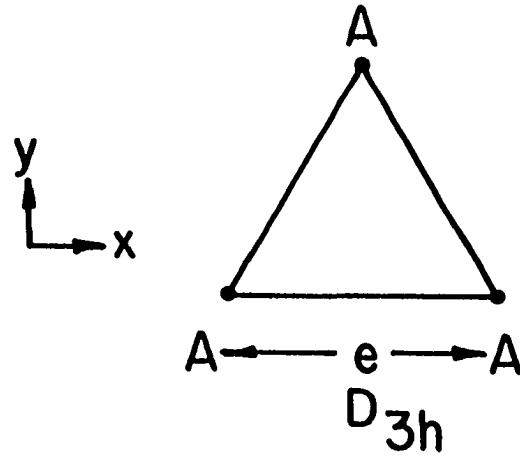
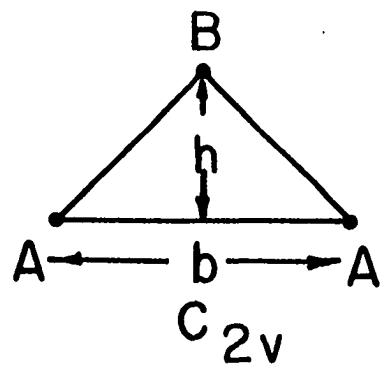
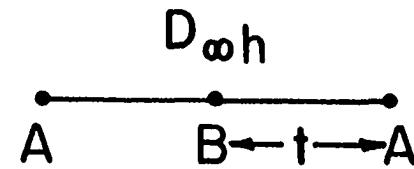


Figure 1. Parameters for molecule geometries

tions. For Na_3^+ , only the equilateral geometry was optimized because of the large amount of time needed per geometry to compute the energy (see Table 77 in Appendix C). The equilibrium geometry of H_3^+ was also determined in order that consistent results for all homonuclear triatomics would be available with the bases described here. Edge lengths for the three homopolar triatomic ions are given in Table 26. For H_3^+ and Li_3^+ these values are within 1% of the best optimal values previously reported (see Table 25).

Dihydrogen-metal ions

The ions LiH_2^+ and NaH_2^+ were also subjected to extensive geometry optimization in the A bases. LiH_2^+ had been treated previously (see Table 25) in considerable detail and was found to have an isosceles conformation consisting of a hydrogen molecule located at some distance from the Li atom. Electrons are distributed in such a way that the Li atom is essentially a positive ion. Lester's calculation (97) is of comparable accuracy to ours and since all nuclear configurations were carefully considered by him, we restricted our geometry searches for both of the ions to the isosceles geometry characterized by base and height parameters b and h. A geometry optimum was successfully found in the present work, essentially duplicating that of Lester for LiH_2^+ . A rationale of the geometries may be made in terms of the very slight

binding experienced with respect to the separated species. The stabilities are to be discussed later in more detail.

Monohydrogen-metal ions

The ions Li_2H^+ and Na_2H^+ also fall into a separate category with respect to geometry. Again Li_2H^+ was computed previously (see Table 25), also with less accurate bases, and its shape was determined to be linear symmetric. We established the equilibrium linear symmetric geometry and then tried deviations from it in both bases but only higher energies were obtained. For Na_2H^+ a similar variation was attempted using the A basis for both symmetric linear and symmetric bent systems. The linear again was found the lowest in energy.

Lithium-sodium ions

The final two triatomic species, NaLi_2^+ and Na_2Li^+ , have not been treated previously in either the experimental or theoretical literature. Therefore extensive geometry optimization had to be done. For NaLi_2^+ , in view of a possible similarity to the MH_2^+ ions, an elongated isosceles triangular geometry was taken to be the initial guess. However, the bond distances were found to be less extreme as compared to MH_2^+ ions. A possible π -type valence orbital ground state configuration was tried but was found to be of higher energy.

On the other hand, the similarity of Na_2Li^+ to the pre-

vious M_2H^+ ions suggested the possibility of an optimum linear symmetric conformation. In the A basis a stable linear symmetric configuration with $t=5.74$ a.u. was found. However, lower energies were obtained for an isosceles or bent geometry.

Another possible ion ($NaLiH^+$) composed of all the atoms, although not computed, is predicted to be linear with the hydrogen atom in the middle on the basis of the present results.

Diatomc molecules and atomic ions

In order that predictions as to the stabilities of the above triatomic ions may be made, it is necessary to have data pertaining to dissociation products of the ions computed with the same bases. Consequently, all possible neutral homo- and heteronuclear diatomic molecules and positive atomic ions consisting of the atoms hydrogen, lithium and sodium were calculated in both bases. Diatomic bond lengths were optimized. A tabulation of experimental and previous theoretical geometries in Table 25 shows that those obtained here are, at worst, the same to within a few percent.

Comparison of bond lengths

It is instructive to compare the six possible interatomic distances in the various molecules. The following is a list of all optimum bond lengths in Angstroms, com-

puted with the larger basis (B) :

	H-H	Li-H	Na-H	Li-Li	Na-Li	Na-Na
H_2	0.734					
H_3^+	0.869					
Li_2			2.778			
Li_3^+			3.043			
Na_2					3.108	
Na_3^+					3.557	
NaLi				2.962		
Na_2Li^+				3.307	3.601	
$NaLi_2^+$			3.029	3.338		
LiH	1.619					
Li_2H^+	1.674		3.348			
NaH			1.907			
Na_2H^+			2.002			4.004
LiH_2^+	0.743	2.103				
NaH_2^+	0.741		2.517			

These are arranged so that the bond length increases in going from left to right in each row and from top to bottom in each column.

The following observations can be made. Although the bond lengths increase from H_2 to Li_2 to Na_2 , those of Na_2 and Li_2 are very close to each other. The heteronuclear diatomics have bond lengths intermediate to the corresponding homonuclear diatomics. Consequently, NaLi is very similar to Na_2 and Li_2 .

The homonuclear triatomics have slightly longer bond lengths than the corresponding diatomics. This is presumably so because the diatomics are neutral, whereas the triatomics are positive ions and, hence, embody larger interatomic elec-

trostatic repulsions. Because of the closeness of the Na_2 , Li_2 , and NaLi bond lengths, the heteronuclear triatomics made from Na and Li are nearly equilateral, in order to approximately maintain these interatomic distances.

If the triatomics Li_2H^+ and Na_2H^+ would preserve the diatomic bondlengths, they would be nearly, but not entirely, linear, i.e. very slightly bent. In fact, they are linear and both the M-H and the M-M bonds are stretched. The reason for this may be as follows. Hydrogen is about twice as electronegative as Li and Na. In agreement with this, the population analysis of the bonding orbital, to be discussed later, yields a charge of about 1.5 on the hydrogen and 0.25 on each of the metal atoms. The additional electrostatic repulsion between the metal atoms may well account for the linearity and the concomitant lengthening of the M-M bonds.

In LiH_2^+ and NaH_2^+ , the electronegativity difference has an even stronger effect, since two hydrogens are pitted against one metal atom. Consequently, the bond orbital population on Li and Na is only 0.04 and 0.004 respectively. It is therefore not surprising that the metal-hydrogen distances are considerably elongated beyond those found in LiH and NaH. The unusual shape of these molecule ions, if indeed they are stable, corresponds to the fact that they really are just weak complexes between a hydrogen molecule and a metal ion.

Finally, there is a more subtle, but quite consistent

regularity among all homonuclear and heteronuclear triatomics. Namely, the A-B bond lengths decrease in going from ABH^+ to ABLi^+ to ABNa^+ . This, again, may be attributed to the decreasing electronegativity in going from H to Li to Na, which leads to a corresponding increase in the fraction of the bonding electrons available for the AB bond and, hence, to stronger AB bonding. This too is borne out by the population analyses of the bond orbitals.

Energies

An extensive set of total molecular energies expressed in atomic units (Hartrees) is displayed in Table 25 in order to compare the best results obtained here with other SCF results wherever available in the literature. Energies and geometries computed with basis B are tabulated. Of primary interest is the performance of the new pseudo-scaled even-tempered atomic orbital basis. It is therefore relevant to discuss the quality of the results in terms of the relative magnitudes of the energy obtained with the present basis. Very accurate treatments have been made previously for the atoms and diatomic molecules in terms of sets of many Slater-type functions. The present diatomic calculations with Gaussians do not attempt to compete with them, but the accurate values rather serve as benchmarks for our results. A certain consistency may be observed. For Li^+ our result

lacks about 0.0035 a.u. while for Na^+ the discrepancy is 0.075 a.u. Likewise, the linear combination of Slater-type orbital results on diatomic molecules are estimated as being close to Hartree-Fock accuracy. Our result on LiH falls 0.0061 a.u. short. For Li_2 , NaH , NaLi , and Na_2 the discrepancies are 0.0078, 0.078, 0.081, and 0.154 a.u. respectively. Such deviations seem substantial by chemical standards, however, they are approximately constant with respect to the included atoms, the values for H, Li, and Na respectively being about 0.0026, 0.0035, and 0.0749 a.u. This consistency in the defect of present energies from accurate Hartree-Fock energies is important since for chemical quantities one is generally interested in energy differences between related molecular systems. In lieu of energy differences between systems computed to Hartree-Fock accuracy it is sufficient to take energies which deviate from the Hartree-Fock by known amounts. It is expected that this constant-atomic-defect-property carries over to the triatomic ions where the comparison with more accurate calculations cannot be made. We believe that this constancy in atomic deviations comes about because our bases are designed to describe in detail the changes in going from the atoms to the molecules, in particular in the valence electrons. Because of this, the atomic basis inadequacies remain the same when the atom is incorporated in different molecules.

The SCF energies obtained for H_3^+ and LiH_2^+ are somewhat short of the best values in the literature; 0.0004 and 0.0038 a.u. respectively. For all others the values are the best and often the only ones yet obtained. For all molecules, the results are listed in Table 26 for both bases.

Wavefunctions

The wavefunctions for all molecular species are given for both bases A and B in Tables 27-60. Each column headed by a symmetry designation and an orbital energy corresponds to one molecular orbital. The orbital designation corresponds to the irreducible representation of the molecular point group to which it belongs. Rows in each column are labelled by the atomic orbital designations for the pseudo-scaled even-tempered Gaussian atomic orbital basis functions corresponding to those of Tables 13-18.

In order to interpret the orbitals correctly, they must be referred to external reference axes as shown in Figure 1. The linear systems are oriented along the z-axis while the triangular systems lie in the xy-plane. Moreover, the numbering of the atoms in a triatomic is such that the two equal atoms lying symmetric to the y-axis come first, the other atom coming last. This is the case for both hetero- and homonuclear triatomic ions. The linear triatomics are numbered similarly, the atom at the molecular midpoint being listed

last. Of the two symmetrically placed atoms, the one placed at a more negative value of the x-coordinate is taken first. Calculation of integrals is with respect to a coordinate system of uniform handedness centered at each nucleus and displaced parallel to a master reference frame with no rotation. Therefore, the orientation of all p atomic orbitals contained in the basis is such that all positive lobes point in the same direction.

At the head of each wavefunction table is a list of energy quantities along with the value of V/E and the computed norm of the wavefunction. The quantity V/E is an indication of the satisfaction of the molecular virial theorem

$$2T + V + \underline{x} \cdot (\partial E / \partial \underline{x}) = \Delta(\underline{t}) ,$$

due to Wassermann (49). If the geometry is an optimum, the derivatives of the energy with respect to the internal coordinate variations $(\partial E / \partial x_i)$ is zero, causing the third term to be zero. The quantity $\Delta(\underline{t})$ represents deviations resulting from incomplete variation of orbital scale factors \underline{t} which are not explicitly included here. If V/E is perfectly equal to 2.0, then the parameters \underline{x} and \underline{t} must be assumed to be fully optimum. Values obtained here are in line with that to be expected as a consequence of comparison with other literature values. The norm is given as an indicator of calculational accuracy. If a basis is used which has linear depen-

dence or near linear dependence, the matrices of expectation values are ill-conditioned. We have noticed that in our computer program, such ill-conditioning leads to a deviation of the norm of the final wavefunction from the correct value of unity. Thus a good value of the norm indicates no problem with linear dependence.

It can be observed that in certain cases some inner shell levels switch in going from the A basis to the B basis. This occurs for NaH_2^+ , NaLi , Na_2Li^+ , and Na_3^+ . The levels in question (see Tables 47 - 50, and 57 - 60) are nearly degenerate, corresponding to the degenerate orbitals in the isolated atom. In the molecular environment they become slightly non-degenerate and the consequent splitting of the orbital energies is different in the separate calculations. The absolute ordering of these orbital energies appears to be a consequence of the basis and little physical significance is attributed to it. Furthermore, orbital energy differences in cases where switching occurs are of negligible magnitude (a maximum of 0.0003 a.u. or 0.2 kcal/mole).

Population Analyses

In 1955, Mulliken recommended a method of partitioning the electronic charge populations of LCAO-MO wavefunctions into net atom, overlap, and gross atom components (98-101). The basic definitions involved are:

$$\text{NAP}(i, A) = \sum_{kl} c_{ik}^A c_{il}^A s_{kl}$$

$$\text{OP}(i, AB) = \sum_{kl} c_{ik}^A c_{il}^B s_{kl}$$

and

$$\text{GAP}(i, A) = \text{NAP}(i, A) + (1/2) \sum_{B \neq A} \text{OP}(i, AB) .$$

The quantity $\text{NAP}(i, A)$ is the net atom population due to orbital i at atom A . The $\text{OP}(i, AB)$ is the overlap population of orbital i for the bond between atoms A and B . The $\text{GAP}(i, A)$ is the gross atom population of orbital i at atom A , one-half of the AB overlap population being added to the net atom population for every other atom in the molecule. The sums on k and l are over basis functions located on the atoms in question and the c 's are molecular orbital coefficients. The quantity s_{kl} is the overlap integral between the respective basis functions denoted by k and l . These definitions are constructed such that a sum over all atoms results in

$$\sum_A \text{GAP}(i, A) = N(i)$$

where $N(i)$ is the orbital occupation number. For the closed-shell Hartree-Fock wavefunctions used here, two electrons oc-

cupy each orbital such that $N(i)=2.0$. Summing instead over all orbitals i gives the total NAP, the total OP, or the total GAP for each atom or bond. The GAP and the OP are the atom and bond populations ordinarily referred to. Further summing the total GAP over all atoms gives the total number of electrons for the molecule.

While this description may not be ideal, (the factor of one-half in the definition of the overlap population being somewhat arbitrary), it is sufficient to give a general picture of the distribution of electrons in lieu of detailed density maps, expectation values of charge distribution properties, etc. For the molecules calculated here, there is always just one valence orbital. Furthermore, the core orbital distributions are such that the core orbital net atom population is essentially equal to the core orbital gross atom population, with the overlap population due to core orbitals essentially negligible. In order to display the Mulliken population analyses for the molecules calculated here, it was felt adequate to sum these quantities over all core orbitals and present them for the core and valence orbitals separately along with totals. Tables 61-75 contain these population analyses for both basis A and B for all molecules including diatomics in order of increasing molecular weight.

Of prime interest are the valence electron distributions for the triatomic ions. It is convenient to break the entire

group of molecules down into the classes described before. Populations in terms of the B bases are discussed unless otherwise noted.

Homonuclear triatomic ions

The homonuclear triatomic ions of H, Li, and Na are fairly uniform. The net atom populations are close to 0.30 while the overlap populations are close to 0.36. Na_3^+ deviates the most from this trend. However, comparison of the respective homonuclear diatomics reveals a similar trend with the sodium quantities again the farthest off. If the diatomic valence orbital overlap populations are compared to those of the triatomics, a smooth trend is observed. In the sequence where X = H, Li, Na, the ratio $\text{OP}(\text{X}_2)/\text{OP}(\text{X}_3^+)$ equals 2.32, 2.15, and 2.00. This relative comparison is further indication that our basis is adequate with respect to property differences rather than absolute values.

Dihydrogen-metal ions

In both LiH_2^+ and NaH_2^+ the two valence electrons reside primarily on the two H atoms and have "diatomic" charge populations very close to that of the isolated H_2 molecule. Very little of the electrons stray to the heavier atom.

Monohydrogen-metal ions

These linear symmetric "protonated homonuclear diatomics" are most similar in gross atom population. The valence orbital is essentially resident at the proton site contributing approximately 3/4 of the electrons. Though the valence orbital overlap populations are reasonably different, a comparison with those of LiH and NaH reveals an identical trend as in the case of homonuclear triatomics. For X = Li and Na, the ratio of valence overlap populations, $O_p(XH)/O_p(X_2H^+)$, equals 2.03 and 2.37 respectively.

Sodium-lithium ions

Besides the obvious homonuclear triatomics, the species $NaLi_2^+$ and Na_2Li^+ correspond most closely to a three-center bonding situation. The bias for electron attraction is uniformly in favor of the Li atoms as in NaLi but not overly so as noted for hydrogen. The valence electrons seem to be distributed reasonably evenly around the whole triangular framework.

Dissociation Energy

A question of particular interest in treating an unknown molecule by ab initio methods is the predicted stability with respect to dissociation. In the case of known molecules, the calculated dissociation energies are good tests for our atomic orbital basis sets, since the energy changes are essen-

tially due to valence orbitals and the design of the atomic orbital basis was meant to account for such valence shell deformations.

Furthermore, the presently considered triatomic ions are particularly interesting because, for them, adequate values of the dissociation energies can be expected in the Hartree-Fock approximation. For many chemical reactions, e.g. the dissociations of many diatomics, the Hartree-Fock approximation can predict only poor reaction energies, because the neglected correlation energy changes character in the course of the reaction. Therefore, it does not cancel out in the difference between the energy of the products and that of the reactants. This is so in particular, if the number of paired electrons in the products is different from that in the reactants. The situation is more favorable if reactants and products have an equal number of paired electrons. Thus, for example, the energy difference between HF and $(H^+ + F^-)$ can be calculated more accurately than that between HF and $(H + F)$ by the Hartree-Fock approximation. However, since in fact, the dissociation proceeds to $(H + F)$, the Hartree-Fock approximation cannot yield a useful internuclear distance dependence for the ground state energy of HF under dissociation. Only if all doubly filled orbitals of the reactants deform continuously into doubly occupied orbitals of the products, can the Hartree-Fock dissociation curve be

useful in the sense that it differs only by an approximate constant from the actual energy curve. An example is the reaction $\text{He}_2 \rightarrow 2\text{He}$. Another example is the dissociation of the triatomic ions considered here into a neutral diatomic and an atomic ion: $\text{ABC}^+ \rightarrow \text{AB} + \text{C}^+$. In this process the doubly filled three-center valence orbital goes continuously into the doubly-occupied diatomic valence orbital. As an illustration of the ability of the Hartree-Fock approximation to describe this process, Table 76 and Figure 2 show the results of calculations made for Li_3^+ in several very elongated isosceles conformations. The triangle base length is held fixed at the Li_2 equilibrium distance. The gross atomic populations given in Table 76 show clearly how the symmetric triatomic bond orbital deforms into a diatomic bond orbital while the energy approaches that of Li_2 plus Li^+ .

In the foregoing reasoning, the correlation energy has been approximated as the sum of intra-orbital pair correlation energies. The implication is that the inter-orbital correlation can be neglected. This again is a favorable feature of the particular systems considered here. In general, molecular correlation energies are not the sum of intra-orbital pair correlation energies. However, separated pair calculations of Miller and Ruedenberg (102) have shown that in all four-electron atom systems (Li^- to Ne^{+6}), 92% of the correlation energy is recovered as the sum of the pair corre-

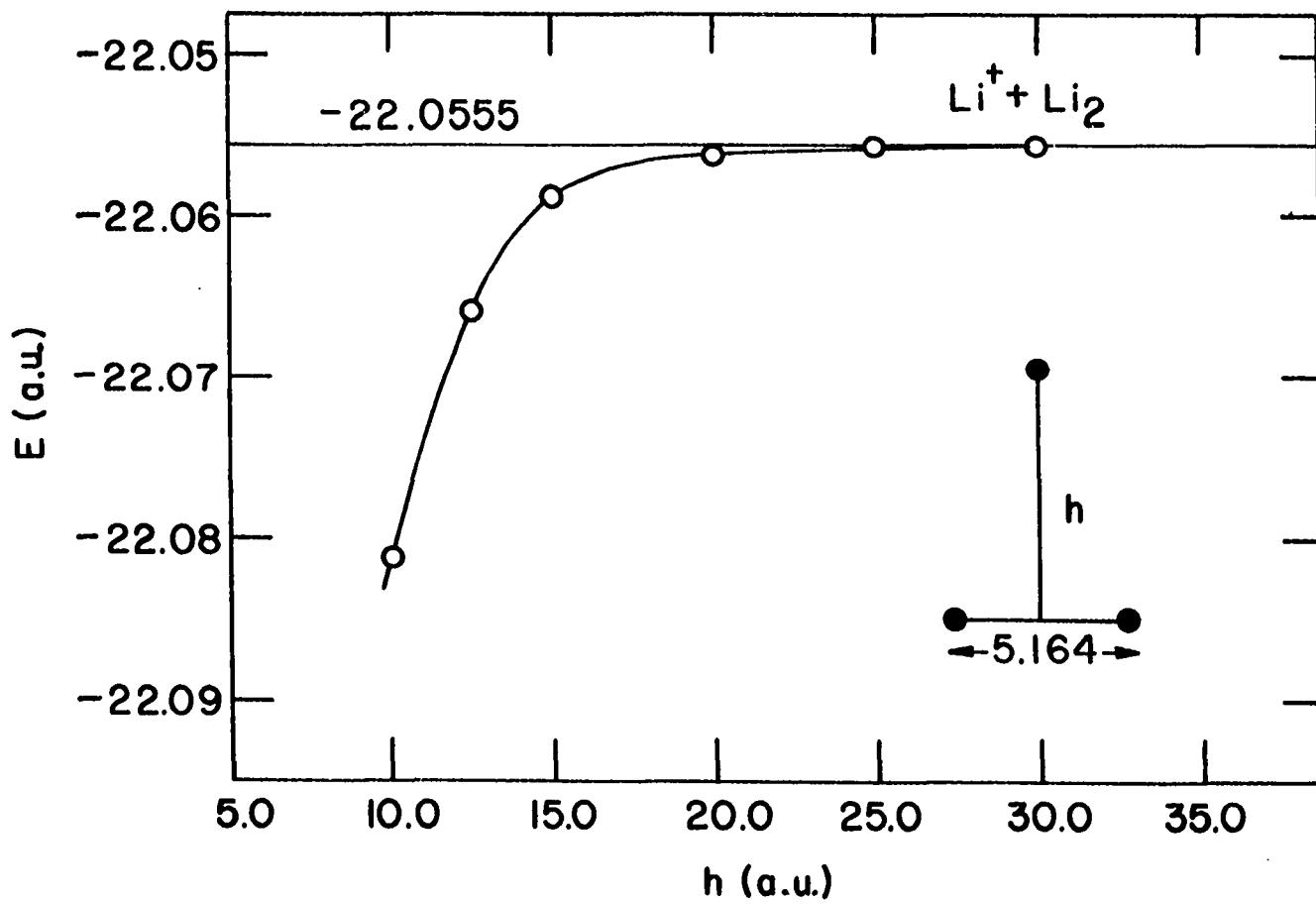


Figure 2. Dissociation energy curve for Li_3^+

lation energies of the inner orbital (about 0.042 a.u.) and that of the outer orbital (about 0.045 a.u.) for Be. Similar results have been obtained for LiH by Mehler et al. (103). It is to be expected that this additivity of inner and outer shell correlation energies remains valid for systems like Li_2 and Li_3^+ which consist of disjointed inner shells and only one valence orbital (104). These systems are probably optimal for description by a separated pair wavefunction. Similar expectations hold for Na_2 , Na_3^+ , etc.

The question arises, whether the triatomic ion ABC^+ dissociates into $(\text{AB} + \text{C}^+)$ or into $(\text{AB}^+ + \text{C})$. The Hartree-Fock approximation can describe the former well but would be poor at describing the latter. This question is equivalent to the question which of the two sets of dissociation products has a lower energy. It is well known experimentally and theoretically (93), that $\text{H}_2 + \text{H}^+$ is more stable than $\text{H} + \text{H}_2^+$. Furthermore, accurate, near Hartree-Fock, self consistent-field calculations are available for the alternate products in some cases so that one may compare them. If the dissociation of Li_2H^+ is considered, the possible dissociation paths are to $\text{Li}_2 + \text{H}^+$, $\text{Li}_2^+ + \text{H}$, $\text{LiH} + \text{Li}^+$, or $\text{LiH}^+ + \text{Li}$. Respective total energies of the products are -14.87179, -15.20971, -15.22372, and -15.16216 Hartrees where individual diatomic energies are from the compendium of Krauss (105) and the atom energies are from the list of Clementi (46). Clearly, the

products $\text{LiH} + \text{Li}^+$ are favored. On the other hand, NaLi_2^+ has the possible products $\text{NaLi} + \text{Li}^+$, $\text{NaLi}^+ + \text{Li}$, $\text{Li}_2 + \text{Na}^+$, or $\text{Li}_2^+ + \text{Na}$ with respective product energies of -176.52985, -176.57132, -176.54855, and -176.56860 Hartrees. Energies for NaLi and NaLi^+ are from Bertoncini et al. (94). Now, although it appears that the two lowest energy paths correspond to formation of the diatomic ions, there is a correlation energy difference between these products and those from paths which lead to neutral diatomics plus atomic ions. The energies of species which have paired valence electrons lack about -0.042 Hartrees of intra-pair correlation energy while this defect is absent in the case where products each have a single, unpaired valence electron. Therefore, if this quantity is added to the energies of the proper species, the total product energies, in the same sequence, become -176.57185, -176.57132, -176.59055, and -176.56860 Hartrees. Therefore, the paths leading to neutral diatomics, which are correctly described qualitatively by Hartree-Fock theory, are in fact predicted to be the lower energy dissociation paths.

We proceed to the quantitative consideration of the results for the various triatomic ions. The uncorrected dissociation energy,

$$D = \sum_P E(P) - \sum_R E(R) ,$$

is the sum of the energies of the dissociation products less the energy of the reactants, i.e. the undissociated molecules. Values computed for D are presented in Table 77 for all triatomic species treated here. Certain corrections may be applied to these values if needed.

Correlation correction

The accurate Hartree-Fock wavefunction accounts for approximately 99.5% of the absolute electronic energy of a molecule. The other 0.5% is called the correlation energy according to common usage. This energy is not trivial in a chemical sense since it usually is greater than typical bond energies. However, in the molecules considered here, 90% of the defect due to correlation energy can be accounted for in equal parts by the intrapair correlation contributions of each electron pair. Since the pairing of electrons is being preserved between the reactants and the products of the dissociation reactions considered here, a large cancellation of the correlation defect will occur if the dissociation energy is computed using Hartree-Fock or approximate Hartree-Fock energies. A test of this hypothesis may be made by comparing accurate Hartree-Fock and accurate correlated results. Csizmadia et al. (106) obtained the necessary values for H_3^+ and H_2 . The electronic energy for H^+ is zero since there are no electrons. The dissociation energy, D, is found to be

0.1676 and 0.1658 Hartrees for correlated and uncorrelated wavefunctions respectively. The difference between these is 0.0018 Hartrees or 1.1 kcal/mole. This is small in comparison to ordinary bond energies which run between 40 and 100 kcal/mole. Therefore, it is assumed that a correlation correction is unnecessary due to an essentially complete cancellation of the correlation effects between products and reactants. The concomitant dissociation energy found with the best wavefunction computed here is 0.1657 which is very close to the accurate result (see Table 26). Even with the poorer wavefunction in the A basis, the dissociation energy is 0.1656 Hartree.

Zero point energy correction

An ab initio calculation for a molecule represents an isolated system in a ground electronic state. The energy computed is the value corresponding to the lowest point on the potential energy curve if the geometry is optimum. In actuality, the uncertainty principle dictates that the system cannot be represented as being at this point but that the lowest energy of a molecule is the above energy plus the sum of zero point energies for each molecular vibrational mode. In the reactions, some of this energy is cancelled by taking the difference, but in general the cancellation is not nearly as complete as that actually found in the case of the corre-

lation energy. The product diatomic molecule has only one degree of freedom or one vibrational mode, while the triatomic species has four or three depending on whether the conformation is linear or non-linear. For quantification, we may refer to the accurate results of Schwartz and Schaad (93) who computed some zero point energies. They found 6.29 and 12.70 kcal/mole for H_2 and H_3^+ respectively, with a consequent difference for the dissociation reaction of 6.41 kcal/mole. This is a reasonably large fraction of the energy when one considers the uncorrected dissociation energy which is about 100 kcal/mole for H_3^+ . Corresponding quantities from less accurate calculations on Li_3^+ (95) and for Li_2 from experiment (107) place the respective zero point energies at 0.60 and 0.50 kcal/mole. The absolute values of the zero point energies and their differences are, in this case, very small and clearly negligible. The large change from the case of the homonuclear hydrogen molecules is due to the large change in component atom weights. The frequency of vibration, v_o , is proportional to the inverse of the square root of the reduced mass, μ , so that for the larger atomic weights, the zero point energy, $hv_o/2$, with $v_o = (2\pi)^{-1} (k/\mu)^{1/2}$, values are highly diminished since the force constants k change only little. For this reason we choose to consider the zero point energy effect on the dissociation energy as negligible except possibly where the hydrogen atom is involved and dominant.

Summary

The corrections which may be applied to the dissociation energy as computed directly from absolute ground state energies are largely cancelled by the differencing involved in the definition of dissociation energy or are negligible with respect to the total uncorrected dissociation energy. Therefore, the stability of the triatomic ions will be discussed in relation to the uncorrected results, shown in Table 77, keeping in mind that for marginal cases, the corrections may yet be invoked to govern the conclusions when the necessary data and conditions are given. The largest non-cancelling correction discussed, that of zero point energy, cannot be computed since not enough of each potential energy surface was obtained in order to be able to ascertain or estimate the force constants for the various vibrational modes.

Other corrections to be considered for certain experimental situations are due to population of higher quantum vibrational and electronic states. These are usually not highly populated unless there are unusually low lying states or some kind of high energy situation is involved. There is also the rotational and translational kinetic energy, usually taken proportional to $RT/2$ per degree of freedom. Finally, there is a pressure-volume term PV for a gas-phase. Depending on the situation, this might be estimated by considering the gas ideal so that $PV=RT$.

Homonuclear triatomic ions

According to the discussion above, we find all the homonuclear triatomic ions to be predicted stable. The best wavefunctions (basis B) for hydrogen, lithium, and sodium produce respective dissociation energies of 103.7, 41.8, and 37.8 kcal/mole. There exists a large disparity between the dissociation energies of sodium as computed from the A and B bases. The A basis yields 62.5 kcal/mole. This is in contrast to the equal A and B values for the hydrogen case. This is presumably related to the adequacy of the various bases involved. It would appear that the A basis is already quite good for hydrogen, but less good for sodium. It is in some ways reassuring that a large difference is found since it shows that the wavefunctions are different and that one is substantially better than the other. Parenthetically, we may remark that Pitzer (108) has reported empirical bond energies, extrapolated to 0°K for homopolar bonds of the same atoms which are 103.2, 26, and 17.8 kcal/mole respectively. Our results are quantitatively in line with these values.

Dihydrogen-metal ions

The reaction path has two alternate routes, leading either to the hydrogen molecule plus metal ion or the hydride plus a proton. The equilibrium geometries of the species (Table 26) highly suggest the former of the two possibili-

ties, being nearly that to begin with. The gross atom populations (Tables 64 and 69) reflect a situation close to that of the dissociated species. Moreover, the former is also energetically the most favorable. Dissociation energies found for LiH_2^+ and NaH_2^+ are low, being respectively 5.2 and 2.9 kcal/mole. It would seem that since these values are of such small magnitude, the effect of certain corrections might prove overwhelming and throw the reaction into reverse. The conclusion here would be that the ions are probably difficult to form even if marginally stable. A closer accounting of all energy terms would be necessary to make a final prediction. Certain experimental evidence in support of this will be discussed.

Monohydrogen-metal ions

It is well known theoretically and experimentally that $\text{LiH} + \text{Li}^+$ is more stable than $\text{Li}_2 + \text{H}^+$. Dissociation energies resulting in metal hydride plus metal ion are 57.2 and 51.4 kcal/mole for Li_2H^+ and Na_2H^+ respectively. Pitzer (108) recorded bond energies of Li-H and Na-H of 58 and 47 kcal/mole. These again show that our results are reasonable. We predict these ions to be stable due to the large magnitude of the dissociation energy. The zero point energy correction discussed above would be small since the metal atoms dominate and imply a large reduced mass.

Sodium-lithium ions

The two species NaLi_2^+ and Na_2Li^+ both have alternate reaction paths which are not dramatically different energy-wise. Those paths which are energetically favored are where a positive sodium ion separates from the triatomic species rather than the lithium ion. Although the respective dissociation energies of 31.2 and 33.6 kcal/mole for this case are of great enough magnitude that the species are predicted to be stable, the effect of various corrections and/or experimental conditions may change the preferred low energy dissociation path.

Experimental Discussion

Outside of H_3^+ , there appears to be no experimental literature concerning the triatomic ions as such. However, some of these are known in mass spectra. Both Li_3^+ and Na_3^+ have been observed in spark spectra (109) where molecule lifetimes are about 1-15 microsec. Using the symmetric ($A1'$) vibrational frequency estimate of 192 cm^{-1} or the doubly degenerate (E') vibration frequency estimate of 104 cm^{-1} as given by Pfeiffer and Ellison (95), one finds that after formation, the molecule undergoes of the order of 10^6 to 10^7 vibration cycles before it is collected for identification. Therefore, the lifetime is of such duration that one may safely say that there was indeed a stable ion. The opinion has also been

given (109) that the ions NaLi_2^+ and Na_2Li^+ could be identified in the same situation if experiments were set up to do so. An interesting correlation may be drawn from mass spectrometric results. It is established (109) that metal-monohydrides are often detected while the metal-dihydrides are very rare. While this information is not from observations of alkali metal spectra, the correlation with present results is clear. It would seem reasonable from our arguments concerning the geometry that in any situation where two hydrogen atoms are involved in a three-center bond, their attraction for each other would be very great. Therefore, subsequent bonding of the H_2 molecule with any other atoms would be minimal. In other words, the energy of the two electrons in H_2 is so low compared to the available valence level of other species, that there is only little interaction.

Besides the mass spectroscopic evidence, one might wonder whether there are other situations in which the ions could play an important part. There is certain evidence that the ions could be present in the highly conductive molten alkali-metal alkali-halide solutions. Reviews by Corbett (110) and Bredig (111) cite measurements which show that although specific conductance increases with metal concentration, the equivalent conductance initially decreases. Some have ascribed this to the possible formation of metal diatomics and the resulting trapping of otherwise free conduction

electrons. The homonuclear triatomic species, M_3^+ , treated here would also appear to be in accord with this argument, the diatomic forming bonds with the free metal ions of the salt. Spin resonance measurements on the lithium-lithium iodide solution suggest that electrons are paired. In addition, it is found (112) that molten Na-NaCl solutions are largely diamagnetic and become only slightly paramagnetic as metal concentration increases. Such properties relating to the electron pairing are also consistent with M_3^+ as a possible participating species. Though there seems to be no evidence for presence of the triatomic ions in particular, they would not appear to be excluded by any evidence.

Alkali-metal liquid ammonia solutions are another situation in which the triatomic ions might exist. However, less is known about the nature of the concentrated solutions than about the molten salts. In high concentration, the metal solutions are excellent conductors and, in contrast to the dilute solutions, have no unpaired electrons (113,114). Here too, the evidence does not exclude the possibility of participation of the triatomic ions.

Table 25. Comparison with literature SCF results

Geometry Params.	Energy	Basis ^a	Year	Reference
H_2				
r=1.402	-	Exp.	-	Herzberg (107)
r=1.4	-1.13342	LCGTO(ON)	1967	Schwartz & Schaad (115)
r=1.4	-1.1335	LCGTO	1970	Csizmadia et al. (106)
r=1.387	-1.133128	LCGTO	1971	This work
H_3^+				
e=1.6405	-1.29993	LCGTO(ON)	1967	Schwartz & Schaad (93)
e=1.66	-1.2993	LCGTO	1970	Csizmadia et al. (106)
e=1.643	-1.298797	LCGTO	1971	This work
Li^+				
-	-7.22140	LCGTO(ON)	1970	Janoschek (116)
-	-7.23641	LCSTO	1965	Clementi (46)
-	-7.232922	LCGTO	1971	This work
LiH				
r=3.015	-	Exp.	-	Herzberg (107)
r=3.2260	-6.5727	FSGO	1967	Frost (117)
r=3.02	-7.985970	LCGTO	1966	Csizmadia (118)
r=3.015	-7.98731	LCSTO	1967	Cade & Huo (119)
r=3.059	-7.981232	LCGTO	1971	This work

^a

Abbreviations are as follows:

Exp.	Experimental (Conversion: 0.52917 a.u./Å°)
DIM	Diatomics in molecules model-non SCF (120)
FSGO	Floating spherical Gaussian orbital model-non SCF (117)
LCSTO	Linear combination of Slater-type orbitals
LCGTO	Linear combination of Gaussian-type orbitals
ON	Off-nuclear Gaussians used

Table 25. (Continued)

Geometry Params.	Energy	Basis ^a	Year	Reference
LiH_2^+				
b=1.4	A.T. ^b	DIM	1967	Wu & Ellison (121)
h=4.95				
b=1.507	-6.920	FSGO	1970	Ray (96)
h=3.352				
b=1.4	-8.37826	LCGTO	1970	Lester (97)
h=3.765				
b=1.405	-8.374427	LCGTO	1971	This work
h=3.912				
Li_2				
r=5.051	-	Exp.	-	Herzberg (107)
r=5.155	-14.83163	LCGTO(ON)	1970	Janoschek (116)
r=5.37	-14.87179	LCSTO	1967	Cade & Wahl (105)
r=5.250	-14.863972	LCGTO	1971	This work
Li_2H^+				
t=3.4	A.T.	DIM	1967	Wu & Ellison (121)
t=3.078	-12.618	FSGO	1970	Ray (96)
t=3.14	-15.289	LCGTO(ON)	1967	Diercksen & Preuss (122)
t=3.1635	-15.305500	LCGTO	1971	This work
Li_3^+				
e=6.18	A.T.	DIM	1965	Pfeiffer & Ellison (95)
e=5.887	-18.292	FSGO	1970	Ray (96)
e=5.769	-22.12376	LCGTO(ON)	1970	Janoschek (116)
e=5.751	-22.163685	LCGTO	1971	This work

^b

Approximate theory - No absolute energy value is obtained for a valid comparison

Table 25. (Continued)

Geometry Params.	Energy	Basis ^a	Year	Reference
Na ⁺				
-	- 161.67676	LCSTO	1965	Clementi (46)
-	- 161.601805	LCGTO	1971	This work
NaH				
r=3.567	-	Exp.	-	Herzberg (107)
r=3.566	- 162.3928	LCSTO	1967	Cade & Huo (123)
r=3.603	- 162.314134	LCGTO	1971	This work
NaLi				
r=5.5	- 169.29344	LCSTO	1970	Bertoncini et al. (94)
r=5.598	- 169.212455	LCGTO	1971	This work
Na ₂				
r=5.819	-	Exp.	-	Herzberg (107)
r=5.8178	- 323.7116	LCSTO	1967	LMSS (105)
r=5.874	- 323.557963	LCGTO	1971	This work

Table 26. Optimum molecular geometries and energies

Molecule	Point Group	BASIS	A	BASIS	B
		Geometry	Energy	Geometry	Energy
H ₂	D _{∞h}	r=1.391	-1.130829	r=1.387	-1.133128
H ₃ ⁺	D _{3h}	e=1.650	-1.296411	e=1.643	-1.298797
Li ⁺	O ₃	-	-7.221335	-	-7.232922
LiH	C _{∞v}	r=3.034	-7.968505	r=3.059	-7.981232
LiH ₂ ⁺	C _{2v}	b=1.415 h=3.879	-8.361201	b=1.405 h=3.912	-8.374427
Li ₂	D _{∞h}	r=5.164	-14.834200	r=5.250	-14.863972
Li ₂ H ⁺	D _{∞h}	t=3.165	-15.283033	t=3.1635	-15.305500
Li ₃ ⁺	D _{3h}	e=5.812	-22.127484	e=5.751	-22.163685

Table 26. (Continued)

Molecule	Point Group	BASIS A		BASIS B	
		Geometry	Energy	Geometry	Energy
Na ⁺	O ₃	-	-161.391975	-	-161.601805
NaH	C _{∞v}	r=3.518	-162.088936	r=3.603	-162.314134
NaH ₂ ⁺	C _{2v}	b=1.400 h=4.700	-162.528050	b=1.400 h=4.705	-162.739526
NaLi	C _{∞v}	r=5.228	-168.970398	r=5.598	-169.212455
NaLi ₂ ⁺	C _{2v}	b=5.650 h=5.485	-176.279469	b=5.725 h=5.622	-176.515658
Na ₂	D _{∞h}	r=5.412	-323.056571	r=5.874	-323.557963
Na ₂ H ⁺	D _{∞h}	t=3.699	-323.573029	t=3.783	-323.997984
Na ₂ Li ⁺	C _{2v}	b=6.525 h=4.886	-330.424927	b=6.805 h=5.242	-330.867980
Na ₃ ⁺	D _{3h}	e=6.133	-484.548381	e=6.722	-485.220160

Table 27. Basis A wavefunction and energy terms for H₂

Molecular Energy =	-1.1308295313
Kinetic Energy =	1.1303431375
Potential Energy =	-2.2611726688
Virial Ratio =	1.9995698788
Elec-Elec Repulsion =	0.6608994966
Elec-Nucl Attraction =	-3.6409794264
Nucl-Nucl Repulsion =	0.7189072610
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital 1A1g
Basis -0.594419

H 1s	0.535536
1s'	0.002943
1s''	-0.002414
2px	0.0
2px'	0.0
2py	0.0
2py'	0.0
2pz	0.026489
2pz'	0.008429
H 1s	0.535536
1s'	0.002943
1s''	-0.002414
2px	0.0
2px'	0.0
2py	0.0
2py'	0.0
2pz	-0.026489
2pz'	-0.008429

Table 28. Basis B wavefunction and energy terms for H₂

Molecular Energy =	-1.1331280499
Kinetic Energy =	1.1326324704
Potential Energy =	-2.2657605203
Virial Ratio =	1.9995626448
Elec-Elec Repulsion =	0.6609395375
Elec-Nucl Attraction =	-3.6476805913
Nucl-Nucl Repulsion =	0.7209805335
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital 1A1g
Basis -0.596585

H	1s	0.535813
	1s'	0.001602
	1s''	-0.005896
	2px	0.0
	2px'	0.0
	2py	0.0
	2py'	0.0
	2pz	0.024979
	2pz'	0.011380
H	1s	0.535813
	1s'	0.001602
	1s''	-0.005896
	2px	0.0
	2px'	0.0
	2py	0.0
	2py'	0.0
	2pz	-0.024979
	2pz'	-0.011380

Table 29. Basis A wavefunction and energy terms for H₃⁺

Molecular Energy =	-1.2964116485
Kinetic Energy =	1.2856631466
Potential Energy =	-2.5820747951
Virial Ratio =	1.9917090363
Elec-Elec Repulsion =	0.7001979399
Elec-Nucl Attraction =	-5.1004545531
Nucl-Nucl Repulsion =	1.8181818182
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital 1A1
Basis -1.207198

H	1s	0.376118
	1s'	0.060327
	1s''	0.004536
	2px	-0.044112
	2px'	-0.004142
	2py	0.025468
	2py'	0.002392
	2pz	0.0
	2pz'	0.0
H	1s	0.376118
	1s'	0.060327
	1s''	0.004536
	2px	0.044112
	2px'	0.004142
	2py	0.025468
	2py'	0.002392
	2pz	0.0
	2pz'	0.0
H	1s	0.376118
	1s'	0.060327
	1s''	0.004536
	2px	0.0
	2px'	0.0
	2py	-0.050936
	2py'	-0.004783
	2pz	0.0
	2pz'	0.0

Table 30. Basis B wavefunction and energy terms for H₃⁺

Molecular Energy =	-1.2987971168
Kinetic Energy =	1.2953939885
Potential Energy =	-2.5941911053
Virial Ratio =	1.9973797845
Elec-Elec Repulsion =	0.7024864521
Elec-Nucl Attraction =	-5.1226057376
Nucl-Nucl Repulsion =	1.8259281802
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital 1A1
Basis -1.211119

H	1s	0.381434
	1s'	0.058052
	1s''	0.002863
	2px	0.032814
	2px'	0.014786
	2py	0.018945
	2py'	0.008537
	2pz	0.0
	2pz'	0.0
H	1s	0.381434
	1s'	0.058052
	1s''	0.002863
	2px	-0.032814
	2px'	-0.014786
	2py	0.018945
	2py'	0.008537
	2pz	0.0
	2pz'	0.0
H	1s	0.381434
	1s'	0.058052
	1s''	0.002863
	2px	0.0
	2px'	0.0
	2py	-0.037890
	2py'	-0.017074
	2pz	0.0
	2pz'	0.0

Table 31. Basis A wavefunction and energy terms for Li⁺

Molecular Energy =	-7.2213356810
Kinetic Energy =	7.1882479951
Potential Energy =	-14.4095836762
Virial Ratio =	1.9954180657
Elec-Elec Repulsion =	1.6432000918
Elec-Nucl Attraction =	-16.0527837679
Nucl-Nucl Repulsion =	0.0
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital 1s
Basis -2.789068

Li 1s	0.992428
1s'	-0.036679
2s	0.002569
2s'	0.005074
2s''	0.007335

Table 32. Basis B wavefunction and energy terms for Li⁺

Molecular Energy =	-7.2329222582
Kinetic Energy =	7.2263051198
Potential Energy =	-14.4592273780
Virial Ratio =	1.9990851363
Elec-Elec Repulsion =	1.6503269876
Elec-Nucl Attraction =	-16.1095543656
Nucl-Nucl Repulsion =	0.0
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital 1s
Basis -2.791298

Li 1s	0.998267
1s'	-0.038969
2s	0.002522
2s'	-0.001592
2s''	0.002328

Table 33. Basis A wavefunction and energy terms for LiH

Molecular Energy =	-7.9685051214
Kinetic Energy =	7.9446666529
Potential Energy =	-15.9131717743
Virial Ratio =	1.9970084140
Elec-Elec Repulsion =	3.4778463903
Elec-Nucl Attraction =	-20.3798118364
Nucl-Nucl Repulsion =	0.9887936717
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

	Orbital	1A1	2A1
	Basis	-2.441669	-0.298057

Li	1s	0.990937	-0.130114
	1s'	-0.036723	0.012100
	2s	0.002834	0.347585
	2s'	0.006378	0.002468
	2s''	0.007637	-0.003465
	2px	0.0	0.0
	2px'	0.0	0.0
	2py	0.0	0.0
	2py'	0.0	0.0
	2pz	-0.004677	0.220743
	2pz'	-0.007075	0.051894
H	1s	-0.005124	-0.670123
	1s'	0.000754	0.079784
	1s''	-0.000235	0.005495
	2px	0.0	0.0
	2px'	0.0	0.0
	2py	0.0	0.0
	2py'	0.0	0.0
	2pz	-0.002177	-0.019215
	2pz'	-0.000904	-0.002201

Table 34. Basis B wavefunction and energy terms for LiH

Molecular Energy =	-7.9812329381
Kinetic Energy =	7.9690643039
Potential Energy =	-15.9502972419
Virial Ratio =	1.9984753441
Elec-Elec Repulsion =	3.4614059763
Elec-Nucl Attraction =	-20.3924158694
Nucl-Nucl Repulsion =	0.9807126512
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

		1A1	2A1
	Basis	-2.450591	-0.299679
Li	1s	-0.995268	-0.131772
	1s'	0.038415	0.013917
	2s	0.008195	0.320950
	2s'	0.005297	0.008293
	2s''	-0.003598	-0.003625
	2px	0.0	0.0
	2px'	0.0	0.0
	2py	0.0	0.0
	2py'	0.0	0.0
	2pz	-0.011668	-0.198249
	2pz'	-0.010222	-0.062991
H	1s	0.016104	-0.689617
	1s'	-0.008059	0.105624
	1s''	-0.001933	0.014297
	2px	0.0	0.0
	2px'	0.0	0.0
	2py	0.0	0.0
	2py'	0.0	0.0
	2pz	-0.007529	0.022835
	2pz'	0.005407	0.002006

Table 35. Basis A wavefunction and energy terms for LiH₂⁺

Molecular Energy =	-8.3612012085
Kinetic Energy =	8.3152658786
Potential Energy =	-16.6764670871
Virial Ratio =	1.9945061327
Elec-Elec Repulsion =	3.3569906032
Elec-Nucl Attraction =	-22.2618580007
Nucl-Nucl Repulsion =	2.2284003104
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

		1A1	2A1
	Basis	-2.767027	-0.849278
H	1s	-0.000948	-0.522794
	1s'	0.000090	-0.007031
	1s''	0.000179	0.001406
	2px	0.000319	-0.040793
	2px'	-0.000263	-0.001821
	2py	0.000569	-0.028187
	2py'	-0.000017	0.007769
	2pz	0.0	0.0
	2pz'	0.0	0.0
H	1s	-0.000948	-0.522794
	1s'	0.000090	-0.007031
	1s''	0.000179	0.001406
	2px	-0.000319	0.040793
	2px'	0.000263	0.001821
	2py	0.000569	-0.028187
	2py'	-0.000017	0.007769
	2pz	0.0	0.0
	2pz'	0.0	0.0
Li	1s	0.992202	-0.047476
	1s'	-0.036665	0.005618
	2s	0.002968	0.023037
	2s'	0.005626	0.007170
	2s''	0.007339	-0.001099
	2px	0.0	0.0
	2px'	0.0	0.0
	2py	-0.000329	0.020212
	2py'	-0.000090	0.022311
	2pz	0.0	0.0
	2pz'	0.0	0.0

Table 36. Basis B wavefunction and energy terms for LiH₂⁺

Molecular Energy =	-8.3744274235
Kinetic Energy =	8.3621883814
Potential Energy =	-16.7366158049
Virial Ratio =	1.9985385219
Elec-Elec Repulsion =	3.3561632554
Elec-Nucl Attraction =	-22.3141180156
Nucl-Nucl Repulsion =	2.2213389553
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

	Orbital	1A1	2A1
	Basis	-2.770741	-0.849061
H	1s	-0.001784	0.523300
	1s'	0.000365	0.007379
	1s''	0.000024	-0.003608
	2px	-0.000311	0.034882
	2px'	0.000156	0.007037
	2py	-0.001443	0.023703
	2py'	0.000637	-0.005548
	2pz	0.0	0.0
	2pz'	0.0	0.0
H	1s	-0.001784	0.523300
	1s'	0.000365	0.007379
	1s''	0.000024	-0.003608
	2px	0.000311	-0.034882
	2px'	-0.000156	-0.007037
	2py	-0.001443	0.023703
	2py'	0.000637	-0.005548
	2pz	0.0	0.0
	2pz'	0.0	0.0
Li	1s	0.997855	0.046055
	1s'	-0.038871	-0.005889
	2s	0.000810	-0.020888
	2s'	-0.002160	-0.015477
	2s''	0.002802	0.001533
	2px	0.0	0.0
	2px'	0.0	0.0
	2py	-0.001931	-0.021765
	2py'	-0.001879	-0.028893
	2pz	0.0	0.0
	2pz'	0.0	0.0

Table 37. Basis A wavefunction and energy terms for Li₂

Molecular Energy =	-14.8342009936
Kinetic Energy =	14.8324641739
Potential Energy =	-29.6666651676
Virial Ratio =	1.9998829179
Elec-Elec Repulsion =	6.5175716687
Elec-Nucl Attraction =	-37.9270718479
Nucl-Nucl Repulsion =	1.7428350116
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital	1A1g	1A2u	2A1g
basis	-2.430178	-2.429932	-0.169622
Li 1s	0.701209	-0.700920	-0.131031
1s'	-0.026096	0.025894	0.009110
2s	0.002997	0.003217	0.557299
2s'	0.004331	-0.004978	-0.019885
2s''	0.005364	-0.005249	-0.005455
2px	0.0	0.0	0.0
2px'	0.0	0.0	0.0
2py	0.0	0.0	0.0
2py'	0.0	0.0	0.0
2pz	0.001048	-0.005201	-0.087466
2pz'	0.002074	-0.001119	-0.027565
Li 1s	0.701209	0.700920	-0.131031
1s'	-0.026096	-0.025894	0.009109
2s	0.002997	-0.003217	0.557299
2s'	0.004331	0.004978	-0.019885
2s''	0.005364	0.005249	-0.005455
2px	0.0	0.0	0.0
2px'	0.0	0.0	0.0
2py	0.0	0.0	0.0
2py'	0.0	0.0	0.0
2pz	-0.001048	-0.005201	0.087466
2pz'	-0.002074	-0.001119	0.027565

Table 38. Basis B wavefunction and energy terms for Li₂

Molecular Energy =	-14.8639724534
Kinetic Energy =	14.8581295966
Potential Energy =	-29.7221020500
Virial Ratio =	1.9996069115
Elec-Elec Repulsion =	6.4163280795
Elec-Nucl Attraction =	-37.8527158438
Nucl-Nucl Repulsion =	1.7142857143
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

	Orbital	1A1g	1A2u	2A1g
	Basis	-2.451490	-2.451291	-0.178185
Li	1s	0.705324	0.706152	-0.126912
	1s'	-0.027690	-0.027974	0.010004
	2s	0.002180	0.011196	0.541081
	2s'	-0.000238	-0.003818	-0.032930
	2s''	0.001740	0.002988	-0.002360
	2px	0.0	0.0	0.0
	2px'	0.0	0.0	0.0
	2py	0.0	0.0	0.0
	2py'	0.0	0.0	0.0
	2pz	0.000009	-0.003111	-0.099404
	2pz'	0.001038	0.004889	-0.035338
Li	1s	0.705324	-0.706152	-0.126912
	1s'	-0.027690	0.027974	0.010004
	2s	0.002180	-0.011196	0.541081
	2s'	-0.000238	0.003818	-0.032930
	2s''	0.001740	-0.002988	-0.002360
	2px	0.0	0.0	0.0
	2px'	0.0	0.0	0.0
	2py	0.0	0.0	0.0
	2py'	0.0	0.0	0.0
	2pz	-0.000009	-0.003111	0.099404
	2pz'	-0.001038	0.004889	0.035338

Table 39. Basis A wavefunction and energy terms for Li_2H^+

Molecular Energy =	-15.2830335297
Kinetic Energy =	15.2026670240
Potential Energy =	-30.4857005538
Virial Ratio =	1.9947414559
Elec-Elec Repulsion =	6.9002508775
Elec-Nucl Attraction =	-40.7034869763
Nucl-Nucl Repulsion =	3.3175355450
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital	1A1g	1A2u	2A1g
Basis	-2.646612	-2.646480	-0.557067
Li 1s	0.701031	-0.700920	0.102502
1s'	-0.025932	0.025841	-0.011796
2s	0.002996	0.001296	-0.128438
2s'	0.004474	-0.004019	-0.008117
2s''	0.005275	-0.005262	0.002566
2px	0.0	0.0	0.0
2px'	0.0	0.0	0.0
2py	0.0	0.0	0.0
2py'	0.0	0.0	0.0
2pz	0.001087	-0.004717	0.097686
2pz'	0.002296	-0.002970	0.046199
Li 1s	0.701031	0.700920	0.102502
1s'	-0.025932	-0.025841	-0.011796
2s	0.002996	-0.001296	-0.128438
2s'	0.004474	0.004019	-0.008117
2s''	0.005275	0.005262	0.002566
2px	0.0	0.0	0.0
2px'	0.0	0.0	0.0
2py	0.0	0.0	0.0
2py'	0.0	0.0	0.0
2pz	-0.001087	-0.004717	-0.097686
2pz'	-0.002296	-0.002970	-0.046199
H 1s	-0.003445	0.0	0.754874
1s'	-0.000271	0.0	-0.107262
1s''	-0.000251	0.0	-0.006519
2px	0.0	0.0	0.0
2px'	0.0	0.0	0.0
2py	0.0	0.0	0.0
2py'	0.0	0.0	0.0
2pz	0.0	-0.003507	0.0
2pz'	0.0	0.000438	0.0

Table 40. Basis B wavefunction and energy terms for Li₂H⁺

Molecular Energy =	-15.3055002391
Kinetic Energy =	15.2798066906
Potential Energy =	-30.5853069297
Virial Ratio =	1.9983212866
Elec-Elec Repulsion =	6.9094097303
Elec-Nucl Attraction =	-40.8138252422
Nucl-Nucl Repulsion =	3.3191085823
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital	1A1g	1A2u	2A1g
Basis	-2.650342	-2.650196	-0.557062
Li 1s	-0.704190	-0.705149	0.102553
1s'	0.027170	0.027512	-0.012403
2s	0.005062	-0.004338	-0.134973
2s'	0.003124	0.004047	-0.028873
2s''	-0.002653	-0.002603	0.005123
2px	0.0	0.0	0.0
2px'	0.0	0.0	0.0
2py	0.0	0.0	0.0
2py'	0.0	0.0	0.0
2pz	-0.006457	0.000019	0.105906
2pz'	-0.005598	-0.005341	0.068135
Li 1s	-0.704190	0.705149	0.102553
1s'	0.027170	-0.027512	-0.012403
2s	0.005062	0.004338	-0.134973
2s'	0.003124	-0.004047	-0.028873
2s''	-0.002653	0.002603	0.005123
2px	0.0	0.0	0.0
2px'	0.0	0.0	0.0
2py	0.0	0.0	0.0
2py'	0.0	0.0	0.0
2pz	0.006457	0.000019	-0.105906
2pz'	0.005598	-0.005341	-0.068135
H 1s	0.019253	0.0	0.721837
1s'	-0.009630	0.0	-0.097211
1s''	-0.002391	0.0	-0.006964
2px	0.0	0.0	0.0
2px'	0.0	0.0	0.0
2py	0.0	0.0	0.0
2py'	0.0	0.0	0.0
2pz	0.0	-0.004973	0.0
2pz'	0.0	0.003448	0.0

Table 41. Basis A wavefunction and energy terms for Li₃⁺

Molecular Energy =	-22.1274843420
Kinetic Energy =	22.0192776781
Potential Energy =	-44.1467620201
Virial Ratio =	1.9951098524
Elec-Elec Repulsion =	10.2990972481
Elec-Nucl Attraction =	-59.0914201767
Nucl-Nucl Repulsion =	4.6455609085
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital	1A1'	1E'	1E'	2A1'
Basis	-2.625838	-2.625735	-2.625735	-0.359666
Li 1s	0.572593	0.698065	0.409850	-0.110459
1s'	-0.021246	-0.025757	-0.015123	0.009653
2s	0.002770	-0.002232	-0.001310	0.358423
2s'	0.003726	0.004945	0.002903	-0.003548
2s''	0.004334	0.005234	0.003073	-0.003958
2px	-0.000673	-0.004239	-0.001414	0.126110
2px'	-0.001104	-0.001007	-0.000726	0.027374
2py	0.000388	0.001363	0.002662	-0.072810
2py'	0.000637	0.000717	0.000188	-0.015805
2pz	0.0	0.0	0.0	0.0
2pz'	0.0	0.0	0.0	0.0
Li 1s	0.572593	-0.703973	0.399617	-0.110459
1s'	-0.021246	0.025975	-0.014745	0.009653
2s	0.002770	0.002251	-0.001278	0.358423
2s'	0.003726	-0.004987	0.002831	-0.003548
2s''	0.004334	-0.005278	0.002996	-0.003958
2px	0.000673	-0.004259	0.001351	-0.126110
2px'	0.001104	-0.001017	0.000711	-0.027374
2py	0.000388	-0.001402	0.002642	-0.072810
2py'	0.000637	-0.000720	0.000177	-0.015805
2pz	0.0	0.0	0.0	0.0
2pz'	0.0	0.0	0.0	0.0
Li 1s	0.572593	0.005908	-0.809467	-0.110459
1s'	-0.021246	-0.000218	0.029868	0.009653
2s	0.002770	-0.000019	0.002588	0.358423
2s'	0.003726	0.000042	-0.005734	-0.003548
2s''	0.004334	0.000044	-0.006069	-0.003958
2px	0.0	-0.001854	-0.000014	0.0
2px'	0.0	0.000232	0.000002	0.0
2py	-0.000777	-0.000037	0.005047	0.145619
2py'	-0.001275	-0.000010	0.001427	0.031609
2pz	0.0	0.0	0.0	0.0
2pz'	0.0	0.0	0.0	0.0

Table 42. Basis B wavefunction and energy terms for Li₃⁺

Molecular Energy =	-22.1636852931
Kinetic Energy =	22.1415516840
Potential Energy =	-44.3052369771
Virial Ratio =	1.9990013570
Elec-Elec Repulsion =	10.3696124386
Elec-Nucl Attraction =	-59.3696850964
Nucl-Nucl Repulsion =	4.6948356808
Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital	1A1'	1E'	1E'	2A1'
Basis	-2.627572	-2.627454	-2.627454	-0.361974
Li 1s	0.575855	0.002698	-0.815287	-0.112149
1s'	-0.022563	-0.000107	0.032187	0.010819
2s	0.001550	0.000044	-0.013238	0.349466
2s'	-0.000314	-0.000014	0.004178	0.021688
2s''	0.001209	0.000011	-0.003453	-0.008883
2px	-0.000266	-0.000953	0.003409	-0.131400
2px'	0.000760	0.000756	-0.004218	-0.045748
2py	-0.000153	0.001625	0.001976	-0.075864
2py'	0.000439	-0.001278	-0.002441	-0.026413
2pz	0.0	0.0	0.0	0.0
2pz'	0.0	0.0	0.0	0.0
Li 1s	0.575855	-0.707408	0.405307	-0.112149
1s'	-0.022563	0.027928	-0.016001	0.010819
2s	0.001550	-0.011487	0.006581	0.349466
2s'	-0.000314	0.003625	-0.002077	0.021688
2s''	0.001209	-0.002996	0.001717	-0.008883
2px	0.000266	-0.002493	0.002513	0.131400
2px'	-0.000760	0.003292	-0.002742	0.045748
2py	-0.000153	0.002520	0.000436	-0.075864
2py'	0.000439	-0.002753	0.000095	-0.026413
2pz	0.0	0.0	0.0	0.0
2pz'	0.0	0.0	0.0	0.0
Li 1s	0.575855	0.704710	0.409980	-0.112149
1s'	-0.022563	-0.027822	-0.016186	0.010819
2s	0.001550	0.011443	0.006657	0.349466
2s'	-0.000314	-0.003611	-0.002101	0.021688
2s''	0.001209	0.002985	0.001736	-0.008883
2px	0.0	-0.000947	0.001628	0.0
2px'	0.0	0.000747	-0.001284	0.0
2py	0.000307	0.003406	0.001981	0.151727
2py'	-0.000877	-0.004212	-0.002450	0.052825
2pz	0.0	0.0	0.0	0.0
2pz'	0.0	0.0	0.0	0.0

Table 43. Basis A wavefunction and energy terms for Na⁺

Molecular Energy =	-161.3919755053	Elec-Elec Repulsion =	62.9117542176
Kinetic Energy =	161.0124558890	Elec-Nucl Attraction =	-385.3161856120
Potential Energy =	-322.4044313943	Nucl-Nucl Repulsion =	0.0
Virial Ratio =	1.9976484604	Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

Orbital	1s	2s	2py	2px	2pz
Basis	-40.751529	-3.083888	-1.801565	-1.801565	-1.801565
Na	1s	0.997939	-0.245053	0.0	0.0
	2s	-0.047528	0.992269	0.0	0.0
	2s'	0.029848	0.019275	0.0	0.0
	3s	0.020016	0.015280	0.0	0.0
	3s'	0.042515	0.031979	0.0	0.0
	3s''	-0.030091	-0.023332	0.0	0.0
	2px	0.0	0.0	0.0	-1.000959
	2px'	0.0	0.0	0.0	0.006507
	3px	0.0	0.0	0.0	-0.002229
	3px'	0.0	0.0	0.0	-0.001276
	2py	0.0	0.0	-1.000959	0.0
	2py'	0.0	0.0	0.006507	0.0
	3py	0.0	0.0	-0.002229	0.0
	3py'	0.0	0.0	-0.001276	0.0
	2pz	0.0	0.0	0.0	-1.000959
	2pz'	0.0	0.0	0.0	0.006507
	3pz	0.0	0.0	0.0	-0.002229
	3pz'	0.0	0.0	0.0	-0.001276

Table 44. Basis B wavefunction and energy terms for Na⁺

Molecular Energy =	-161.6018052724	Elec-Elec Repulsion =	63.1748535046		
Kinetic Energy =	161.5127492148	Elec-Nucl Attraction =	-386.2894079917		
Potential Energy =	-323.1145544872	Nucl-Nucl Repulsion =	0.0		
Virial Ratio =	1.9994489167	Wavefunction Norm =	1.0000000000		
Coefficient Vectors and Eigenvalues					
Orbital	1s	2s	2py	2pz	2px
Basis	-40.745661	-3.073127	-1.798230	-1.798230	-1.798230
Na	1s	0.999463	0.244073	0.0	0.0
	2s	-0.005212	-1.029041	0.0	0.0
	2s'	0.004205	0.001001	0.0	0.0
	3s	0.002578	-0.003088	0.0	0.0
	3s'	0.004816	0.002978	0.0	0.0
	3s''	0.007503	-0.000850	0.0	0.0
	2px	0.0	0.0	0.0	1.000604
	2px'	0.0	0.0	0.0	-0.000945
	3px	0.0	0.0	0.0	0.001258
	3px'	0.0	0.0	0.0	0.001574
	2py	0.0	0.0	1.000604	0.0
	2py'	0.0	0.0	-0.000945	0.0
	3py	0.0	0.0	0.001258	0.0
	3py'	0.0	0.0	0.001574	0.0
	2pz	0.0	0.0	0.0	1.000604
	2pz'	0.0	0.0	0.0	-0.000945
	3pz	0.0	0.0	0.0	0.001258
	3pz'	0.0	0.0	0.0	0.001574

Table 45. Basis & wavefunction and energy terms for NaH

Molecular Energy =	-162.0889363333	Elec-Elec Repulsion =	69.3754893377			
Kinetic Energy =	161.8420560821	Elec-Nucl Attraction =	-396.4332583307			
Potential Energy =	-323.9309924154	Nucl-Nucl Repulsion =	3.1267765776			
Virial Ratio =	1.9984768840	Wavefunction Norm =	1.0000000000			
Coefficient Vectors and Eigenvalues						
Orbital	1A1	2A1	1E1x	1E1y	3A1	4A1
Basis	-40.432719	-2.773838	-1.489304	-1.489304	-1.487830	-0.247117
Na	1s -0.997938	-0.244941	0.0	0.0	-0.006374	-0.026000
	2s 0.047602	0.991344	0.0	0.0	0.029465	0.110701
	2s' -0.029885	0.019500	0.0	0.0	0.000164	0.010524
	3s -0.019801	0.014906	0.0	0.0	0.000479	-0.316587
	3s' -0.042627	0.032945	0.0	0.0	-0.006336	-0.011969
	3s'' 0.030113	-0.023526	0.0	0.0	0.001222	-0.019698
	2px 0.0	0.0	0.999009	0.0	0.0	0.0
	2px' 0.0	0.0	-0.007181	0.0	0.0	0.0
	3px 0.0	0.0	-0.001674	0.0	0.0	0.0
	3px' 0.0	0.0	-0.001557	0.0	0.0	0.0
	2py 0.0	0.0	0.0	0.999009	0.0	0.0
	2py' 0.0	0.0	0.0	-0.007181	0.0	0.0
	3py 0.0	0.0	0.0	-0.001674	0.0	0.0
	3py' 0.0	0.0	0.0	-0.001557	0.0	0.0
	2pz -0.000016	0.020443	0.0	0.0	-0.999066	-0.100012
	2pz' -0.000380	-0.002628	0.0	0.0	0.006232	0.002847
	3pz 0.000511	-0.001867	0.0	0.0	0.009148	-0.130837
	3pz' 0.000206	-0.000728	0.0	0.0	0.003472	-0.028085
H	1s -0.000598	0.004522	0.0	0.0	0.002407	-0.756513
	1s' 0.000281	-0.001455	0.0	0.0	0.006295	0.137725
	1s'' 0.000078	-0.000388	0.0	0.0	0.000403	0.015362
	2px 0.0	0.0	-0.000468	0.0	0.0	0.0
	2px' 0.0	0.0	-0.000225	0.0	0.0	0.0
	2py 0.0	0.0	0.0	-0.000468	0.0	0.0
	2py' 0.0	0.0	0.0	-0.000225	0.0	0.0
	2pz -0.000215	0.001871	0.0	0.0	0.011081	-0.021248
	2pz' 0.000025	-0.000352	0.0	0.0	-0.004244	0.023074

Table 46. Basis B wavefunction and energy terms for NaH

Molecular Energy =	-162.3141339130	Elec-Elec Repulsion =	69.3318129185						
Kinetic Energy =	162.2451204983	Elec-Nucl Attraction =	-396.9440787092						
Potential Energy =	-324.5592544113	Nucl-Nucl Repulsion =	3.0530113794						
Virial Ratio =	1.9995748157	Wavefunction Norm =	1.0000000000						
Coefficient Vectors and Eigenvalues									
Orbital	1A1	2A1	1E1x	1E1y	3A1	4A1			
Basis	-40.448084	-2.783071	-1.506296	-1.506296	-1.505019	-0.268901			
Na	1s 0.999459 -0.244014 0.0 0.0 -0.005585 -0.022828	2s -0.005297 1.028395 0.0 0.0 0.024913 0.124813	2s' 0.004216 -0.001016 0.0 0.0 0.000235 -0.000161	3s 0.002214 0.001595 0.0 0.0 -0.001767 -0.323578	3s' 0.004570 -0.004138 0.0 0.0 -0.006862 -0.018964	3s'' 0.007563 0.000936 0.0 0.0 -0.003214 0.011174			
	2px 0.0 0.0 0.999476 0.0 0.0 0.0	2px' 0.0 0.0 -0.001836 0.0 0.0 0.0	3px 0.0 0.0 -0.001751 0.0 0.0 0.0	3px' 0.0 0.0 -0.001406 0.0 0.0 0.0	2py 0.0 0.0 0.999476 0.0 0.0 0.0	2py' 0.0 0.0 -0.001836 0.0 0.0 0.0			
	3py 0.0 0.0 -0.001751 0.0 0.0 0.0	3py' 0.0 0.0 -0.001406 0.0 0.0 0.0	2pz 0.000081 0.018313 0.0 0.0 -0.998915 -0.089687	2pz' 0.000226 -0.002613 0.0 0.0 0.000666 0.001042	3pz -0.000273 -0.000894 0.0 0.0 0.004422 -0.124508	3pz' -0.000476 -0.002149 0.0 0.0 0.002590 -0.024923			
H	1s 0.000375 0.002903 0.0 0.0 0.004345 -0.740228	1s' -0.000173 -0.000613 0.0 0.0 0.004525 0.149502	1s'' 0.000015 0.000188 0.0 0.0 0.000853 0.025058	2px 0.0 0.0 0.000399 0.0 0.0 0.0	2px' 0.0 0.0 -0.000845 0.0 0.0 0.0	2py 0.0 0.0 0.0 0.000399 0.0 0.0	2py' 0.0 0.0 0.0 -0.000845 0.0 0.0	2pz -0.000540 -0.003561 0.0 0.0 0.000864 0.028161	2pz' 0.000272 0.001258 0.0 0.0 -0.002407 -0.002806

Table 47. Basis A wavefunction and energy terms for NaN_2^+

Molecular Energy =	-162.5280502590	Elec-Elec Repulsion =	67.8918816522
Kinetic Energy =	162.1415627885	Elec-Nucl Attraction =	-397.9055642254
Potential Energy =	-324.6696130475	Nucl-Nucl Repulsion =	5.3440695256
Virial Ratio =	1.9976220260	Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

	Orbital	1A1	2A1	1B1	3A1	1B2	4A1
	Basis	-40.739771	-3.072715	-1.790389	-1.790319	-1.790305	-0.806620
H	1s	-0.000404	0.003141	-0.011044	-0.005176	0.0	-0.527327
	1s'	0.000305	-0.001943	0.004977	0.000744	0.0	-0.005072
	1s''	0.000054	-0.000254	0.000174	-0.000276	0.0	0.002200
	2px	0.000193	-0.001441	-0.004501	0.001797	0.0	-0.036655
	2px'	-0.000177	0.001107	0.002058	-0.001161	0.0	-0.003734
	2py	-0.000289	0.002021	0.001717	-0.001554	0.0	-0.021299
	2py'	0.000132	-0.000767	-0.000599	-0.000437	0.0	0.006945
	2pz	0.0	0.0	0.0	0.0	0.000754	0.0
	2pz'	0.0	0.0	0.0	0.0	-0.000451	0.0
H	1s	-0.000404	0.003141	0.011044	-0.005176	0.0	-0.527327
	1s'	0.000305	-0.001943	-0.004977	0.000744	0.0	-0.005072
	1s''	0.000054	-0.000254	-0.000174	-0.000276	0.0	0.002200
	2px	-0.000193	0.001441	-0.004501	-0.001797	0.0	0.036655
	2px'	0.000177	-0.001107	0.002058	0.001161	0.0	0.003734
	2py	-0.000289	0.002021	-0.001717	-0.001554	0.0	-0.021299
	2py'	0.000132	-0.000767	0.000599	-0.000437	0.0	0.006945
	2pz	0.0	0.0	0.0	0.0	0.000754	0.0
	2pz'	0.0	0.0	0.0	0.0	-0.000451	0.0
Na	1s	-0.997935	-0.245042	0.0	0.000439	0.0	-0.004542
	2s	0.047600	0.991590	0.0	-0.001657	0.0	0.021483
	2s'	-0.029885	0.019500	0.0	-0.000110	0.0	0.001685
	3s	-0.019561	0.012337	0.0	0.002087	0.0	-0.020797
	3s'	-0.042650	0.032832	0.0	0.000132	0.0	0.000975
	3s''	0.030103	-0.023665	0.0	0.000079	0.0	-0.002478
	2px	0.0	0.0	1.000837	0.0	0.0	0.0
	2px'	0.0	0.0	-0.006601	0.0	0.0	0.0
	3px	0.0	0.0	0.002740	0.0	0.0	0.0
	3px'	0.0	0.0	0.001033	0.0	0.0	0.0
	2py	0.000011	-0.001332	0.0	-1.000937	0.0	0.034279
	2py'	0.000020	0.000091	0.0	0.006525	0.0	-0.000565
	3py	-0.000681	0.004558	0.0	-0.004843	0.0	0.012450
	3py'	0.000021	0.000062	0.0	-0.001089	0.0	0.003401
	2pz	0.0	0.0	0.0	0.0	-1.000859	0.0
	2pz'	0.0	0.0	0.0	0.0	0.006582	0.0
	3pz	0.0	0.0	0.0	0.0	-0.002422	0.0
	3pz'	0.0	0.0	0.0	0.0	-0.001082	0.0

Table 48. Basis B wavefunction and energy terms for NaH₂⁺

Molecular Energy =	-162.7395264268	Elec-Elec Repulsion =	68.1484448403
Kinetic Energy =	162.6468094894	Elec-Nucl Attraction =	-398.8740368164
Potential Energy =	-325.3863359162	Nucl-Nucl Repulsion =	5.3392560599
Virial Ratio =	1.9994302740	Wavefunction Norm =	1.0000000000
Coefficient Vectors and Eigenvalues			
	Orbital 1A1 2A1 1B1 1B2 3A1 4A1		
	Basis -40.734156 -3.062219 -1.787368 -1.787292 -1.787215 -0.806919		
H	1s 0.000104 -0.000590 -0.003059 0.0 0.001668 0.532276 1s' 0.000009 -0.000072 0.001308 0.0 -0.000075 0.001410 1s'' 0.000015 -0.000165 0.000028 0.0 -0.000152 -0.005520 2px -0.000200 0.001395 -0.000979 0.0 0.001002 0.033129 2px' 0.000127 -0.000806 0.000297 0.0 -0.000352 0.007509 2py -0.000110 0.000720 -0.000375 0.0 0.000741 0.022947 2py' 0.000078 -0.000534 0.000191 0.0 -0.000300 -0.008345 2pz 0.0 0.0 0.0 0.000004 0.0 0.0 2pz' 0.0 0.0 0.0 -0.000024 0.0 0.0		
H	1s 0.000104 -0.000590 0.003059 0.0 0.001668 0.532276 1s' 0.000009 -0.000072 -0.001308 0.0 -0.000075 0.001410 1s'' 0.000015 -0.000165 -0.000028 0.0 -0.000152 -0.005520 2px 0.000200 -0.001395 -0.000979 0.0 -0.001002 -0.033129 2px' -0.000127 0.000806 0.000297 0.0 0.000352 -0.007509 2py -0.000110 0.000720 0.000375 0.0 0.000741 0.022947 2py' 0.000078 -0.000534 -0.000191 0.0 -0.000300 -0.008345 2pz 0.0 0.0 0.0 0.000004 0.0 0.0 2pz' 0.0 0.0 0.0 -0.000024 0.0 0.0		
Na	1s -0.999461 -0.244067 0.0 0.0 -0.000436 0.004269 2s 0.005242 1.028952 0.0 0.0 0.001838 -0.026479 2s' -0.004218 -0.001029 0.0 0.0 0.000003 0.000609 3s -0.002517 0.002658 0.0 0.0 -0.000924 0.009510 3s' -0.004749 -0.003471 0.0 0.0 -0.000883 0.007707 3s'' -0.007546 0.000956 0.0 0.0 -0.000255 -0.001895 2px 0.0 0.0 -1.000520 0.0 0.0 0.0 2px' 0.0 0.0 0.001062 0.0 0.0 0.0 3px 0.0 0.0 -0.001113 0.0 0.0 0.0 3px' 0.0 0.0 -0.001398 0.0 0.0 0.0 2py -0.000010 -0.001358 0.0 0.0 1.000498 -0.033718 2py' 0.000038 0.000146 0.0 0.0 -0.000882 0.000339 3py -0.000025 0.000093 0.0 0.0 0.001524 -0.000932 3py' -0.000165 0.000936 0.0 0.0 0.002006 -0.009632 2pz 0.0 0.0 0.0 1.000529 0.0 0.0 2pz' 0.0 0.0 0.0 -0.001040 0.0 0.0 3pz 0.0 0.0 0.0 0.001077 0.0 0.0 3pz' 0.0 0.0 0.0 0.001383 0.0 0.0		

Table 49. Basis & wavefunction and energy terms for NaLi

Molecular Energy =	-168.9703983319	Elec-Elec Repulsion =	74.8971367794					
Kinetic Energy =	168.7266968680	Elec-Nucl Attraction =	-418.9063972432					
Potential Energy =	-337.6970951999	Nucl-Nucl Repulsion =	6.3121652640					
Virial Ratio =	1.9985577269	Wavefunction Norm =	1.0000000000					
Coefficient Vectors and Eigenvalues								
Orbital	1A1	2A1	3A1	1E1x	1E1y	4A1	5A1	
Basis	-40.430689	-2.770020	-2.390222	-1.484455	-1.484455	-1.484451	-0.148720	
Na	1s	-0.997935	-0.245135	-0.000136	0.0	0.0	0.003388	-0.027968
	2s	0.047754	0.990983	0.000357	0.0	0.0	-0.014462	0.129785
	2s'	-0.029906	0.019640	0.000001	0.0	0.0	0.000020	0.017562
	3s	-0.018726	0.006644	-0.001462	0.0	0.0	0.009615	-0.267942
	3s'	-0.042676	0.032225	-0.000188	0.0	0.0	0.004606	0.019594
	3s''	0.030103	-0.023587	0.0	0.0	0.0	-0.000613	-0.028826
	2px	0.0	0.0	0.0	0.999220	0.0	0.0	0.0
	2px'	0.0	0.0	0.0	-0.006832	0.0	0.0	0.0
	3px	0.0	0.0	0.0	-0.001269	0.0	0.0	0.0
	3px'	0.0	0.0	0.0	-0.001084	0.0	0.0	0.0
	2py	0.0	0.0	0.0	0.0	0.999220	0.0	0.0
	2py'	0.0	0.0	0.0	0.0	-0.006832	0.0	0.0
	3py	0.0	0.0	0.0	0.0	-0.001269	0.0	0.0
	3py'	0.0	0.0	0.0	0.0	-0.001084	0.0	0.0
	2pz	-0.000037	0.010476	0.000325	0.0	0.0	0.999119	-0.046662
	2pz'	-0.000180	-0.001268	-0.000036	0.0	0.0	-0.006688	0.000174
	3pz	0.001193	-0.007663	-0.000585	0.0	0.0	0.002704	0.051830
	3pz'	0.000057	-0.000617	0.000086	0.0	0.0	-0.000935	0.003826
Li	1s	0.000075	-0.0000757	0.991593	0.0	0.0	-0.001684	-0.128113
	1s'	-0.000030	0.000230	-0.036912	0.0	0.0	0.000046	0.003600
	2s	0.001384	-0.009805	0.003021	0.0	0.0	0.008544	0.828570
	2s'	-0.000127	0.0000752	0.006364	0.0	0.0	0.001439	-0.043159
	2s''	0.000001	0.000061	0.007574	0.0	0.0	-0.000041	-0.006297
	2px	0.0	0.0	0.0	-0.002584	0.0	0.0	0.0
	2px'	0.0	0.0	0.0	0.000212	0.0	0.0	0.0
	2py	0.0	0.0	0.0	0.0	-0.002584	0.0	0.0
	2py'	0.0	0.0	0.0	0.0	0.000212	0.0	0.0
	2pz	0.001472	-0.010328	-0.002025	0.0	0.0	0.006748	0.250555
	2pz'	-0.000029	0.000142	-0.002400	0.0	0.0	0.001814	-0.003064

Table 50. Basis B wavefunction and energy terms for NaLi

Molecular Energy =	-169.2124552969	Elec-Elec Repulsion =	74.4483564915					
Kinetic Energy =	169.1425396804	Elec-Nucl Attraction =	-418.6983139554					
Potential Energy =	-338.3549949773	Nucl-Nucl Repulsion =	5.8949624866					
Virial Ratio =	1.9995868176	Wavefunction Norm =	1.0000000000					
Coefficient Vectors and Eigenvalues								
Orbital	1A1	2A1	3A1	4A1	1E1x	1E1y	5A1	
Basis	-40.443180	-2.776407	-2.446015	-1.498893	-1.498766	-1.498766	-0.167503	
Na	1s	-0.999476	-0.244085	-0.000044	-0.002415	0.0	0.0	-0.025185
	2s	0.005182	1.029719	0.000440	0.010822	0.0	0.0	0.138039
	2s'	-0.004206	-0.000946	-0.000022	0.000050	0.0	0.0	0.000448
	3s	-0.003648	0.010482	0.001029	0.000145	0.0	0.0	-0.408552
	3s'	-0.004691	-0.003888	-0.000373	-0.003848	0.0	0.0	-0.013347
	3s''	-0.007611	0.000890	0.000318	-0.001431	0.0	0.0	0.016002
	2px	0.0	0.0	0.0	0.0	0.999464	0.0	0.0
	2px'	0.0	0.0	0.0	0.0	-0.001351	0.0	0.0
	3px	0.0	0.0	0.0	0.0	-0.001856	0.0	0.0
	3px'	0.0	0.0	0.0	0.0	-0.001161	0.0	0.0
	2py	0.0	0.0	0.0	0.0	0.0	0.999464	0.0
	2py'	0.0	0.0	0.0	0.0	0.0	-0.001351	0.0
	3py	0.0	0.0	0.0	0.0	0.0	-0.001856	0.0
	3py'	0.0	0.0	0.0	0.0	0.0	-0.001161	0.0
	2pz	-0.300026	0.007790	-0.000080	-0.999227	0.0	0.0	-0.039085
	2pz'	-0.300108	-0.001140	0.000153	0.001202	0.0	0.0	0.000645
	3pz	-0.3000687	0.004977	-0.000674	0.003722	0.0	0.0	0.000668
	3pz'	0.000137	-0.000422	-0.001289	0.001404	0.0	0.0	-0.020882
Li	1s	-0.000039	0.000314	0.997766	0.001313	0.0	0.0	-0.118911
	1s'	-0.000003	-0.000143	-0.039230	-0.000128	0.0	0.0	0.006022
	2s	-0.001068	0.007044	0.003832	0.001294	0.0	0.0	0.696118
	2s'	0.000230	-0.001632	-0.001210	-0.000802	0.0	0.0	-0.075160
	2s''	-0.000151	0.000765	0.002860	0.000413	0.0	0.0	0.003413
	2px	0.0	0.0	0.0	0.0	-0.000003	0.0	0.0
	2px'	0.0	0.0	0.0	0.0	-0.000197	0.0	0.0
	2py	0.0	0.0	0.0	0.0	0.0	-0.000003	0.0
	2py'	0.0	0.0	0.0	0.0	0.0	-0.000191	0.0
	2pz	-0.000752	0.005097	-0.000595	0.001294	0.0	0.0	0.162469
	2pz'	0.000281	-0.001840	-0.002390	-0.000934	0.0	0.0	0.007075

Table 51. Basis A wavefunction and energy terms for NaLi₂⁺

Molecular Energy =	-176.2794698430	Elec-Elec Repulsion =	80.2168346290					
Kinetic Energy =	175.8723279799	Elec-Nucl Attraction =	-444.6589019225					
Potential Energy =	-352.1517978229	Nucl-Nucl Repulsion =	12.2902694706					
Virial Ratio =	1.9976903614	Wavefunction Norm =	1.0000000000					
Coefficient Vectors and Eigenvalues								
Orbital	1A1	2A1	3A1	1B1	2B1	1B2	4A1	5A1
Basis	-40.624468	-2.961253	-2.607688	-2.607596	-1.677540	-1.677212	-1.676845	-0.343851
Li 1s	0.0	-0.000164	0.701259	-0.701023	-0.000009	0.0	-0.000512	-0.111926
1s'	0.000301	0.000082	-0.026025	0.025884	0.000089	0.0	0.000025	0.008003
2s	-0.000497	-0.003588	0.002761	0.002093	-0.004740	0.0	0.003402	0.458529
2s'	0.000206	0.001414	0.004552	-0.004954	0.000557	0.0	-0.000009	-0.010752
2s''	-0.000020	0.000059	0.005317	-0.005262	0.000050	0.0	-0.000111	-0.004238
2px	0.000253	0.001781	0.001024	-0.004490	0.002074	0.0	-0.001800	-0.138841
2px'	-0.00069	-0.000402	0.001585	-0.001081	0.000097	0.0	-0.000475	-0.024521
2py	0.000710	0.005053	0.000817	-0.000885	0.003541	0.0	-0.003370	-0.125696
2py'	-0.000080	-0.000525	0.000693	-0.000758	-0.000181	0.0	-0.000949	-0.005258
2pz	0.0	0.0	0.0	0.0	0.0	0.001302	0.0	0.0
2pz'	0.0	0.0	0.0	0.0	0.0	-0.000338	0.0	0.0
Li 1s	0.0	-0.000164	0.701259	0.701023	0.000009	0.0	-0.000512	-0.111926
1s'	0.000301	0.000082	-0.026025	-0.025884	-0.000089	0.0	0.000025	0.008003
2s	-0.000497	-0.003588	0.002761	-0.002093	0.004740	0.0	0.003402	0.458529
2s'	0.000206	0.001414	0.004552	0.004954	-0.000557	0.0	-0.000009	-0.010752
2s''	-0.000020	0.000059	0.005317	0.005262	-0.000050	0.0	-0.000111	-0.004238
2px	-0.000253	-0.001781	-0.001024	-0.004490	0.002074	0.0	0.001800	0.138841
2px'	0.000069	0.000402	-0.001585	-0.001081	0.000097	0.0	0.000475	0.024521
2py	0.000710	0.005053	0.000817	0.000885	-0.003541	0.0	-0.003370	-0.125696
2py'	-0.000080	-0.000525	0.000693	0.000758	0.000181	0.0	-0.000949	-0.005258
2pz	0.0	0.0	0.0	0.0	0.0	0.001302	0.0	0.0
2pz'	0.0	0.0	0.0	0.0	0.0	-0.000338	0.0	0.0
Na 1s	0.997933	-0.245075	-0.000051	0.0	0.0	0.0	0.002679	-0.020363
2s	-0.047696	0.991035	0.000078	0.0	0.0	0.0	-0.011493	0.094004
2s'	0.029908	0.019625	-0.000018	0.0	0.0	0.0	-0.000010	0.011608
3s	0.019052	0.008833	-0.001451	0.0	0.0	0.0	0.006992	-0.193187
3s'	0.042704	0.032811	-0.000230	0.0	0.0	0.0	0.003491	0.009614
3s''	-0.030102	-0.023719	-0.000002	0.0	0.0	0.0	-0.000468	-0.019261
2px	0.0	0.0	0.0	0.000022	0.999817	0.0	0.0	0.0
2px'	0.0	0.0	0.0	-0.000010	-0.006860	0.0	0.0	0.0
3px	0.0	0.0	0.0	-0.001131	0.002233	0.0	0.0	0.0
3px'	0.0	0.0	0.0	-0.000167	-0.000358	0.0	0.0	0.0
2py	-0.000032	-0.008315	-0.000053	0.0	0.0	0.0	-1.000196	0.054539
2py'	-0.000144	0.001010	0.000001	0.0	0.0	0.0	0.006522	-0.001393
3py	0.000925	0.005945	0.000683	0.0	0.0	0.0	-0.003746	0.039667
3py'	0.000021	-0.000334	0.000304	0.0	0.0	0.0	-0.000348	0.010224
2pz	0.0	0.0	0.0	0.0	0.0	-0.999943	0.0	0.0
2pz'	0.0	0.0	0.0	0.0	0.0	0.006777	0.0	0.0
3pz	0.0	0.0	0.0	0.0	0.0	-0.000380	0.0	0.0
3pz'	0.0	0.0	0.0	0.0	0.0	0.000159	0.0	0.0

Table 52. Basis 8 wavefunction and energy terms for NaLi₂⁺

Molecular Energy =	-176.5156587092	Elec-Elec Repulsion =	80.1649101851					
Kinetic Energy =	176.4100295343	Elec-Nucl Attraction =	-445.1242502822					
Potential Energy =	-352.9256882435	Nucl-Nucl Repulsion =	12.0336518536					
Virial Ratio =	1.9994015875	Wavefunction Norm =	1.0000000000					
Coefficient Vectors and Eigenvalues								
Orbital	1A1	2A1	3A1	1B1	2B1	1B2	4A1	5A1
Basis	-40.621456	-2.953082	-2.620740	-2.620664	-1.676880	-1.676542	-1.676352	-0.346484
Li 1s	-0.000010	-0.000166	-0.705395	-0.706045	-0.000220	0.0	-0.000475	-0.111951
1s'	-0.000007	0.000061	0.027655	0.027870	0.000027	0.0	0.000048	0.010048
2s	-0.000531	-0.003463	-0.002281	-0.010827	-0.000537	0.0	-0.000547	0.395255
2s'	0.000115	0.000815	0.000673	0.003457	0.000309	0.0	0.000358	0.007601
2s''	-0.000070	-0.000274	-0.001725	-0.002911	-0.000073	0.0	-0.000095	-0.007844
2px	0.000162	0.001063	0.000109	0.003619	0.000239	0.0	0.000086	-0.130966
2px'	-0.000042	-0.000261	-0.001092	-0.004056	-0.000116	0.0	-0.000262	-0.041112
2py	0.000450	0.003020	-0.000005	0.000403	0.000383	0.0	0.000756	-0.079319
2py'	-0.000087	-0.000518	-0.000958	-0.001043	-0.000264	0.0	-0.000207	-0.019134
2pz	0.0	0.0	0.0	0.0	0.0	-0.000021	0.0	0.0
2pz'	0.0	0.0	0.0	0.0	0.0	-0.000102	0.0	0.0
Li 1s	-0.000010	-0.000166	-0.705395	0.706045	0.000220	0.0	-0.000475	-0.111951
1s'	-0.000007	0.000061	0.027655	-0.027870	-0.000027	0.0	0.000048	0.010048
2s	-0.000531	-0.003463	-0.002281	0.010827	0.000537	0.0	-0.000547	0.395255
2s'	0.000115	0.000815	0.000673	-0.003457	-0.000309	0.0	0.000358	0.007601
2s''	-0.000070	-0.000274	-0.001725	0.002911	0.000073	0.0	-0.000095	-0.007844
2px	-0.000162	-0.001063	-0.000109	0.003619	0.000239	0.0	-0.000086	0.130966
2px'	0.000042	0.000261	0.001092	-0.004056	-0.000116	0.0	0.000262	0.041112
2py	0.000450	0.003020	-0.000005	-0.000403	-0.000383	0.0	0.000756	-0.079319
2py'	-0.000087	-0.000518	-0.000958	0.001043	0.000264	0.0	-0.000207	-0.019134
2pz	0.0	0.0	0.0	0.0	0.0	-0.000021	0.0	0.0
2pz'	0.0	0.0	0.0	0.0	0.0	-0.000102	0.0	0.0
Na 1s	-0.999470	0.244046	0.000007	0.0	0.0	0.0	0.002312	-0.019549
2s	0.005196	-1.029532	-0.000289	0.0	0.0	0.0	-0.010324	0.107037
2s'	-0.004219	0.001040	-0.000009	0.0	0.0	0.0	-0.000046	-0.000546
3s	-0.003608	-0.010051	-0.001251	0.0	0.0	0.0	-0.000148	-0.297979
3s'	-0.004651	0.004159	-0.000164	0.0	0.0	0.0	0.003773	-0.023310
3s''	-0.007619	-0.001006	-0.000264	0.0	0.0	0.0	0.001412	0.011426
2px	0.0	0.0	0.0	-0.000130	-0.999757	0.0	0.0	0.0
2px'	0.0	0.0	0.0	0.000110	0.001389	0.0	0.0	0.0
3px	0.0	0.0	0.0	0.001188	0.001367	0.0	0.0	0.0
3px'	0.0	0.0	0.0	-0.000428	0.000471	0.0	0.0	0.0
2py	0.000029	0.007496	-0.000209	0.0	0.0	0.0	-0.999808	0.050798
2py'	0.000100	-0.001117	0.000181	0.0	0.0	0.0	0.000920	-0.001322
3py	0.0003716	0.005160	-0.000590	0.0	0.0	0.0	0.002351	0.101656
3py'	-0.000130	-0.000406	-0.000606	0.0	0.0	0.0	0.000004	0.036995
2pz	0.0	0.0	0.0	0.0	0.0	0.999571	0.0	0.0
2pz'	0.0	0.0	0.0	0.0	0.0	-0.001308	0.0	0.0
3pz	0.0	0.0	0.0	0.0	0.0	-0.000710	0.0	0.0
3pz'	0.0	0.0	0.0	0.0	0.0	-0.000208	0.0	0.0

Table 53. Basis A wavefunction and energy terms for Na₂

Molecular Energy =	-323.0565713457	Elec-Elec Repulsion =	156.0389785637					
Kinetic Energy =	322.8009575123	Elec-Nucl Attraction =	-824.2542309989					
Potential Energy =	-645.8575288580	Nucl-Nucl Repulsion =	22.3577235772					
Virial Ratio =	1.9992087645	Wavefunction Norm =	1.0000000000					
Coefficient Vectors and Eigenvalues								
Orbital	1A1g	1A1g	2A1g	2A2u	3A1g	1E1ux	1E1uy	1E1gx
Basis	-40.357951	-40.357978	-2.700318	-2.700138	-1.414035	-1.413639	-1.413639	-1.413016
Na 1s	-0.705651	0.705660	-0.173308	-0.173387	-0.002355	0.0	0.0	0.0
2s	0.033789	-0.033665	0.701710	0.701005	0.010476	0.0	0.0	0.0
2s'	-0.021154	0.021124	0.013834	0.013963	0.000056	0.0	0.0	0.0
3s	-0.013180	0.014366	0.012514	0.004315	-0.000442	0.0	0.0	0.0
3s'	-0.030212	0.030162	0.022834	0.022713	-0.002502	0.0	0.0	0.0
3s''	0.021315	-0.021305	-0.016371	-0.016679	0.000562	0.0	0.0	0.0
2px	0.0	0.0	0.0	0.0	0.0	0.705527	0.0	0.706126
2px'	0.0	0.0	0.0	0.0	0.0	-0.004873	0.0	-0.004946
3px	0.0	0.0	0.0	0.0	0.0	-0.004121	0.0	-0.001494
3px'	0.0	0.0	0.0	0.0	0.0	-0.001956	0.0	-0.001560
2py	0.0	0.0	0.0	0.0	0.0	0.0	0.705527	0.0
2py'	0.0	0.0	0.0	0.0	0.0	0.0	-0.004873	0.0
3py	0.0	0.0	0.0	0.0	0.0	0.0	-0.004121	0.0
3py'	0.0	0.0	0.0	0.0	0.0	0.0	-0.001956	0.0
2pz	0.0000002	0.000025	0.008401	0.008429	-0.704342	0.0	0.0	0.0
2pz'	-0.000174	0.000148	-0.001124	-0.000997	0.004694	0.0	0.0	0.0
3pz	0.001207	0.000017	0.000688	-0.007725	0.003758	0.0	0.0	0.0
3pz'	0.000206	0.000038	0.000512	-0.001400	0.003255	0.0	0.0	0.0
Na 1s	0.705651	0.705660	-0.173308	0.173387	-0.002355	0.0	0.0	0.0
2s	-0.033789	-0.033665	0.701710	-0.701005	0.010476	0.0	0.0	0.0
2s'	0.021154	0.021124	0.013834	-0.013962	0.000056	0.0	0.0	0.0
3s	0.013180	0.014346	0.012514	-0.004314	-0.000442	0.0	0.0	0.0
3s'	0.030212	0.030162	0.022834	-0.022713	-0.002502	0.0	0.0	0.0
3s''	-0.021315	-0.021305	-0.016371	0.016679	0.000562	0.0	0.0	0.0
2px	0.0	0.0	0.0	0.0	0.0	0.705527	0.0	-0.706126
2px'	0.0	0.0	0.0	0.0	0.0	-0.004873	0.0	0.004946
3px	0.0	0.0	0.0	0.0	0.0	-0.004121	0.0	0.001494
3px'	0.0	0.0	0.0	0.0	0.0	-0.001956	0.0	0.001560
2py	0.0	0.0	0.0	0.0	0.0	0.0	0.705527	0.0
2py'	0.0	0.0	0.0	0.0	0.0	0.0	-0.004873	0.0
3py	0.0	0.0	0.0	0.0	0.0	0.0	-0.004121	0.0
3py'	0.0	0.0	0.0	0.0	0.0	0.0	-0.001956	0.0
2pz	0.000002	-0.000025	-0.008401	0.008429	0.704342	0.0	0.0	0.0
2pz'	-0.000174	-0.000148	0.001124	-0.000997	-0.004694	0.0	0.0	0.0
3pz	0.001207	-0.000017	-0.000688	-0.007725	0.003758	0.0	0.0	0.0
3pz'	0.000206	-0.000038	0.000512	-0.001400	-0.003255	0.0	0.0	0.0

Table 53. (Continued)

	Orbital	1E1gY	3A2u	4A1g
	Basis	-1.413016	-1.410150	-0.093837
Na	1s	0.0	0.003048	-0.036936
	2s	0.0	-0.013399	0.140231
	2s'	0.0	0.000140	0.014733
	3s	0.0	0.009702	-0.608983
	3s'	0.0	0.004788	-0.031279
	3s''	0.0	-0.000586	-0.030949
	2px	0.0	0.0	0.0
	2px'	0.0	0.0	0.0
	3px	0.0	0.0	0.0
	3px'	0.0	0.0	0.0
	2py	0.706126	0.0	0.0
	2py'	-0.004946	0.0	0.0
	3py	-0.001494	0.0	0.0
	3py'	-0.001560	0.0	0.0
	2pz	0.0	0.707313	-0.045943
	2pz'	0.0	-0.004716	0.001067
	3pz	0.0	-0.001865	-0.059661
	3pz'	0.0	0.000307	-0.013767
Na	1s	0.0	-0.003048	-0.036936
	2s	0.0	-0.013399	0.140231
	2s'	0.0	-0.000140	0.014733
	3s	0.0	-0.009702	-0.608983
	3s'	0.0	-0.004788	-0.031279
	3s''	0.0	-0.000586	-0.030949
	2px	0.0	0.0	0.0
	2px'	0.0	0.0	0.0
	3px	0.0	0.0	0.0
	3px'	0.0	0.0	0.0
	2py	-0.706126	0.0	0.0
	2py'	0.004946	0.0	0.0
	3py	0.001494	0.0	0.0
	3py'	0.001560	0.0	0.0
	2pz	0.0	0.707313	0.045943
	2pz'	0.0	-0.004716	-0.001067
	3pz	0.0	0.001865	0.059661
	3pz'	0.0	0.000307	0.013767

Table 54. Basis 3 wavefunction and energy terms for Na_2

Molecular Energy =	-323.5579633245	Elec-Elec Repulsion =	153.2715669623					
Kinetic Energy =	323.4560968202	Elec-Nucl Attraction =	-820.8848780434					
Potential Energy =	-647.0140601447	Nucl-Nucl Repulsion =	20.5992509363					
Virial Ratio =	1.9996851677	Wavefunction Norm =	1.0000000000					
Coefficient Vectors and Eigenvalues								
Orbital	1A2u	1A1g	2A1g	2A2u	3A1g	121ux	121uy	121gx
Basis	-40.428816	-40.428807	-2.762316	-2.762251	-1.485405	-1.484614	-1.484614	-1.484337
Na								
1s	-0.706737	0.706733	-0.172633	-0.172608	0.001460	0.0	0.0	0.0
2s	0.003668	-0.003686	0.727640	0.728202	-0.006262	0.0	0.0	0.0
2s'	-0.002980	0.002948	-0.000609	-0.000638	-0.000052	0.0	0.0	0.0
3s	-0.002619	0.001746	0.001943	0.007582	0.000933	0.0	0.0	0.0
3s'	-0.003353	0.003365	-0.002346	-0.002532	0.002260	0.0	0.0	0.0
3s''	-0.005403	0.005269	0.000290	0.000683	0.001087	0.0	0.0	0.0
2px	0.0	0.0	0.0	0.0	0.0	0.706533	0.0	0.706795
2px'	0.0	0.0	0.0	0.0	0.0	-0.000947	0.0	-0.000968
3px	0.0	0.0	0.0	0.0	0.0	-0.001466	0.0	-0.001438
3px'	0.0	0.0	0.0	0.0	0.0	-0.001079	0.0	-0.000883
2py	0.0	0.0	0.0	0.0	0.0	0.0	0.706533	0.0
2py'	0.0	0.0	0.0	0.0	0.0	0.0	-0.000947	0.0
3py	0.0	0.0	0.0	0.0	0.0	0.0	-0.001466	0.0
3py'	0.0	0.0	0.0	0.0	0.0	0.0	-0.001079	0.0
2pz	0.000008	-0.000050	0.005283	0.005016	0.705669	0.0	0.0	0.0
2pz'	-0.000101	0.000033	-0.000886	-0.000642	-0.000766	0.0	0.0	0.0
3pz	-0.000598	0.000029	0.000394	0.004256	-0.001945	0.0	0.0	0.0
3pz'	0.000105	0.000004	0.000158	-0.000256	-0.001533	0.0	0.0	0.0
Na								
1s	0.706737	0.706733	-0.172633	0.172608	0.001460	0.0	0.0	0.0
2s	-0.003668	-0.003686	0.727640	-0.728202	-0.006262	0.0	0.0	0.0
2s'	0.002980	0.002948	-0.000609	0.000638	-0.000052	0.0	0.0	0.0
3s	0.002619	0.001746	0.001943	-0.007582	0.000933	0.0	0.0	0.0
3s'	0.003353	0.003365	-0.002346	0.002532	0.002260	0.0	0.0	0.0
3s''	-0.005403	0.005269	0.000290	-0.000683	0.001087	0.0	0.0	0.0
2px	0.0	0.0	0.0	0.0	0.0	0.706533	0.0	-0.706795
2px'	0.0	0.0	0.0	0.0	0.0	-0.000947	0.0	0.000968
3px	0.0	0.0	0.0	0.0	0.0	-0.001466	0.0	0.001438
3px'	0.0	0.0	0.0	0.0	0.0	-0.001079	0.0	0.000883
2py	0.0	0.0	0.0	0.0	0.0	0.0	0.706533	0.0
2py'	0.0	0.0	0.0	0.0	0.0	0.0	-0.000947	0.0
3py	0.0	0.0	0.0	0.0	0.0	0.0	-0.001466	0.0
3py'	0.0	0.0	0.0	0.0	0.0	0.0	-0.001079	0.0
2pz	0.000008	-0.000050	-0.005283	0.005016	-0.705669	0.0	0.0	0.0
2pz'	-0.000101	-0.000033	0.000986	-0.000642	0.000766	0.0	0.0	0.0
3pz	-0.000598	-0.000029	-0.000394	0.004256	0.001945	0.0	0.0	0.0
3pz'	0.000105	-0.000004	-0.000158	-0.000256	0.001533	0.0	0.0	0.0

Table 54. (Continued)

		Orbital Basis	1E1g _y -1.484337	3A2u -1.483743	4A1g -0.153585
Na	1s	0.0	0.001729	0.027109	
	2s	0.0	-0.007871	-0.129977	
	2s'	0.0	-0.000034	-0.002218	
	3s	0.0	0.000143	0.579522	
	3s'	0.0	0.002645	0.006062	
	3s''	0.0	0.000841	-0.015154	
	2px	0.0	0.0	0.0	
	2px'	0.0	0.0	0.0	
	3px	0.0	0.0	0.0	
	3px'	0.0	0.0	0.0	
	2py	0.706795	0.0	0.0	
	2py'	-0.000968	0.0	0.0	
	3py	-0.001438	0.0	0.0	
	3py'	-0.000883	0.0	0.0	
	2pz	0.0	0.707350	0.035611	
	2pz'	0.0	-0.000968	-0.000893	
	3pz	0.0	-0.003078	0.050904	
	3pz'	0.0	-0.000917	0.015784	
Na	1s	0.0	-0.001729	0.027109	
	2s	0.0	0.007871	-0.129977	
	2s'	0.0	0.000034	-0.002218	
	3s	0.0	-0.000143	0.579522	
	3s'	0.0	-0.002645	0.006062	
	3s''	0.0	-0.000841	-0.015154	
	2px	0.0	0.0	0.0	
	2px'	0.0	0.0	0.0	
	3px	0.0	0.0	0.0	
	3px'	0.0	0.0	0.0	
	2py	-0.706795	0.0	0.0	
	2py'	0.000968	0.0	0.0	
	3py	0.001438	0.0	0.0	
	3py'	0.000883	0.0	0.0	
	2pz	0.0	0.707350	-0.035611	
	2pz'	0.0	-0.000968	0.000893	
	3pz	0.0	-0.003078	-0.050904	
	3pz'	0.0	-0.000917	-0.015784	

Table S5. Basis & wavefunction and energy terms for Na_2H^+

Molecular Energy =	-323.5730299124	Elec-Elec Repulsion =	150.5254112128
Kinetic Energy =	322.8810075939	Elec-Nucl Attraction =	-819.2827139421
Potential Energy =	-646.4540375063	Nucl-Nucl Repulsion =	22.3033252230
Virial Ratio =	1.9978613103	Wavefunction Norm =	1.0000000000

Coefficient Vectors and Eigenvalues

	Orbital	1s ² u	1s ¹ g	2s ² u	2s ¹ g	1s ¹ u ^x	1s ¹ u ^y	1s ¹ u ^x	1s ¹ g ^x	1s ¹ g ^y
	Basis	-40.619616	-40.619594	-2.956961	-2.956885	-1.673967	-1.673967	-1.673947	-1.673947	-1.673947
Na	1s	0.705651	0.705646	-0.173196	-0.173231	0.0	0.0	0.0	0.0	0.0
	2s	-0.033644	-0.033640	0.701089	0.701136	0.0	0.0	0.0	0.0	0.0
	2s'	0.021172	0.021109	0.014008	0.013630	0.0	0.0	0.0	0.0	0.0
	3s	0.014419	0.013860	0.012987	0.008953	0.0	0.0	0.0	0.0	0.0
	3s'	-0.021313	-0.021280	0.024668	0.022585	0.0	0.0	0.0	0.0	0.0
	2px	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2px'	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3px	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3px'	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2py	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3py	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3py'	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2pz	0.06140	0.00025	-0.00011	0.008601	-0.008755	0.0	0.0	0.0	0.0
	3pz	-0.00043	-0.00063	0.000270	-0.002333	0.0	0.0	0.0	0.0	0.0
	3pz'	-0.00240	-0.00248	0.001171	-0.01128	0.0	0.0	0.0	0.0	0.0
Na	1s	-0.705651	-0.705646	0.173196	-0.173231	0.0	0.0	0.0	0.0	0.0
	2s	0.033644	0.033640	-0.701089	0.701136	0.0	0.0	0.0	0.0	0.0
	2s'	0.021172	0.021109	0.014008	0.013630	0.0	0.0	0.0	0.0	0.0
	3s	0.014419	0.013860	0.012987	0.008953	0.0	0.0	0.0	0.0	0.0
	3s'	-0.030370	-0.03028	0.024668	0.022585	0.0	0.0	0.0	0.0	0.0
	3s''	0.021313	-0.021280	0.016939	-0.016548	0.0	0.0	0.0	0.0	0.0
	2px	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2px'	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3px	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3px'	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2py	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3py	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3py'	0.0	0.00025	0.00011	0.008601	-0.008755	0.0	0.0	0.0	0.0
	2pz	0.000140	-0.000176	-0.001045	0.001114	0.0	0.0	0.0	0.0	0.0
	3pz	-0.00043	-0.000463	0.000270	-0.002333	0.0	0.0	0.0	0.0	0.0
	3pz'	-0.00208	-0.00240	0.001171	-0.01128	0.0	0.0	0.0	0.0	0.0
H	1s	0.0	-0.001107	0.0	-0.008383	0.0	0.0	0.0	0.0	0.0
	1s'	0.0	-0.00625	0.0	-0.003618	0.0	0.0	0.0	0.0	0.0
	1s''	0.0	-0.00137	0.0	-0.00641	0.0	0.0	0.0	0.0	0.0
	2px	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2px'	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2py	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2py'	0.0	0.000684	0.0	0.005075	0.0	0.0	0.0	0.0	0.0
	2pz	-0.000533	0.0	-0.003283	0.0	0.0	0.0	0.0	0.0	0.0

Table 55. (Continued)

		Orbital	3A1g	3A2g	4A1g
	Basis	-1.673222	-1.672909	-0.480457	
Na	1s	0.002689	-0.002686	0.016856	
	2s	-0.012273	0.012887	-0.076105	
	2s'	0.000208	0.000644	-0.007987	
	3s	0.002345	0.007223	0.152918	
	3s'	0.004135	0.000540	-0.002577	
	3s''	-0.000836	-0.001117	0.013362	
	2px	0.0	0.0	0.0	
	2px'	0.0	0.0	0.0	
	3px	0.0	0.0	0.0	
	3px'	0.0	0.0	0.0	
	2py	0.0	0.0	0.0	
	2py'	0.0	0.0	0.0	
	3py	0.0	0.0	0.0	
	3py'	0.0	0.0	0.0	
	2pz	0.706746	-0.707886	0.087769	
	2pz'	-0.004343	0.004597	-0.002651	
	3pz	-0.000948	0.007605	0.082671	
	3pz'	0.000575	0.003886	0.016230	
Na	1s	0.002689	0.002686	0.016856	
	2s	-0.012273	-0.012887	-0.076105	
	2s'	0.000208	-0.000644	-0.007987	
	3s	0.002345	-0.007223	0.152918	
	3s'	0.004135	-0.000540	-0.002577	
	3s''	-0.000836	-0.001117	0.013362	
	2px	0.0	0.0	0.0	
	2px'	0.0	0.0	0.0	
	3px	0.0	0.0	0.0	
	3px'	0.0	0.0	0.0	
	2py	0.0	0.0	0.0	
	2py'	0.0	0.0	0.0	
	3py	0.0	0.0	0.0	
	3py'	0.0	0.0	0.0	
	2pz	-0.706746	-0.707886	-0.087769	
	2pz'	0.004343	0.004597	0.002651	
	3pz	0.000948	0.007605	-0.082671	
	3pz'	-0.000575	0.003886	-0.016230	
H	1s	-0.011308	0.0	0.774122	
	1s'	-0.002454	0.0	-0.133253	
	1s''	-0.000067	0.0	-0.012517	
	2px	0.0	0.0	0.0	
	2px'	0.0	0.0	0.0	
	2py	0.0	0.0	0.0	
	2py'	0.0	0.0	0.0	
	2pz	0.0	0.020210	0.0	
	2pz'	0.0	-0.011383	0.0	

Table 56. Basis B wavefunction and energy terms for Na₂H⁺

Molecular Energy =	-323.9979842976	Elec-Elec Repulsion =	150.3919009855					
Kinetic Energy =	323.8169033912	Elec-Nucl Attraction =	-820.0148774927					
Potential Energy =	-647.8148876888	Nucl-Nucl Repulsion =	21.8080888184					
Virial Ratio =	1.9994411048	Wavefunction Norm =	1.0000000000					
Coefficient Vectors and Eigenvalues								
Orbital	1A2t	1A1g	2A2u	2A1g	1E1ux	1E1uy	1E1gx	1E1gy
Basis	-40.62211 -40.622101	-2.954032	-2.954004	-1.678562	-1.678562	-1.678532	-1.678532	
Na	1s	-0.706715	-0.706725	0.172578	-0.172543	0.0	0.0	0.0
	2s	0.003633	0.003717	-0.727122	0.727362	0.0	0.0	0.0
	2s'	-0.003002	-0.002986	0.000868	-0.000714	0.0	0.0	0.0
	3s	-0.001459	-0.001644	-0.000644	0.001434	0.0	0.0	0.0
	3s'	-0.002861	-0.003377	0.005253	-0.002095	0.0	0.0	0.0
	3s''	-0.005412	-0.005345	-0.000732	0.000736	0.0	0.0	0.0
	2px	0.0	0.0	0.0	0.0	0.707061	0.0	-0.707107
	2pz'	0.0	0.0	0.0	0.0	-0.001112	0.0	0.001126
	3px	0.0	0.0	0.0	0.0	-0.000325	0.0	0.000171
	3px'	0.0	0.0	0.0	0.0	-0.000188	0.0	-0.000021
	2py	0.0	0.0	0.0	0.0	0.0	0.707061	0.0
	2py'	0.0	0.0	0.0	0.0	-0.001112	0.0	0.001126
	3py	0.0	0.0	0.0	0.0	-0.000325	0.0	0.000171
	3py'	0.0	0.0	0.0	0.0	-0.000188	0.0	-0.000021
	2pz	-0.000049	-0.000016	-0.008240	0.008091	0.0	0.0	0.0
	2pz'	-0.000087	-0.000122	0.001199	-0.001083	0.0	0.0	0.0
	3pz	0.000078	0.000187	-0.000231	-0.000742	0.0	0.0	0.0
	3pz'	0.000675	0.000203	0.003560	-0.000792	0.0	0.0	0.0
Na	1s	0.706715	-0.706725	-0.172578	-0.172543	0.0	0.0	0.0
	2s	-0.003333	0.003717	0.727122	0.727362	0.0	0.0	0.0
	2s'	0.003002	-0.002986	-0.000868	-0.000714	0.0	0.0	0.0
	3s	0.001459	-0.001644	0.000644	0.001434	0.0	0.0	0.0
	3s'	0.002861	-0.003377	0.005253	-0.002095	0.0	0.0	0.0
	3s''	0.005412	-0.005345	0.000732	0.000736	0.0	0.0	0.0
	2px	0.0	0.0	0.0	0.0	0.707061	0.0	0.707107
	2px'	0.0	0.0	0.0	0.0	-0.001112	0.0	-0.001126
	3px	0.0	0.0	0.0	0.0	-0.000325	0.0	-0.000171
	3px'	0.0	0.0	0.0	0.0	-0.000188	0.0	-0.000021
	2py	0.0	0.0	0.0	0.0	0.0	0.707061	0.0
	2py'	0.0	0.0	0.0	0.0	-0.001112	0.0	-0.001126
	3py	0.0	0.0	0.0	0.0	-0.000325	0.0	-0.000171
	3py'	0.0	0.0	0.0	0.0	-0.000188	0.0	-0.000021
	2pz	-0.000049	0.000016	-0.008240	-0.008091	0.0	0.0	0.0
	2pz'	-0.000087	0.000122	0.001199	0.001083	0.0	0.0	0.0
	3pz	0.000078	-0.000187	-0.000231	0.000742	0.0	0.0	0.0
	3pz'	0.000675	-0.000203	0.003560	0.000792	0.0	0.0	0.0
H	1s	0.0	-0.000372	0.0	0.002998	0.0	0.0	0.0
	1s'	0.0	0.000180	0.0	-0.000646	0.0	0.0	0.0
	1s''	0.0	-0.000033	0.0	0.000338	0.0	0.0	0.0
	2px	0.0	0.0	0.0	0.0	-0.000112	0.0	0.0
	2px'	0.0	0.0	0.0	0.0	-0.000682	0.0	0.0
	2py	0.0	0.0	0.0	0.0	0.000112	0.0	0.0
	2py'	0.0	0.0	0.0	0.0	-0.000682	0.0	0.0
	2pz	0.001444	0.0	0.008560	0.0	0.0	0.0	0.0
	2pz'	-0.000865	0.0	-0.004247	0.0	0.0	0.0	0.0

Table 56. (Continued)

	Orbital	311g	312u	411g
	Basis	-1.677769	-1.677378	-0.485503
Na	1s	-0.002450	0.002523	0.014991
	2s	0.010731	-0.011396	-0.089750
	2s'	0.000098	-0.000073	0.000870
	3s	-0.001366	0.000173	0.131866
	3s'	-0.003267	0.003410	0.015149
	3s''	-0.001478	0.001374	-0.007585
	2px	0.0	0.0	0.0
	2px'	0.0	0.0	0.0
	3px	0.0	0.0	0.0
	3px'	0.0	0.0	0.0
	2py	0.0	0.0	0.0
	2py'	0.0	0.0	0.0
	3py	0.0	0.0	0.0
	3py'	0.0	0.0	0.0
	2pz	-0.706678	0.707246	0.079401
	2pz'	0.000451	-0.000441	-0.000885
	3pz	0.000857	-0.001999	0.058202
	3pz'	-0.000061	0.000104	0.020780
Na	1s	-0.002450	-0.002523	0.014991
	2s	0.010731	0.011396	-0.089750
	2s'	0.000098	0.000073	0.000870
	3s	-0.001366	-0.000173	0.131866
	3s'	-0.003267	-0.003410	0.015149
	3s''	-0.001478	-0.001374	-0.007585
	2px	0.0	0.0	0.0
	2px'	0.0	0.0	0.0
	3px	0.0	0.0	0.0
	3px'	0.0	0.0	0.0
	2py	0.0	0.0	0.0
	2py'	0.0	0.0	0.0
	3py	0.0	0.0	0.0
	3py'	0.0	0.0	0.0
	2pz	0.706678	0.707246	-0.079401
	2pz'	-0.000451	-0.000441	0.000885
	3pz	-0.000857	-0.001999	-0.058202
	3pz'	0.000061	0.000104	-0.020780
H	1s	0.007912	0.0	0.799930
	1s'	0.002951	0.0	-0.168462
	1s''	0.000963	0.0	-0.024971
	2px	0.0	0.0	0.0
	2px'	0.0	0.0	0.0
	2py	0.0	0.0	0.0
	2py'	0.0	0.0	0.0
	2pz	0.0	0.000245	0.0
	2pz'	0.0	0.002652	0.0

Table 57. Basis A wavefunction and energy terms for Na_2Li^+

Table 57. (Continued)

	Orbital	1A2	5A1	4B1	6A1
	Basis	-1.662358	-1.662082	-1.661651	-0.326630
Na	1s	0.0	-0.001997	0.002121	0.022414
	2s	0.0	0.008723	-0.008952	-0.099679
	2s'	0.0	-0.000079	-0.000130	-0.011993
	3s	0.0	-0.004121	0.006511	0.245445
	3s'	0.0	-0.002924	0.002279	-0.004485
	3s''	0.0	0.000495	-0.000179	0.020734
	2px	0.0	-0.477801	0.595080	0.047759
	2px'	0.0	0.003037	-0.003950	-0.001532
	3px	0.0	-0.000217	0.004299	0.066821
	3px'	0.0	0.000061	0.000326	0.014082
	2py	0.0	-0.520930	0.382591	0.028823
	2py'	0.0	0.003449	-0.002494	-0.000244
	3py	0.0	-0.001787	0.001257	-0.012252
	3py'	0.0	0.000013	0.000040	0.001349
	2pz	-0.707006	0.0	0.0	0.0
	2pz'	0.004827	0.0	0.0	0.0
	3pz	-0.000486	0.0	0.0	0.0
	3pz'	0.000320	0.0	0.0	0.0
Na	1s	0.0	-0.001997	-0.002121	0.022414
	2s	0.0	0.008723	0.008952	-0.099676
	2s'	0.0	-0.000079	0.000129	-0.011995
	3s	0.0	-0.004121	-0.006511	0.245444
	3s'	0.0	-0.002923	-0.002279	-0.004488
	3s''	0.0	0.000494	0.000179	0.020736
	2px	0.0	0.477801	0.595080	-0.047759
	2px'	0.0	-0.003037	-0.003950	0.001532
	3px	0.0	0.000217	0.004299	-0.066821
	3px'	0.0	-0.000061	0.000326	-0.014082
	2py	0.0	-0.520930	-0.382591	0.028823
	2py'	0.0	0.003449	0.002494	-0.000244
	3py	0.0	-0.001787	-0.001257	-0.012252
	3py'	0.0	0.000013	-0.000040	0.001349
	2pz	0.707006	0.0	0.0	0.0
	2pz'	-0.004827	0.0	0.0	0.0
	3pz	0.000486	0.0	0.0	0.0
	3pz'	-0.000320	0.0	0.0	0.0
Li	1s	0.0	0.001013	0.0	0.113072
	1s'	0.0	-0.000021	0.0	-0.004464
	2s	0.0	-0.008019	0.0	-0.656361
	2s'	0.0	-0.000518	0.0	0.024590
	2s''	0.0	0.000183	0.0	0.004727
	2px	0.0	0.0	0.003189	0.0
	2px'	0.0	0.0	0.001242	0.0
	2py	0.0	-0.006294	0.0	-0.282548
	2py'	0.0	-0.001824	0.0	-0.006825
	2pz	0.0	0.0	0.0	0.0
	2pz'	0.0	0.0	0.0	0.0

Table 58. Basis B wavefunction and energy terms for Na_2Li^+

Table 53. (Continued)

		Orbital Basis	5A ₁ -1.668717	1A ₂ -1.668665	4B ₁ -1.668369	6A ₁ -0.332554
Na	1s	-0.001548	0.0	0.001622	0.020272	
	2s	0.006867	0.0	-0.007252	-0.108564	
	2s'	0.000035	0.0	-0.000031	0.000137	
	3s	0.000069	0.0	0.000238	0.331075	
	3s'	-0.002574	0.0	0.002597	0.020187	
	3s''	-0.000943	0.0	0.000992	-0.011776	
	2px	-0.541197	0.0	0.653107	0.041445	
	2px'	0.000509	0.0	-0.000629	-0.001231	
	3px	0.001069	0.0	-0.001533	0.089229	
	3px'	-0.000107	0.0	-0.000204	0.027116	
	2py	-0.454598	0.0	0.271086	0.025058	
	2py'	0.000472	0.0	-0.000227	-0.000620	
	3py	0.001440	0.0	-0.000526	0.040312	
	3py'	0.000163	0.0	0.000052	0.017256	
	2pz	0.0	-0.707030	0.0	0.0	
	2pz'	0.0	0.000920	0.0	0.0	
	3pz	0.0	0.000530	0.0	0.0	
	3pz'	0.0	0.000160	0.0	0.0	
Na	1s	-0.001548	0.0	-0.001622	0.020272	
	2s	0.006867	0.0	-0.007252	-0.108564	
	2s'	0.000035	0.0	0.000031	0.000137	
	3s	0.000069	0.0	-0.000238	0.331075	
	3s'	-0.002574	0.0	-0.002597	0.020187	
	3s''	-0.000943	0.0	-0.000992	-0.011776	
	2px	-0.541197	0.0	0.653107	0.041445	
	2px'	-0.000509	0.0	-0.000629	0.001231	
	3px	-0.001069	0.0	-0.001533	-0.089229	
	3px'	0.000107	0.0	-0.000204	-0.027116	
	2py	-0.454598	0.0	-0.271086	0.025058	
	2py'	0.000472	0.0	-0.000227	-0.000620	
	3py	0.001440	0.0	-0.000526	0.040312	
	3py'	0.000163	0.0	-0.00052	0.017256	
	2pz	0.0	0.707030	0.0	0.0	
	2pz'	0.0	-0.000920	0.0	0.0	
	3pz	0.0	-0.000530	0.0	0.0	
	3pz'	0.0	-0.000160	0.0	0.0	
Li	1s	-0.000801	0.0	0.0	0.109959	
	1s'	-0.000093	0.0	0.0	-0.008481	
	2s	0.001632	0.0	0.0	-0.468636	
	2s'	-0.000765	0.0	0.0	0.013832	
	2s''	0.000226	0.0	0.0	-0.005399	
	2px	0.0	0.0	-0.000701	0.0	
	2px'	0.0	0.0	0.00047	0.0	
	2py	0.001197	0.0	0.0	-0.167017	
	2py'	-0.000691	0.0	0.0	-0.032078	
	2pz	0.0	0.0	0.0	0.0	
	2pz'	0.0	0.0	0.0	0.0	

Table 59. Basis 1 wavefunction and energy terms for Na_2^+

Table 59. (Continued)

	Orbital Basis	1d ²	3d ¹	1E ¹	1E ¹	1A ²	4E ¹	4E ¹	4I ¹
Na	1s	0-0	-0.001854	0-0	0-0	0-0	0-0	0-0	-0.002835
	2s	0-0	-0.008521	0-0	0-0	0-0	-0.003318	0-0	-0.07563
	2s'	0-0	-0.00102	0-0	0-0	0-0	-0.00031	0-0	-0.012062
	3s	0-0	-0.002827	0-0	0-0	0-0	-0.002511	0-0	-0.010204
	3s'	0-0	-0.002582	0-0	0-0	0-0	-0.000928	0-0	-0.009124
	3s''	0-0	-0.000618	0-0	0-0	-0.00079	-0.000289	-0-0	-0.023220
	2px	0-0	-0.499071	0-0	0-0	-0.088619	-0.262231	0-0	-0.051078
	2px'	0-0	-0.003181	0-0	0-0	-0.002029	-0.01786	-0-0	-0.004270
	3px	0-0	-0.002107	0-0	0-0	-0.000928	-0.002342	-0-0	-0.004054
	3px'	0-0	-0.001091	0-0	0-0	-0.000221	-0.000325	-0-0	-0.00449
	2py	0-0	-0.288139	0-0	0-0	-0.495903	-0.029523	-0-0	-0.023033
	2py'	0-0	-0.001837	0-0	0-0	-0.004829	-0.423498	-0-0	-0.029491
	3py	0-0	-0.001216	0-0	0-0	-0.001607	-0.001280	-0-0	-0.061249
	3py'	0-0	-0.000630	0-0	0-0	-0.000383	-0.000244	-0-0	-0.013298
	2pz	0-0	-0.576748	0-0	-0.703178	-0.414059	0-0	0-0	0-0
	2pz'	0-0	-0.003938	0-0	-0.00473	-0.002869	0-0	0-0	0-0
	3pz	0-0	-0.002173	0-0	-0.000336	-0.000021	0-0	0-0	0-0
	3pz'	0-0	-0.000553	0-0	-0.000766	-0.000451	0-0	0-0	0-0
Ba	1s	0-0	-0.001854	0-0	-0.000745	0-0	-0.00742	-0-0	-0.027563
	2s	0-0	-0.008521	0-0	0-0	-0.012105	-0.003157	-0-0	-0.107031
	2s'	0-0	-0.000102	0-0	0-0	-0.000112	-0.00029	0-0	-0.010204
	3s	0-0	-0.002827	0-0	0-0	-0.000157	-0.002388	-0-0	-0.432207
	3s'	0-0	-0.002582	0-0	0-0	-0.003385	-0.000883	-0-0	-0.024843
	3s''	0-0	-0.000618	0-0	0-0	-0.000290	-0.000076	-0-0	-0.021572
	2px	0-0	-0.499071	0-0	-0.288619	-0.650525	-0.253766	-0-0	-0.051078
	2px'	0-0	-0.003181	0-0	-0.002029	-0.000295	-0.001731	0-0	-0.001630
	3px	0-0	-0.002107	0-0	-0.000928	-0.000085	-0.002286	-0-0	-0.106087
	3px'	0-0	-0.001091	0-0	-0.000221	-0.000453	-0.000318	-0-0	-0.023033
	2py	0-0	-0.288139	0-0	-0.499073	-0.42612	-0.355059	-0-0	-0.029491
	2py'	0-0	-0.001837	0-0	-0.003515	-0.012821	-0.000321	-0-0	-0.000941
	3py	0-0	-0.001216	0-0	-0.001607	-0.003048	-0.001321	-0-0	-0.061249
	3py'	0-0	-0.000630	0-0	-0.000383	-0.000375	-0.000249	-0-0	-0.013298
	2pz	0-0	-0.576748	0-0	-0.701775	-0.401941	0-0	0-0	0-0
	2pz'	0-0	-0.003938	0-0	-0.004922	-0.002786	0-0	0-0	0-0
	3pz	0-0	-0.002173	0-0	-0.000336	-0.000020	0-0	0-0	0-0
	3pz'	0-0	-0.000653	0-0	-0.000774	-0.000438	0-0	0-0	0-0
	Na	1s	-0.001854	0-0	-0.000745	0-0	-0.002093	-0-0	-0.027563
	2s	0-0	-0.008521	0-0	0-0	-0.008787	-0.000904	-0-0	-0.107032
	2s'	0-0	-0.000102	0-0	0-0	-0.000081	-0.000082	-0-0	-0.010204
	3s	0-0	-0.002827	0-0	0-0	-0.006646	-0.006736	-0-0	-0.432207
	3s'	0-0	-0.002582	0-0	0-0	-0.002458	-0.002490	-0-0	-0.024843
	3s''	0-0	-0.000618	0-0	0-0	-0.000211	-0.000213	-0-0	-0.021572
	2px	0-0	-0.576748	0-0	-0.703178	-0.414059	-0.00453	-0-0	-0.029491
	2px'	0-0	-0.003674	0-0	-0.000928	-0.000085	-0.002286	-0-0	-0.000941
	3px	0-0	-0.002432	0-0	-0.000459	-0.000841	-0.000829	-0-0	-0.000941
	3px'	0-0	-0.001259	0-0	-0.001856	-0.001682	-0.001660	-0-0	-0.000941
	2py	0-0	-0.576748	0-0	-0.701775	-0.401941	0-0	0-0	0-0
	2py'	0-0	-0.003674	0-0	-0.004922	-0.002786	0-0	0-0	0-0
	3py	0-0	-0.002432	0-0	-0.000336	-0.000020	0-0	0-0	0-0
	3py'	0-0	-0.001259	0-0	-0.000774	-0.000438	0-0	0-0	0-0
	2pz	0-0	-0.576748	0-0	-0.703178	-0.414059	-0.00453	-0-0	-0.029491
	2pz'	0-0	-0.003674	0-0	-0.000928	-0.000085	-0.002286	-0-0	-0.000941
	3pz	0-0	-0.002432	0-0	-0.000459	-0.000841	-0.000829	-0-0	-0.000941
	3pz'	0-0	-0.001259	0-0	-0.001856	-0.001682	-0.001660	-0-0	-0.000941

Table 60. Basis B wavefunction and energy terms for Na_3^+

Table 60. (Continued)

		Orbital	31'	142''	142'	12''	12'	1E''	4E'	4E''	4A1'
Basis	-1.6660114	-1.6660010	-1.659988	-1.6593878	-1.659878	-1.659506	-1.659506	-1.659506	-1.659506	-1.659506	-0.318850
Ma	1s	0.001227	0.0	0.0	0.0	0.0	0.0	0.001803	0.000426	0.000426	0.021278
	2s	-0.005277	0.0	0.0	0.0	0.0	0.0	-0.008060	-0.001904	-0.001904	-0.107359
	2s'	-0.00039	0.0	0.0	0.0	0.0	0.0	-0.000335	-0.00008	-0.00008	-0.00518
	3s	0.000854	0.0	0.0	0.0	0.0	0.0	0.000413	0.000097	0.000097	0.337607
	3s''	0.002054	0.0	0.0	0.0	0.0	0.0	0.002872	0.000678	0.000678	0.017608
	3s'''	0.00780	0.0	0.0	0.0	0.0	0.0	0.001090	0.000257	0.000257	0.011556
	2px	0.495584	0.0	-0.288648	0.0	0.0	0.0	0.693950	0.088276	0.088276	0.39627
	2px'	-0.00500	0.0	-0.00391	0.0	0.0	0.0	-0.00677	-0.00066	-0.00066	-0.01225
	3px	-0.00818	0.0	-0.00462	0.0	0.0	0.0	-0.001671	-0.000014	-0.000014	0.090818
	3px'	-0.00004	0.0	-0.00167	0.0	0.0	0.0	-0.000242	0.000097	0.000097	0.024041
	3s	-0.286435	0.0	-0.499953	0.0	0.0	0.0	-0.361359	-0.216349	-0.216349	0.022879
	2py	-0.000289	0.0	-0.00677	0.0	0.0	0.0	-0.000342	-0.000243	-0.000243	-0.00707
	3py	-0.000472	0.0	-0.00800	0.0	0.0	0.0	-0.000767	-0.000841	-0.000841	0.052434
	3py'	-0.000002	0.0	-0.00289	0.0	0.0	0.0	-0.00060	-0.000289	-0.000289	0.013880
	2pz	-0.577166	0.0	-0.816372	-0.003133	0.0	0.0	0.0	0.0	0.0	0.0
	2pz'	-0.000747	0.0	-0.001064	-0.000004	0.0	0.0	0.0	0.0	0.0	0.0
	3pz	0.0	-0.000539	0.0	-0.000706	-0.000003	0.0	0.0	0.0	0.0	0.0
	3pz'	0.0	-0.000298	0.0	-0.000265	-0.000001	0.0	0.0	0.0	0.0	0.0
Ma	1s	0.001227	0.0	0.0	0.0	0.0	0.0	-0.001270	0.001349	0.001349	0.021278
	2s	-0.005277	0.0	0.0	0.0	0.0	0.0	-0.005779	-0.006028	-0.006028	-0.107359
	2s'	-0.00039	0.0	0.0	0.0	0.0	0.0	-0.00024	-0.000038	-0.000038	-0.00518
	3s	0.000854	0.0	0.0	0.0	0.0	0.0	-0.000291	0.000309	0.000309	0.397607
	3s''	0.002054	0.0	0.0	0.0	0.0	0.0	-0.00024	0.002448	0.002448	0.017608
	3s'''	0.00780	0.0	0.0	0.0	0.0	0.0	-0.000768	0.00815	0.00815	0.011556
	2px	-0.499954	0.0	-0.288648	0.0	0.0	0.0	-0.455781	-0.039627	-0.039627	0.039627
	2px'	-0.000500	0.0	-0.00391	0.0	0.0	0.0	-0.000529	-0.000428	-0.000428	0.01225
	3px	-0.000818	0.0	-0.00462	0.0	0.0	0.0	-0.001387	-0.000931	-0.000931	0.090818
	3px'	-0.00004	0.0	-0.00167	0.0	0.0	0.0	-0.000255	-0.00052	-0.00052	0.024041
	2py	0.286435	0.0	-0.499953	0.0	0.0	0.0	-0.182680	-0.379807	-0.379807	0.022879
	2py'	-0.000289	0.0	-0.00677	0.0	0.0	0.0	-0.000152	-0.000392	-0.000392	-0.00707
	3py	-0.000472	0.0	-0.00800	0.0	0.0	0.0	-0.000178	-0.001125	-0.001125	0.013880
	3py'	-0.000002	0.0	-0.00289	0.0	0.0	0.0	-0.000104	-0.000267	-0.000267	0.013880
	2pz	-0.577166	0.0	-0.705432	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3pz	0.0	-0.000747	0.0	-0.000535	-0.000019	0.0	0.0	0.0	0.0	0.0
	3pz'	0.0	-0.000539	0.0	-0.000355	-0.000061	0.0	0.0	0.0	0.0	0.0
	3pz''	0.0	-0.000298	0.0	-0.000133	-0.0000229	0.0	0.0	0.0	0.0	0.0
Ga	1s	0.001227	0.0	0.0	0.0	0.0	0.0	-0.000533	-0.001774	-0.001774	0.021278
	2s	-0.005277	0.0	0.0	0.0	0.0	0.0	-0.002381	-0.007932	-0.007932	-0.107359
	2s'	-0.00039	0.0	0.0	0.0	0.0	0.0	-0.000034	-0.000034	-0.000034	-0.00518
	3s	0.000854	0.0	0.0	0.0	0.0	0.0	-0.000122	-0.000406	-0.000406	0.397607
	3s''	0.002054	0.0	0.0	0.0	0.0	0.0	-0.000848	-0.002826	-0.002826	0.017608
	3s'''	0.00780	0.0	0.0	0.0	0.0	0.0	-0.000322	-0.001072	-0.001072	0.011556
	2px	0.0	-0.577296	0.0	0.0	0.0	0.0	-0.141040	-0.042193	-0.042193	0.0
	2px'	0.0	-0.000782	0.0	0.0	0.0	0.0	-0.000175	-0.000052	-0.000052	0.0
	3px	0.0	-0.000924	0.0	0.0	0.0	0.0	-0.000710	-0.000213	-0.000213	0.0
	3px'	0.0	-0.000333	0.0	0.0	0.0	0.0	-0.000287	-0.000086	-0.000086	0.0
	2py	-0.576870	0.0	0.0	0.0	0.0	0.0	0.230949	0.769245	0.769245	-0.45757
	2py'	-0.000578	0.0	0.0	0.0	0.0	0.0	-0.000224	-0.00744	-0.00744	0.0314
	3py	0.000945	0.0	0.0	0.0	0.0	0.0	-0.000541	-0.001802	-0.001802	0.10388
	3py'	-0.000005	0.0	0.0	0.0	0.0	0.0	-0.000071	-0.000235	-0.000235	-0.027760
	2pz	0.0	-0.577166	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3pz	0.0	-0.000757	0.0	0.0	0.0	0.0	-0.000528	-0.000923	-0.000923	0.0
	3pz'	0.0	-0.000339	0.0	0.0	0.0	0.0	-0.000350	-0.000612	-0.000612	0.0
			0.0	-0.000131	-0.000230	0.0	0.0	0.0	0.0	0.0	0.0

Table 61. Mulliken population analyses for H₂

Basis A				Basis B		
Net Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
H	0.0	0.57519	0.57519	0.0	0.57578	0.57578
H	0.0	0.57519	0.57519	0.0	0.57578	0.57578
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
H-H	0.0	0.84963	0.84963	0.0	0.84844	0.84844
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
H	0.0	1.0	1.0	0.0	1.0	1.0
H	0.0	1.0	1.0	0.0	1.0	1.0

Table 62. Mulliken population analyses for H₃⁺

Basis A				Basis B		
Net Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
H	0.0	0.29569	0.29569	0.0	0.30142	0.30142
H	0.0	0.29569	0.29569	0.0	0.30142	0.30142
H	0.0	0.29569	0.29569	0.0	0.30142	0.30142
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
H-H	0.0	0.37097	0.37097	0.0	0.36524	0.36524
H-H	0.0	0.37097	0.37097	0.0	0.36524	0.36524
H-H	0.0	0.37097	0.37097	0.0	0.36524	0.36524
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
H	0.0	0.66667	0.66667	0.0	0.66667	0.66667
H	0.0	0.66667	0.66667	0.0	0.66667	0.66667
H	0.0	0.66667	0.66667	0.0	0.66667	0.66667

Table 63. Mulliken population analyses for LiH

Basis A				Basis B			
	Atom	Net	Atom Population		Atom	Population	
Bond	Atom	Core	Valence	Total	Core	Valence	Total
	Li	1.99707	0.34537	2.34244	1.97967	0.29600	2.27567
	H	0.00006	0.91118	0.91125	0.00083	0.97418	0.97501
Overlap Population							
Bond	Core	Valence	Total	Core	Valence	Total	
Li-H	0.00287	0.74344	0.74631	0.01950	0.72982	0.74932	
Gross Atom Population							
Atom	Core	Valence	Total	Core	Valence	Total	
Li	1.99850	0.71709	2.71560	1.98942	0.66091	2.65033	
H	0.00150	1.28291	1.28440	0.01058	1.33909	1.34967	

Table 64. Mulliken population analyses for LiH₂⁺

Basis A				Basis B		
Net Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
H	0.00000	0.55180	0.55181	0.00001	0.55157	0.55159
H	0.00000	0.55180	0.55181	0.00001	0.55157	0.55159
Li	2.00019	0.00627	2.00646	1.99746	0.00676	2.00422
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
H-H	0.00000	0.83503	0.83503	0.00002	0.82450	0.82452
Li-H	-0.00010	0.02755	0.02745	0.00125	0.03279	0.03404
Li-H	-0.00010	0.02755	0.02745	0.00125	0.03279	0.03404
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
H	-0.00005	0.98309	0.98305	0.00065	0.98022	0.98087
H	-0.00005	0.98309	0.98305	0.00065	0.98022	0.98087
Li	2.00009	0.03381	2.03390	1.99871	0.03956	2.03826

Table 65. Mulliken population analyses for Li₂

Basis A				Basis B		
Atom	Net Atom Population			Core	Valence	Total
	Core	Valence	Total			
Li	1.99704	0.62619	2.62323	2.00465	0.59796	2.60261
Li	1.99704	0.62619	2.62323	2.00465	0.59796	2.60261
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
Li-Li	0.00592	0.74763	0.75355	-0.00930	0.80407	0.79477
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Li	2.00000	1.00000	3.00000	2.00000	1.00000	3.00000
Li	2.00000	1.00000	3.00000	2.00000	1.00000	3.00000

Table 66. Mulliken population analyses for Li₂H⁺

Basis A				Basis B		
Atom	Net Atom Population			Core	Valence	Total
	Core	Valence	Total			
Li	1.99643	0.06685	2.06328	1.98925	0.07809	2.06815
Li	1.99643	0.06685	2.06328	1.98925	0.07809	2.06815
H	0.00005	1.16203	1.16208	0.00100	1.06040	1.06140
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
Li-Li	0.00193	0.05964	0.06157	-0.00520	0.06524	0.06004
Li-H	0.00258	0.32232	0.32490	0.01204	0.35909	0.37113
Li-H	0.00258	0.32232	0.32490	0.01204	0.35909	0.37113
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Li	1.99868	0.25783	2.25651	1.99347	0.29026	2.28373
Li	1.99868	0.25783	2.25651	1.99347	0.29026	2.28373
H	0.00263	1.48435	1.48698	0.01305	1.41948	1.43253

Table 67. Mulliken population analyses for Li₃⁺

Basis A				Basis B		
Net Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Li	1.99733	0.29819	2.29552	2.00610	0.29288	2.29898
Li	1.99733	0.29819	2.29552	2.00610	0.29288	2.29898
Li	1.99733	0.29819	2.29552	2.00610	0.29288	2.29898
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
Li-Li	0.00268	0.36847	0.37115	-0.00610	0.37379	0.36769
Li-Li	0.00268	0.36847	0.37115	-0.00610	0.37379	0.36769
Li-Li	0.00268	0.36847	0.37115	-0.00610	0.37379	0.36769
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Li	2.00000	0.66667	2.66667	2.00000	0.66667	2.66667
Li	2.00000	0.66667	2.66667	2.00000	0.66667	2.66667
Li	2.00000	0.66667	2.66667	2.00000	0.66667	2.66667

Table 68. Mulliken population analyses for NaH

Basis A				Basis B		
Net Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Na	10.00923	0.25114	10.26037	9.99792	0.25672	10.25464
H	0.00043	1.18404	1.18447	0.00015	1.14231	1.14246
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
Na-H	-0.00966	0.56482	0.55516	0.00194	0.60096	0.60290
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Na	10.00440	0.53355	10.53795	9.99889	0.55720	10.55609
H	-0.00440	1.46645	1.46205	0.00111	1.44280	1.44391

Table 69. Mulliken population analyses for NaH_2^+

Basis A				Basis B		
Net Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
H	0.00046	0.55995	0.56041	0.00005	0.57020	0.57025
H	0.00046	0.55995	0.56041	0.00005	0.57020	0.57025
Na	9.99644	0.00388	10.00032	9.99785	0.00338	10.00123
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
H-H	-0.00062	0.84511	0.84449	-0.00004	0.85498	0.85494
Na-H	0.00163	0.01555	0.01718	0.00105	0.00062	0.00167
Na-H	0.00163	0.01555	0.01718	0.00105	0.00062	0.00167
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
H	0.00097	0.99028	0.99125	0.00055	0.99800	0.99855
H	0.00097	0.99028	0.99125	0.00055	0.99800	0.99855
Na	9.99806	0.01944	10.01750	9.99890	0.00400	10.00290

Table 70. Mulliken population analyses for NaLi

Basis A				Basis B		
Net Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Na	9.98669	0.18755	10.17424	10.00684	0.34337	10.35021
Li	2.00023	1.47378	3.47401	2.00057	1.01244	3.01301
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
Na-Li	0.01308	0.33867	0.35175	-0.00742	0.64420	0.63678
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Na	9.99323	0.35688	10.35011	10.00313	0.66547	10.66860
Li	2.00677	1.64312	3.64989	1.99687	1.33453	3.33140

Table 71. Mulliken population analyses for NaLi₂⁺

Basis A				Basis B		
Net Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Li	1.99823	0.48316	2.48139	2.00425	0.35762	2.36187
Li	1.99823	0.48316	2.48139	2.00425	0.35762	2.36187
Na	9.99014	0.09671	10.08685	10.00594	0.20507	10.21101
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
Li-Li	0.00371	0.60699	0.61070	-0.00777	0.46918	0.46141
Na-Li	0.00485	0.16499	0.16984	-0.00334	0.30526	0.30192
Na-Li	0.00485	0.16499	0.16984	-0.00334	0.30526	0.30192
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Li	2.00250	0.86915	2.87165	1.99870	0.74483	2.74353
Li	2.00250	0.86915	2.87165	1.99870	0.74483	2.74353
Na	9.99498	0.26171	10.25669	10.00260	0.51033	10.51293

Table 72. Mulliken population analyses for Na₂

Basis A				Basis B		
Net Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Na	9.99475	0.74963	10.74438	10.00332	0.67130	10.67462
Na	9.99475	0.74963	10.74438	10.00332	0.67130	10.67462
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
Na-Na	0.01050	0.50074	0.51124	-0.00663	0.65739	0.65076
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Na	10.00000	1.00000	11.00000	10.00000	1.00000	11.00000
Na	10.00000	1.00000	11.00000	10.00000	1.00000	11.00000

Table 73. Mulliken population analyses for Na₂H⁺

Basis A				Basis B		
Atom	Net Atom Population			Core	Valence	Total
	Core	Valence	Total			
Na	10.01131	0.07737	10.08868	9.99493	0.05950	10.05443
Na	10.01131	0.07737	10.08868	9.99493	0.05950	10.05443
H	0.00159	1.23342	1.23501	0.00037	1.33642	1.33679
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
Na-Na	-0.00091	0.03041	0.02950	-0.00173	0.03572	0.03399
Na-H	-0.01165	0.29071	0.27906	0.00575	0.25443	0.26018
Na-H	-0.01165	0.29071	0.27906	0.00575	0.25443	0.26018
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Na	10.00502	0.23794	10.24296	9.99694	0.20457	10.20151
Na	10.00502	0.23794	10.24296	9.99694	0.20457	10.20151
H	-0.01005	1.52413	1.51408	0.00612	1.59085	1.59697

Table 74. Mulliken population analyses for Na₂Li⁺

Basis A				Basis B		
Net Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Na	9.99126	0.14692	10.13818	10.00490	0.24329	10.24819
Na	9.99126	0.14692	10.13818	10.00490	0.24329	10.24819
Li	2.00020	1.00278	3.00298	2.00111	0.48786	2.48897
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
Na-Na	0.00247	0.07423	0.07670	-0.00132	0.23413	0.23281
Na-Li	0.00741	0.31458	0.32199	-0.00479	0.39571	0.39092
Na-Li	0.00741	0.31458	0.32199	-0.00479	0.39571	0.39092
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Na	9.99620	0.34132	10.33752	10.00185	0.55821	10.56006
Na	9.99620	0.34132	10.33752	10.00185	0.55821	10.56006
Li	2.00761	1.31736	3.32497	1.99632	0.88357	2.87989

Table 75. Mulliken population analyses for Na₃⁺

Basis A				Basis B		
Atom	Net Atom Population			Core	Valence	Total
	Core	Valence	Total			
Na	9.99513	0.40413	10.39926	10.00253	0.33824	10.34077
Na	9.99513	0.40413	10.39926	10.00253	0.33824	10.34077
Na	9.99513	0.40413	10.39926	10.00253	0.33824	10.34077
Overlap Population						
Bond	Core	Valence	Total	Core	Valence	Total
Na-Na	0.00486	0.26254	0.26740	-0.00254	0.32843	0.32589
Na-Na	0.00486	0.26254	0.26740	-0.00254	0.32843	0.32589
Na-Na	0.00486	0.26254	0.26740	-0.00254	0.32843	0.32589
Gross Atom Population						
Atom	Core	Valence	Total	Core	Valence	Total
Na	10.00000	0.66667	10.66667	10.00000	0.66667	10.66667
Na	10.00000	0.66667	10.66667	10.00000	0.66667	10.66667
Na	10.00000	0.66667	10.66667	10.00000	0.66667	10.66667

Table 76. Energy of Li₂•••Li⁺ system (Basis A)

h	Energy	Gross Population	
		Li ₂	Li ⁺
10.0	-22.0813	5.694	2.305
12.5	-22.0660	5.808	2.193
15.0	-22.0588	5.890	2.110
20.0	-22.0564	6.000	2.000
25.0	-22.0559	6.000	2.000
30.0	-22.0557	6.000	2.000
limit of infinite separation	-22.0555	6.0	2.0

Table 77. Triatomic ion dissociation energies

Reaction	D (kcal/mole)	Basis A	Basis B
$H_3^+ \longrightarrow H^+ + H_2$	103.7	103.7	
$LiH_2^+ \longrightarrow Li^+ + H_2$	5.7	5.2	
$Li_2H^+ \longrightarrow Li^+ + LiH$	58.3	57.2	
$Li_3^+ \longrightarrow Li^+ + Li_2$	45.0	41.8	
$NaH_2^+ \longrightarrow Na^+ + H_2$	3.3	2.9	
$NaLi_2^+$ $\overline{\longrightarrow} Na^+ + Li_2$	33.4	31.2	
$NaLi_2^+$ $\overline{\longrightarrow} Li^+ + NaLi$	54.9	44.0	
$Na_2H^+ \longrightarrow Na^+ + NaH$	57.7	51.4	
Na_2Li^+ $\overline{\longrightarrow} Na^+ + NaLi$	39.2	33.6	
Na_2Li^+ $\overline{\longrightarrow} Li^+ + Na_2$	92.1	48.3	
$Na_3^+ \longrightarrow Na^+ + Na_2$	62.5	37.8	

SUMMARY

The concept of an even-tempered atomic orbital basis has been introduced. The basis is shown to be a practical, as well as accurate replacement for traditional atomic orbital bases which depend on many non-linear parameters in an awkward and uneconomical manner. In SCF calculations on atoms, near Hartree-Fock accuracy, as well as calculational efficiency and speed has been obtained. For a wide variety of atoms, full optimization of the few even-tempered basis parameters was carried out economically.

For use in molecular calculations, "contracted" even-tempered Gaussian bases, i.e. linear combinations of even-tempered Gaussians are formed. An efficient method of assembling contracted two-electron integrals made possible the use of a new scheme for the definition of the contracted Gaussian atomic orbitals. It was shown to be energetically very competitive with the best prior method. Using the new contracted even-tempered Gaussian atomic orbital bases, several calculations on positive triatomic alkali ions were made and their significance discussed. The performance of the new basis in the molecular calculations was found to be excellent, showing economy and efficiency and causing no convergence problems.

APPENDIX A: DESCRIPTION OF THE ATOMIC SCF PROGRAM

General Specifications

The program used for all atomic calculations employing exponential basis functions is based on the well-known open-shell matrix Hartree-Fock procedure in terms of symmetry orbitals as given by Roothaan (9) and described following the introduction. It was of interest to find the best even-tempered bases for the atoms and so an optimization of the SCF atomic energy in terms of the even-tempered basis parameters of Eq. 22 was included. A modified version of Powell's program VAG4A (124) for minimization of a non-linear function (125) was used and the accuracy of each parameter x was required to be $2 \cdot 10^{-3} \cdot x(\text{initial})$. The optimization of each symmetry was carried out independently beginning with s and proceeding through the p, d, and f shells before returning to s. Each symmetry proved to be largely independent of the next and often only one pass through each was necessary to achieve a stable energy with an accompanying good virial coefficient V/E.

A set of subroutines is incorporated to compute several atomic properties from the wavefunctions obtained. At completion, the wavefunction is punched onto cards which may be used as input for future runs. A restart feature was incorporated in order that system failure or a poor estimate of

running time on the user's part would result in loss of a minimum amount of computation. Current information is stored on a peripheral device (disk file) at given time intervals and the supervisor program has an optional entry point which permits recovery of this information.

The basis size is limited so that main core size is kept at 128K bytes. This limitation only affects the section which carries out the formation of the \mathcal{Q} and \mathcal{K} symmetry averaged supermatrices. The consequence is that the largest basis for a given symmetry is fifteen exponential-type functions. If fifteen are used for one symmetry, all other symmetries must be twelve or less. A certain array is adjusted to 9260 elements which is $[n(n+1)/2] \times [m(m+1)/2]$ where $n=15$ and $m=12$. A basis of 195 primitives is possible if all four symmetries (s,p,d,f), with degeneracies (1,3,5,7), are used and if the numbers per symmetry are (15,12,12,12).

Sample running times for some typical atoms on an IBM 360/65 are:

Atom	State	Basis	Time/Energy (sec.)
O	³ P	(4,3)	5
Kr	¹ S	(8,6,3)	13
S	³ P	(9,6)	18
Gd ⁺³	⁸ S	(10,8,5,4)	68
Ga	² P	(12,9,4)	75
Rn	¹ S	(12,10,6,3)	92

Matrix Elements

The integrals which represent the matrix elements in the procedure were programmed by this author. The one-electron integrals which are expectation values of one-electron operators \hat{O} , with respect to the primitive exponential basis functions defined in Eq. 19, ($p=1$) . ,

$$\langle X(klm) | \hat{O} | X(k'l'm') \rangle = \int dV X(klm) \hat{O} X(k'l'm')$$

are easily derived and are quite simple by virtue of the limitation to 1s, 2p, 3d, and 4f orbitals. The formulae are; overlap,

$$\langle X(klm) | X(k'l'm') \rangle = (\zeta_p / \zeta_a)^{2l+3} \delta(l,l') \delta(m,m')$$

nuclear attraction,

$$\langle X(klm) | -Z/r | X(k'l'm') \rangle = -Z\zeta_a \langle X(klm) | X(k'l'm') \rangle / (l+1)$$

and kinetic energy,

$$\langle X(klm) | -\Delta/2 | X(k'l'm') \rangle = \zeta_p^2 \langle X(klm) | X(k'l'm') \rangle / 2$$

with

$$\zeta_a = (\zeta_k + \zeta_{k'})/2 \quad \text{and} \quad \zeta_p = (\zeta_k \zeta_{k'})^{1/2} .$$

The symbol $\delta(i,j)$ is the ordinary Kronecker delta.

The expectation values of various powers of r comprise a set of one-electron integrals which form the essential part

of several atomic properties. They can be expressed in a general form. If

$$I(l,n) = \langle X(klm) | r^n | X(k'lm') \rangle$$

it is easily shown that

$$I(l,n) = I(l,0) (\zeta_k + \zeta_{k'})^{-n} (2l+k+2)! / (2l+2)!, \quad (2l+k+2) \geq 0$$

where $I(l,0)$ is the overlap integral given previously.

The two-electron integrals which are the expectation values of the two-electron operator r_{12}^{-1} were programmed from a formula derived from the more general one given by Silver (126). Because of the symmetry restriction in the SCF procedure and because the primitive basis functions $X(klm)$ are carriers for the irreducible representations of the three-dimensional orthogonal group, only integrals

$$[X_1 X_2 | X_3 X_4] = [X(k_1 l_1 m_1) X(k_2 l_2 m_2) | X(k_3 l_3 m_3) X(k_4 l_4 m_4)]$$

where the pairs of quantum numbers $qv = (l_v, m_v)$ have a specific relationship must be computed. Either $q1=q2$ and $q3=q4$ or $q1=q3$ and $q2=q4$ (equivalently $q1=q4$ and $q2=q3$) must be satisfied. All other possibilities lead to integrals which are not required in the present SCF formalism. The special case $q1=q2=q3=q4$ is of course included. The two cases above must be provided for, with the special case treated best using formulae for the second.

Case 1: $q_1=q_2$ and $q_3=q_4$

$$[X_1 X_2 | X_3 X_4] = \frac{r \left(\zeta_p^{12} \right)^{2l_1+3} \left(\zeta_p^{34} \right)^{2l_3+3}}{\zeta_a^{2l_1+2} \zeta_{a'}^{2l_3+2} (\zeta_a + \zeta_{a'})} \\ \times \left\{ \tau_{a'}^{2l_3+2} \sum_{s=0}^{2l_1+1} \tau_a^s B_s + \tau_a^{2l_1+2} \sum_{t=0}^{2l_3+1} \tau_{a'}^t B_t' \right\}$$

with

$$B_s = (n'+s+1)! A(s, n), \quad n = 2l_1+1,$$

$$B_t' = (n+t+1)! A(t, n'), \quad n' = 2l_3+1.$$

Case 2: $q_1=q_3$ and $q_2=q_4$

$$[X_1 X_2 | X_3 X_4] = \frac{r \left(\zeta_p^{13} \right)^{2l_1+3} \left(\zeta_p^{24} \right)^{2l_2+3}}{\left(\zeta_a \zeta_{a'} \right)^{l_1+l_2+2} (\zeta_a + \zeta_{a'})} \\ \times \left\{ \tau_{a'}^{l_1+l_2+2} \sum_{s=s_0}^{l_1+l_2+1} \tau_a^s B_s + \tau_a^{l_1+l_2+2} \sum_{s=s_0}^{l_1+l_2+1} \tau_{a'}^s B_s \right\}$$

with

$$B_s = (n+s+1)! A(s, n), \quad n=l_1+l_2+1.$$

The following definitions serve to complete the two-electron

integral formulae:

$$\zeta_p^{ij} = (\zeta_i \zeta_j)^{1/2}$$

$$\zeta_a = (\zeta_1 + \zeta_2)/2, \quad \zeta_{a'} = (\zeta_3 + \zeta_4)/2,$$

$$\tau_a = [\zeta_a / (\zeta_a + \zeta_{a'})], \quad \tau_{a'} = [\zeta_{a'} / (\zeta_a + \zeta_{a'})],$$

$$A(s, n) = p \sum_{l=1}^{l(\max)} \frac{(n-1)!}{(n-s)!} \sum_{m, m'} \delta(m, m') Q(lmq_1q_2) Q(lmq_3q_4),$$

$$p = \prod_{\mu=1}^4 (-1)^\mu (2l_\mu + 1)^{1/2},$$

$$s^o = l^o = \max\{|l_1 - l_2|, |l_3 - l_4|, m^*\},$$

$$m^* = \min\{|m-|, |m+|\},$$

$$m\pm = \text{sign}(m_1) \text{ sign}(m_2) ||m_1| \pm |m_2||,$$

$$m\pm' = \text{sign}(m_3) \text{ sign}(m_4) ||m_3| \pm |m_4||,$$

$$\text{sign}(m) = m/|m|, \quad \text{sign}(0) = +1,$$

$$l(\max) = \min\{l_1 + l_2, l_3 + l_4, s\},$$

and $(l + l_1 + l_2)$ must be even. The remaining quantities are:

$$Q(lm+q_1q_2) = e^+(m_1, m_2) (-1)^{m_1+m_2} [(1+\delta(0, m_1 m_2))/2]^{1/2} \\ \times \begin{pmatrix} l_1 & l_2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & 1 \\ |m_1| & |m_2| & -(l_1 + l_2) \end{pmatrix}$$

$$Q(lm-q_1q_2) = e^-(m_1, m_2) (-1)^M [(1+\delta(m_1, m_2))/2]^{1/2} \\ \times \begin{pmatrix} l_1 & l_2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & 1 \\ -|m_1| & |m_2| & (l_1 - l_2) \end{pmatrix}$$

where $M = \max\{|m_1|, |m_2|\}$, the symbol

$$\begin{pmatrix} 1 & l' & l'' \\ m & m' & m'' \end{pmatrix}$$

is the Wigner 3j coefficient (127), and e^+ and e^- depend upon the signs of the product, $(m_1 m_2)$, and the sum, $(m_1 + m_2)$, according to

$(m_1 m_2)$	$(m_1 + m_2)$	e^+	e^-
+	+	1	1
+	-	-1	1
-	+	1	-1
-	-	1	1
-	0	1	0
0	+, 0, -	1	0

The sums on m and m' in the definition of $A(s, n)$ are each over the two values m^+ and m^- . The B 's are considered constants and were computed once and for all for each set of possible basis parameters l and m where $0 \leq l \leq 3$ and $-l \leq m \leq l$.

The basic integrals were computed in blocks with respect

to the pairs of quantum numbers l and m , the cycles within the blocks being over the zetas. Because of this, it was easy to include a feature whereby only blocks of integrals which change in successive energy calculations are recomputed. In successive calculations, during an energy optimization, only the parameters of a single symmetry, are changed at one time. Thus only integrals over basis functions belonging to that symmetry will change. This may involve a relatively small fraction of the total number of basic integrals and consequently a large percentage of the integral computation time is saved.

After computation of the necessary basic integrals, a transformation of the naturally symmetry adapted two-electron integral supermatrices to "symmetry averaged" supermatrices

$$g(lpq\bar{l}'rs) = (d_l d_{l'})^{-1} \sum_{mm'} g(lmpq\bar{l}'m'rs)$$

and

$$\kappa(lpq\bar{l}'rs) = (d_l d_{l'})^{-1} \sum_{mm'} \kappa(lmpq\bar{l}'m'rs)$$

is carried out where $d_l=2l+1$ and the m summations run over all partners in the irrep l ($-l \leq m \leq l$). This serves to condense the number of integrals which must be handled during SCF iterations. Then the symmetry averaged supermatrices

$\rho_{(ll')}$ and $\varrho_{(ll')}$ as defined in Eqs. 17 and 18 are found. The coupling coefficients $A_{(ll')}$ and $B_{(ll')}$ in Eq. 18 are required for the open-shell calculations. Single open-shell coefficients used in this work are recorded in Table 78. All double open-shells were double half-filled shells for which the off-diagonal ($l' \neq l$) coefficients are: $A_{(ll')}=0$ and $B_{(ll')}=-1$.

SCF Specifications

The Restricted Hartree-Fock program used is a highly revised version of Wahl's programs which form a part of the BISON system (11). It consists of the procedure which was described in the text and is essentially that of Roothaan and Bagus (9). The Fock equations are formed and solved separately for each symmetry l for both open and closed shells.

In order to aid convergence, a density extrapolation procedure was used. Unlike vector extrapolation, orthonormalization of extrapolated densities is not required. Along with the extrapolation, a density averaging technique was used. In the case where intermediate energies undergo an oscillatory behavior, convergence may often not be obtained. Averaging the prior two density matrices aids in damping the oscillation. The density extrapolation techniques were found satisfactory in all cases except one in which even a prior vector extrapolation technique failed. The atomic state

under consideration, a 1S state of carbon, was not important in this investigation and so the solution of this convergence problem was not pursued.

Table 78. Single open-shell coupling coefficients

l	Configu- ration	State	A	B
0	s^1	2S	0	-1
1	p^1	2P	1	1
1	p^2	3P 1D 1S	$1/4$ $11/20$ 1	$-1/2$ $13/10$ 4
1	p^3	4S 2D 2P	0 $1/5$ $1/3$	-1 $1/5$ 1
1	p^4	3P 1D 1S	$1/16$ $11/80$ $1/4$	$-1/8$ $13/40$ 1
1	p^5	2P	$1/25$	$1/25$
2	d^1	2D	1	1
2	d^5	6S	0	-1
3	f^1	2F	1	1
3	f^7	8S	0	-1

APPENDIX B: NON-ORTHOGONAL SCF ATOMIC ORBITALS

Discussion

The atomic orbitals of hydrogen can be formed by a Schmidt orthogonalization process. The 1s, 2s, 3s,... orbitals result from carrying out this process on the nodeless radial functions $\exp(-r)$, $r \exp(-r/2)$, $r^2 \exp(-r/3)$, etc. Similarly, approximations to canonical SCF atomic orbitals of larger atoms formed from a minimal basis set can be expressed as the result of successive Schmidt orthogonalizations of certain Slater-type atomic orbitals; $\exp(-\zeta_1 r)$, $r \exp(-\zeta_2 r)$, $r^2 \exp(-\zeta_3 r)$, etc. An inspection of the accurate canonical SCF atomic orbitals reveals that they too have similar character with respect to the functional form. The number of nodes increases with the quantum number, the inner lobes appearing to represent those admixtures of orbitals with lower quantum number required for Schmidt orthogonality.

Molecular calculations employing unscaled minimal basis set Slater-type atomic orbitals may be carried out with either these non-orthogonal functions or with the orthogonalized atomic functions. The result is the same. It is most convenient to work with a non-orthogonal basis since such a basis is composed of single analytic functions in terms of which the necessary integrals are usually computed. Hence, an additional transformation of integrals is avoided. If in-

dependent scale factors are to be introduced, then there exists two choices: (1) introduce a scale parameter for each of the non-orthogonal Slater-type atomic orbitals of the minimal basis set, or (2) introduce a scale parameter for each orthogonal canonical SCF atomic orbital. If the latter were done, then the orthogonality between orthogonal functions would not be preserved. Furthermore, this would result in an increase in the number of primitive basis orbitals, because the function comprising an inner lobe of some scaled outer atomic orbital will be different from that comprising the innermost scaled atomic orbital. Linear dependence problems would arise unless the scale factors were made sufficiently different. Moreover, the two scaling procedures would end up energetically different. It is believed here that the first scaling procedure, which is the one usually adopted for practical reasons, would also yield better energies.

A similar situation is to be expected when accurate SCF atomic orbitals are used as basis for minimal basis set molecular calculations, and it seems desirable to be able to form non-orthogonal nodeless radial functions whose Schmidt orthogonalization reproduces the exact canonical SCF atomic orbitals. Any one of these nodeless functions should represent one lobe with the same qualitative character as a term $r^1 \exp(-\zeta r)$. Moreover one would expect that each of these non-orthogonal, nodeless, single-lobe functions would cover a

much smaller domain of the radial coordinate than the original canonical SCF atomic orbitals. Consequently, if accurate SCF atomic orbitals are expressed as linear combinations of primitive exponentials or Gaussians, a smaller number of these primitives will be required to represent the nodeless SCF atomic orbitals to an accuracy equivalent to that in the case of the canonical SCF atomic orbitals. This will lead to a savings in computation time for the calculation of integrals between the SCF atomic orbitals. It stands to reason, that the mentioned advantages will also persist if the minimal basis set is expanded to a larger basis.

Inversion of the Schmidt Process

The desired "deorthogonalization" of the canonical SCF atomic orbitals is conceptually the inverse of the Schmidt orthogonalization. Since the deorthogonalization need only be carried out within a symmetry, only the radial part of the SCF atomic orbital has to be considered. Hence, if $q_i^o(r)$ are the radial parts of the orthogonal canonical orbitals and $q_k(r)$ those of the desired non-orthogonal orbitals, the inverse Schmidt process is expressed by

$$q_k(r) = \sum_{i=1}^k q_i^o(r) c_{ik} \quad (k=2, 3, \dots).$$

where the sum extends only up to k . This implies that the

inverse Schmidt matrix \underline{c} is triangular. The process is however not unique. There are infinitely many non-orthogonal bases whose Schmidt orthogonalization yields the same orthogonal basis. For this reason, some criterion must be introduced.

Criterion

In view of the stated objective, it seems reasonable to demand that each of the nodeless SCF atomic orbitals should have as little absolute overlap as possible with all nodeless SCF atomic orbitals with lower quantum number. Therefore, the radial part of the k -th non-orthogonal SCF atomic orbital $g_k(r)$ is chosen such that

$$(31) \quad D(f) = \int_0^\infty r^2 dr |q_k(r)|^2 f(r)$$

is a minimum where $f(r)$ is a function of the inner shell density. In order to maintain normalization, the usual constraint

$$\int_0^\infty r^2 dr |q_k(r)|^2 = 1$$

is imposed. Several likely functions of the inner shell density were tested for $f(r)$. That one chosen was simply

$$f(r) = r^{-2} \sum_{j=1}^{k-1} |q_j(r)|^2 .$$

The sum above is over all non-orthogonal SCF atomic orbitals of the same symmetry and inner to that one being deorthogonalized (viz. that in Eq. 31). Since the part of the orthogonal orbital which is to be removed in deorthogonalization is that closest to the origin, the factor r^{-2} diminishes the de-emphasis of that region caused by the volume element $r^2 dr$ of Eq. 31.

Computational method

The method is recursive in nature. One proceeds to determine the k -th non-orthogonal atomic orbital after the previous ones $q_2(r) \dots q_{k-1}(r)$ have already been found. Minimization of the integral $D(f)$ of Eq. 31 to determine the k -th non-orthogonal SCF atomic orbital $q_k(r)$ is equivalent to diagonalizing the k -dimensional matrix \underline{D}^k which is the matrix of expectation values of $f(r)$ computed with respect to the orthogonal SCF atomic orbitals

$$(32) \quad D_{ij}^k = \int r^2 dr \ q_i^o(r) \ q_j^o(r) \ f(r).$$

That eigenvector corresponding to the smallest eigenvalue of \underline{D}^k is the k -th non-orthogonal orbital which satisfies the proposed criterion. Hence for each non-orthogonal atomic orbital a new matrix diagonalization is required, which yields one additional non-orthogonal orbital. The procedure consists therefore of a sequence of diagonalizations of matrices

whose orders successively increase by one.

The matrix elements of D^k are found by substituting expressions for $q_i^\circ(r)$ and $f(r)$ into Eq. 32. If

$$q_i^\circ(r) = \sum_l R_l(r) a_{il}^\circ \quad (l=1, 2, \dots, m)$$

and

$$q_i(r) = \sum_l R_l(r) a_{il} \quad (l=1, 2, \dots, m)$$

and $R_l(r)$ is the normalized radial part of the primitive basis functions, then

$$D_{ij}^k = \sum_{stuv} a_{is}^\circ a_{jt}^\circ \left(\sum_{l=1}^{k-1} a_{lu} a_{lv} \right) I(stuv)$$

with

$$I(stuv) = \int R_s(r) R_t(r) R_u(r) R_v(r) dr .$$

A closed form expression for this integral is readily found for $R(r)$ of either exponential or Gaussian form using standard integral tables.

Origin adjustment

The procedure outlined above forms non-orthogonal orbitals which are very much like those desired. However, in most cases some residual nodal behavior still exists near the origin, and furthermore, the quantities $q_k(r)/r^l$ (l being the symmetry quantum number) do not go to zero at the origin. By further forcing the condition

$$(d/dr)^l q_k(r) = 0 \quad (k=2,3,\dots)$$

at the origin, even more acceptable orbitals which have this property may be formed. This is accomplished by adding enough of the innermost SCF atomic orbital of the particular symmetry to bring the value of $q_k(r)/r^l$ to zero at the origin for each new non-orthogonal orbital. Often some slight nodes persist, but the result is quite sufficient for the purposes outlined.

Example

The result of applying this process is illustrated in Table 79. The orthogonal orbitals shown are the three canonical SCF s-atomic orbitals of aluminum in 2P ground state as given by Clementi (46). Each column corresponds to a linear combination of Slater-type orbitals characterized by exponents (ζ) and the principal quantum number n . The non-orthogonal orbitals were formed according to the above proc-

ess and are expressed in terms of the same Slater-type orbitals. The inverse Schmidt matrix which effects the transformation is given. Note that the 1s SCF atomic orbital is left unchanged as prescribed. Radial plots of these orbitals are shown in Figures 3 and 4. These show clearly how much the orbitals are changed by the process and also how much the non-orthogonal orbitals resemble non-orthogonal Slater-type orbitals.

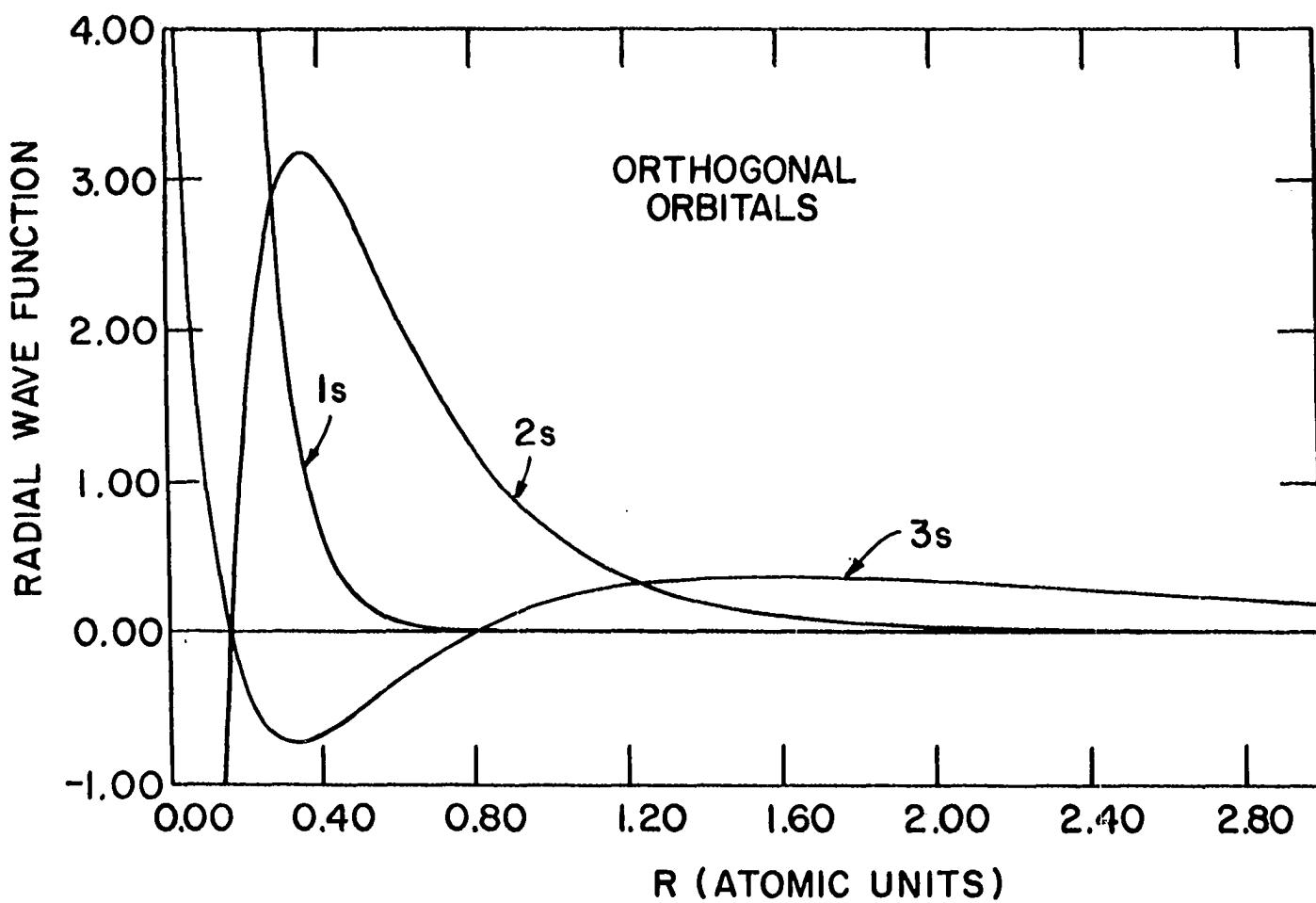


Figure 3. Orthogonal SCF s-atomic orbitals of aluminum

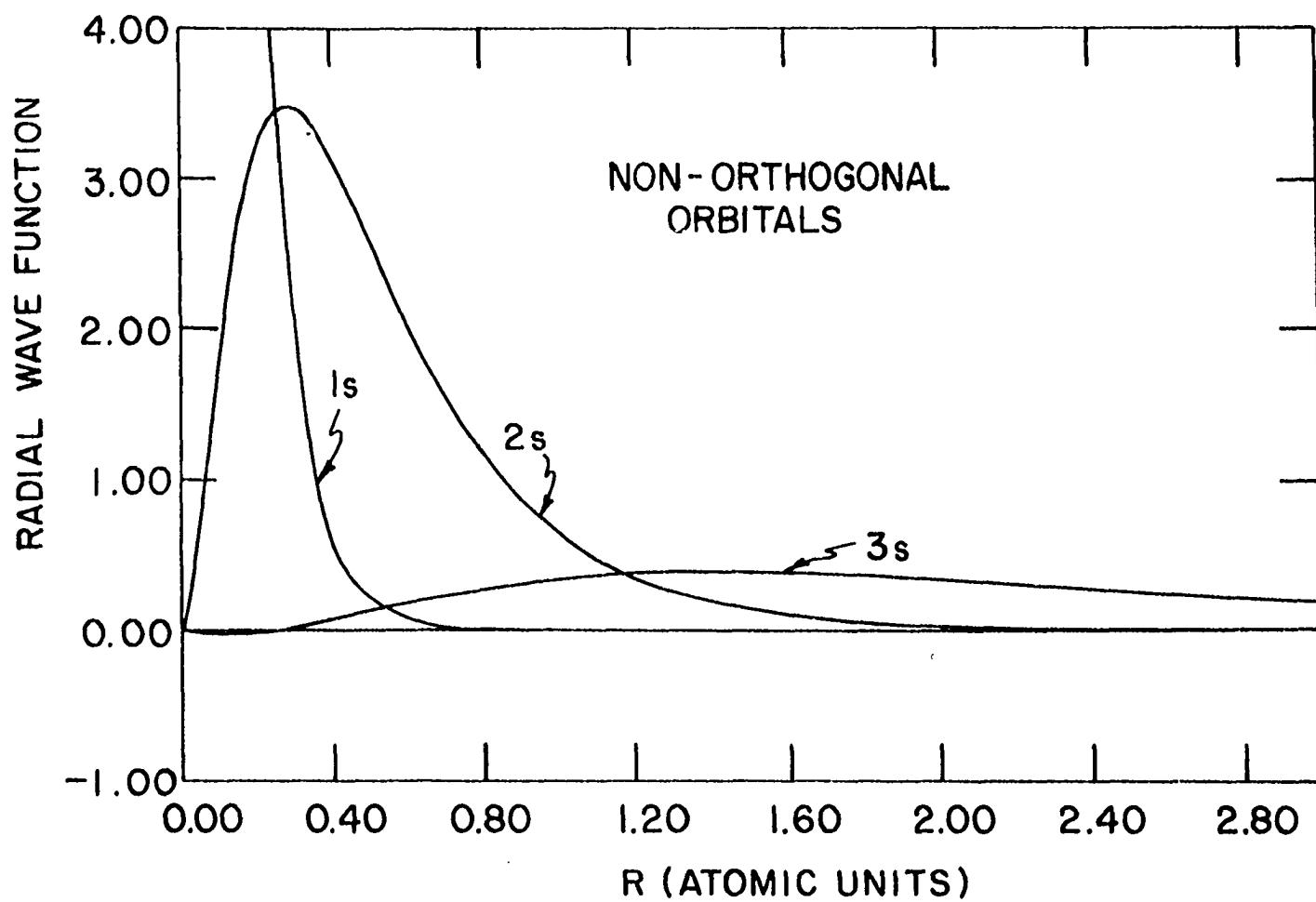


Figure 4. Non-orthogonal SCF s-atomic orbitals of aluminum

Table 79. Orthogonal and non-orthogonal aluminum orbitals

Primitives		Orthogonal Orbitals		
n	Zeta	a°(1s)	a°(2s)	a°(3s)
1	13.00000	0.96741	-0.25135	0.05804
3	14.71120	0.03493	-0.00465	0.00047
3	9.90000	0.01691	0.12045	-0.02622
3	7.10880	-0.00304	0.37640	-0.10400
3	4.74670	0.00178	0.55297	-0.13670
3	2.87680	-0.00048	0.05379	-0.10971
3	1.90290	0.00020	-0.01021	0.57002
3	1.11400	-0.00004	0.00178	0.59412

Inverse Schmidt Matrix				
	1s	2s	3s	
1s°	1.00000	0.25173	0.00279	
2s°		0.96888	0.23516	
3s°			0.97195	

Primitives		Non-Orthogonal Orbitals		
n	Zeta	a(1s)	a(2s)	a(3s)
1	13.00000	0.96741	0.0	0.00000
3	14.71120	0.03493	0.00429	-0.00054
3	9.90000	0.01691	0.12096	0.00289
3	7.10880	-0.00304	0.36392	-0.01258
3	4.74670	0.00178	0.53621	-0.00282
3	2.87680	-0.00048	0.05199	-0.09399
3	1.90290	0.00020	-0.00984	0.55163
3	1.11400	-0.00004	0.00171	0.57788

APPENDIX C: DESCRIPTION OF THE MOLECULAR SCF PROGRAM

In order to test the even-tempered Gaussian basis for molecular calculations, a new integral program, which takes into account certain properties of the basis, had to be designed. The most important and unique feature of the present basis is that several of the contracted functions forming the computational basis have contributions from the same even-tempered Gaussian primitive functions. This fact requires that integrals over contracted functions are formed by a general transformation of the integrals over primitive functions. Transformations of this sort are notoriously time consuming and must be carried out efficiently or else this scheme cannot be competitive with those in which segmented basis contractions are used. Certain algorithmic advantages accrue from the use of an even-tempered basis but all of these have not been provided for here in order that time would not be wasted "tooling up" for an approach which could turn out to be fruitless. Happily this was shown not to be the case. Certain advantages which were left out had been kept in mind so that appropriate modifications can easily be introduced into the programs as time permits.

Matrix Elements

Basic integrals

This package of subroutines was designed to provide integrals over Gaussians of only s and p symmetry. This limited set of Gaussians is alternately represented by normalized "cartesian" Gaussian functions

$$X(\underline{A}, \zeta, l, m, n) = N(\zeta, l, m, n) x_A^l y_A^m z_A^n \exp(-\zeta r_A^2)$$

with

$$0 \leq (l+m+n) \leq 1, \quad r_A^2 = x_A^2 + y_A^2 + z_A^2,$$

$$x_A = x - A_x, \quad y_A = y - A_y, \quad z_A = z - A_z,$$

and

$$N(\zeta, l, m, n) = (2/\pi)^{3/4} \left[\frac{\zeta^{l+m+n+3/2}}{(2l-1)!!(2m-1)!!(2n-1)!!} \right]^{1/2}$$

(in contrast to the real, normalized, "solid spherical" Gaussians of Eq. 19, p=2) and hence, the integral formulae, as given by Taketa et al. (64) and Huzinaga (128) for cartesian Gaussians were employed. In order that computation speed might be a maximum, the formulae were decomposed into special cases as far as was deemed worthwhile.

Contracted integrals

The transformation of primitive to contracted integrals required for use of the contracted functions as calculational basis need not involve all primitive integrals at once.

Primitive functions with distinct values of the set of parameters (\underline{A}, l, m, n) contribute only to contracted functions which may be characterized by the same parameters. Moreover, the relation is also inverse. The integrals then can be divided into groups according to the atomic center (denoted by \underline{A}) and the orbital parameters (l, m, n) which are equivalent to the ordinary angular quantum numbers (l, m). For example, the most general electron repulsion integral would involve orbitals from four centers and four sets of orbital parameters. If each group or "block" of integrals so designated is not overly large, the calculation of primitive integrals and the transformation to contracted integrals may be efficiently done. Each primitive integral is computed and its contribution to each contracted integral in which it appears is made. The primitive integrals are not stored or recomputed. The calculations of blocks of integrals is done within cycles over the atomic centers and orbital parameters of such groups of primitives located on those centers. These cycles also take into account the index symmetries which prevail in one- and two-electron integrals so that only unique blocks are computed.

Because of the quantum number and atomic center cycling, it is easy to administer simple tests in order to determine whether an entire block is identically zero by symmetry. If such is the case, one simply skips the calculation and proceeds to the next block. This results of course, in a savings of considerable computation time.

It is apparent that the algorithm for contracting or transforming the primitive integrals is a very important feature. The procedure used is easiest to describe for the case of one-electron integrals which require a two-index transformation. The generalization to four indices is straightforward.

Suppose that the contracted functions involved are

$$G_i = \sum_{k=1}^{m_i} x_k A_{ki} \quad (i=1, 2, \dots, n_i)$$

and

$$G_j = \sum_{l=1}^{m_j} x_l B_{lj} \quad (j=1, 2, \dots, n_j)$$

Integrals over primitive functions X with a one-electron operator \hat{O} are

$$O_{kl} = \int dv x_k \hat{O} x_l .$$

The contracted integrals may be expressed by

$$(33) \quad O_{ij}' = \int dv G_i \hat{\circ} G_j \\ = \sum_{kl} A_{ki} B_{lj} O_{kl} .$$

This is the form of a general two index transformation, with the matrices A and B rectangular with $m_i \geq n_i$ and $m_j \geq n_j$. Given the matrix of primitive integrals, O, it would be eminently simple to sum Eq. 33 for each element of O' separately. This would involve $2 \cdot m_i \cdot m_j$ multiplications for each element of O' or $2 \cdot m_i \cdot m_j \cdot n_i \cdot n_j$ multiplications altogether, independent of other operations which in general require less time than a multiplication. This method would be highly inefficient timewise and would require keeping the $m_i \cdot m_j$ elements of the matrix O available until all $n_i \cdot n_j$ elements of O' were computed.

The procedure developed for this work is based on the rearrangement of Eq. 33, namely

$$O_{ij}' = \sum_k T_{kj} A_{ki}$$

and

$$T_{kj} = \sum_l B_{lj} O_{kl} .$$

Now, in computing, the order of operations is important. The program structure is given in Figure 5. Two outer program loops on k and l define the indices of O . As soon as O_{kl} is computed, it is used to form values of T_j . Note that T_j need not be indexed by k in the algorithm since k is defined by an outer loop. When the index l is exhausted, the elements of the intermediate vector T are used to form their contribution to O' . From the figure it is easy to see that $n_i \cdot n_j \cdot m_i + n_j \cdot m_i \cdot m_j$ multiplications and $n_j(1+n_i)$ words of storage are required. A comparison to the simple approach mentioned previously shows a tremendous savings in multiplications and likewise in storage. The value of n_i or n_j is typically ≤ 6

Zero array O'

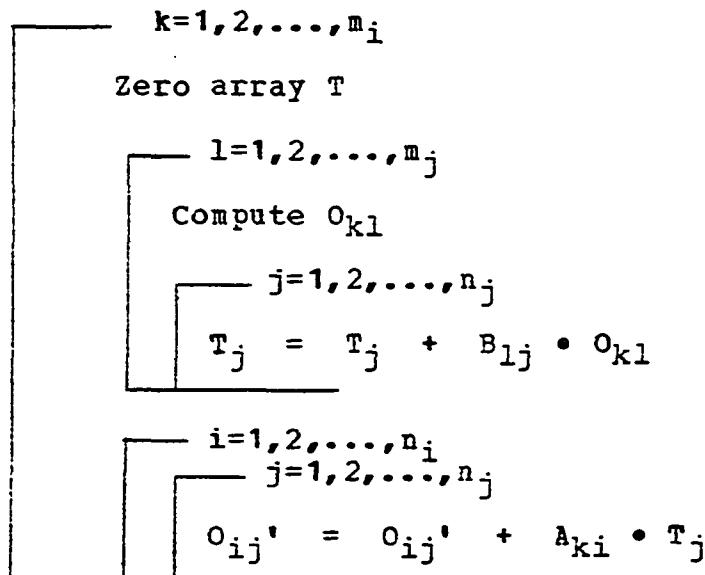


Figure 5. Program structure for a general two-index transformation

while m_i or m_j is ordinarily of the order of 10 for first row atoms to obtain fair accuracy (so-called double-zeta accuracy).

The generalization of this procedure to a four-index transformation

$$I(ijkl) = \sum_{i'j'k'l'} A_{ii} B_{jj} C_{kk} D_{ll} I(i'j'k'l')$$

with

$$i=1, 2, \dots, n_i, \quad i'=1, 2, \dots, m_i, \quad m_i \geq n_i,$$

etc., requires $n_1(1+n_k(1+n_j(1+n_i)))$ words of storage and $n_1 \cdot m_1 \cdot m_k \cdot m_j \cdot m_i + n_1 \cdot n_k \cdot m_k \cdot m_j \cdot m_i + n_1 \cdot n_k \cdot n_j \cdot m_j \cdot m_i + n_1 \cdot n_k \cdot n_j \cdot n_i \cdot m_i$ multiplications. A diagram of the program flow is shown in Figure 6. Arrays I1, I2, and I3 are temporary and correspond to T in the two index procedure. The space necessary for typical values of n is easily allocated in the main or fast core storage area. As a result, no input/output operations other than writing of the finished block of contracted integrals themselves are needed. This must be done in any case.

Zero array I'

$i'=1, 2, \dots, m_i$

Zero array I3

$j'=1, 2, \dots, m_j$

Zero array I2

$k'=1, 2, \dots, m_k$

Zero array I1

$l'=1, 2, \dots, m_l$

Compute $I0 = I(i'j'k'l')$

$l=1, 2, \dots, n_l$

$I1(l) = I1(l) + d_{ll}, \cdot I0$

$k=1, 2, \dots, n_k$

$l=1, 2, \dots, n_l$

$I2(kl) = I2(kl) + c_{kk}, \cdot I1(l)$

$j=1, 2, \dots, n_j$

$k=1, 2, \dots, n_k$

$l=1, 2, \dots, n_l$

$I3(jkl) = I3(jkl) + b_{jj}, \cdot I2(kl)$

$i=1, 2, \dots, n_i$

$j=1, 2, \dots, n_j$

$k=1, 2, \dots, n_k$

$l=1, 2, \dots, n_l$

$I(ijkl)' = I(ijkl)' + a_{ii}, \cdot I3(jkl)$

Figure 6. Program structure for a general four-index transformation

Integral storage

The size of typical molecular bases requires calculation of large numbers of integrals which must be reused. Core sizes exclude the possibility of keeping these numbers close at hand and they must be stored on peripheral devices. There has recently been debate concerning the storage of two-electron integrals. On the one hand, if all integrals are computed and stored, then their order is defined by the order of calculation. Detractors of this method say that it requires the storage of zero integrals which are not used and of which there are often very many (perhaps as much as 25% or more of all unique integrals necessary for a molecule of high symmetry). Furthermore, programs which use the integrals are tied to the specific ordering pattern used. On the other hand, there are those who would store each non-zero integral in arbitrary order along with a label or word giving the four orbital indices and a code for the integral type. The type refers to index symmetries and tells how the integral is used in the calculation. This alleviates the order dependence of programs using the integrals. Detractors of this method would argue that an effectively random order is bad since the use of program loops, which often permit computational code optimization, is not possible. Any non-random ordering of the integrals is a lost advantage. Moreover, additional storage and input/output is needed for the labels. The

labels must also be decoded, requiring the separation of one word into five and is often accomplished by using four division operations. The present program uses the best features of both methods. The ordering of the blocks of integrals is defined by loops and the blocks are stored and retrieved in this order. Zero blocks are not computed, stored, or otherwise used. Furthermore, a set of just ten integers is stored with a block of up to 6^4 integrals. Eight of these integers provide program loop limits to define basis function indices of a block. One integer specifies a particular set of loops to be used for specifying the indices. The last integer is the number of integrals in the block. This is in order to read the block from storage. Although the blocks may be randomly ordered, the integrals in each block are identified by use of a set of loops, allowing the possibility of explicit program optimization. A large share of the zeroes are eliminated and very little space is required for the index information. Integrals stored which are not needed are the ones which are not exactly zero but negligible. These occur most often for large molecules having several non-bonded atoms and are not a consideration in this work. A satisfactory method of storing just part of a block is not directly available with this technique. However, when integrals are used, their values are tested to determine if they are negligible with respect to the required accuracy. If so, work

done with them is avoided.

SCF Specifications

The SCF program used for the molecular calculations is based on the standard Roothaan closed-shell method (7). It differs from the atomic programs by a direct formation of the Fock matrix F from the contracted basis function integrals as opposed to forming the symmetry adapted integrals described in Appendix A. This is sufficient since the program is just for closed-shell calculations.

In order to save computation time, a novel technique was introduced to separate the eigenvalue problem into subproblems according to the point group symmetry. In the case where no symmetry exists, a fast Givens algorithm is used to diagonalize the Fock matrix (Wilkinson, 129). When symmetry allows the problem to be broken up, a Jacobi technique is used because of the possible multiple degeneracies which occur (Greenstadt, 130). With the Jacobi diagonalization, no explicit orthogonalization of degenerate eigenvectors must be done. Though the Jacobi technique is slower, the sizes of matrices corresponding to each subproblem are smaller than the entire basis. This also compensates for its use.

Automatic construction of symmetry adapted basis functions

The Hamiltonian operator describing the interactions of particles comprising a molecule has the symmetry of the point

group to which the molecule belongs. This symmetry is a consequence of the spatial configuration of the atomic nuclei. As a result, the canonical SCF orbitals, which form a solution to the Hartree-Fock equations, belong to irreducible representations or irreps of the point group. It is possible to make linear combinations of certain atomic orbital basis functions which are symmetry adapted functions and which therefore belong to irreps of the point group. If these symmetry adapted functions are used as basis for a molecular calculation, there will be no subsequent mixing among those basis functions belonging to separate irreps. Thus a molecular problem may be broken up into several smaller, largely independent sub-problems, one for each irrep. This is an advantage first of all because each sub-problem requires handling of smaller matrices and hence, less computation time and storage space is needed. For the full problem, all the matrices would be the size of the full basis, but would have many zeroes. Another advantage is that having broken the problem into symmetry species, one may easily select the electronic configuration or state to be determined by assigning electrons to orbitals belonging to the proper irreps.

This investigation makes use of a novel fully automatic scheme for constructing the symmetry adapted basis functions from the atomic orbital basis. It could fail if the original atomic basis were linearly dependent. Such bases are exclud-

ed for other obvious reasons.

The method is based on a general numerical approach to the construction of symmetry orbitals. It requires two operators which are invariant against the symmetry group. Diagonalization of the first yields symmetry adapted functions. Diagonalization or examination of the block diagonal form of the second in the symmetry adapted basis is necessary to recognize those functions belonging to the same irreps.

In the present approach the first operator is taken to be the projection operator in the space of all basis functions, i.e.

$$\rho(x|x') = \sum_{kl} |x_k\rangle s_{kl}^{-1} \langle x_l| .$$

This operator is invariant with respect to all symmetry operations of the molecule since the set of all x 's is naturally assumed to form a basis for a (reducible) representation of the symmetry group, i.e.

$$\hat{\rho} \hat{R} = \hat{R} \hat{\rho}$$

for all symmetry operators \hat{R} . The matrix of this operator becomes

$$\langle x_i | \hat{\rho} | x_j \rangle = \langle x_i | x_j \rangle = s_{ij} .$$

the overlap matrix. Thus the eigenvectors of the overlap matrix represent symmetry adapted orbitals. We modify these

slightly by normalizing them to unity. Thus, they are defined by

$$|X'\rangle = |X\rangle \underline{T}$$

where

$$\underline{T} = \underline{U} \underline{S}^{-1/2}$$

with \underline{U} being the orthogonal matrix ($\underline{U}^+ = \underline{U}^{-1}$) that diagonalizes \underline{S} , i.e.

$$\underline{U}^+ \underline{S} \underline{U} = \underline{S}_D,$$

with \underline{S}_D being the resulting diagonal matrix.

It is apparent that the transformation of the basis, $|X\rangle$, with \underline{T} is identical with Löwdin's canonical orthogonalization (89). Thus the resulting functions are not only symmetry adapted but also mutually orthogonal. Diagonalization of the matrix, $\langle X|\hat{O}|X\rangle$, of other symmetry invariant operators would in general not give orbitals, $|X'\rangle$, which are orthogonal with respect to the overlap.

In order to be able to handle the new functions, $|X'\rangle$, belonging to separate irreps together for each subproblem, they must be grouped with respect to that classification. Now, in the symmetry adapted basis $|X'\rangle$, the matrix of another specific operator $\hat{O}(\neq I)$, having the same molecular point group symmetry is,

$$\underline{O}' = \langle X' | O | X' \rangle = \underline{T}^* \langle X | O | X \rangle \underline{T} = \underline{T}^* \underline{O} \underline{T}.$$

This matrix will now have block form. In general the block form will not be that simple one in which the blocks all lie along the main diagonal. If the new matrix \underline{O}' is now diagonalized, then each eigenvector belongs to an irrep and hence, has contributions from the symmetry adapted functions, $|X'\rangle$, belonging to that irrep. The eigenvectors may therefore be classified into groups with respect to irreps by use of the observation that those belonging to one irrep all have zeroes in the same rows while others have zeroes elsewhere. Finally, the eigenvectors of the matrix which diagonalizes \underline{O}' , i.e. its columns, are permuted so that those associated with a particular irrep are adjacent. This represents a new transformation \underline{V} whose product with \underline{T} , i.e. $(\underline{T}\underline{V})$, when applied to the matrix \underline{U} of any operator \hat{U} computed with respect to the original basis, produces a block diagonal matrix in which the blocks fall along the main diagonal:

$$(\underline{T}\underline{V})^* \underline{U} (\underline{T}\underline{V}) = \begin{bmatrix} \underline{U}_1 & & & \\ & \vdots & & 0 \\ & & \underline{U}_2 & \\ & & & \vdots \\ 0 & & & \underline{U}_3 \end{bmatrix}$$

In particular, if \underline{U} is the Fock matrix, the problem of Eq. 7

is directly transformed to an ordinary eigenvalue equation and to convenient block form.

The operator $\hat{O} = \hat{H}_1$ was used here to gain another advantage. Since \hat{H}_1 can be considered as a very rough approximation to the Fock operator, its eigenvectors can be used as starting vectors for the self-consistent-field process. The ordering of the eigenvalues of \hat{H}_1 enable one to make an automatic assignment of a ground state configuration. This facility has proved useful in most cases. However, the omission of the important electron repulsion term and the resulting lack of nuclear screening by inner electrons has the consequence that the valence shells in particular are not well approximated. This leads to incorrect ground state configuration assignments and as such, the assignments must be adjusted.

It is apparent that because of the block diagonal form, one does not need a very precise diagonalization of \underline{Q} in order to recognize orbitals of the same irrep. In practice, diagonalization of the matrix of \hat{O} was carried out by a Jacobi technique such that off-diagonal elements were brought to values approximately 10^{-3} • (average diagonal element).

General Specifications

The programs were designed to require a small amount of core storage so that under the present multiprogramming envi-

ronment, short runs could be made frequently rather than only during periods of light work load. The program and primary I/O (read/write) buffers require 128K bytes of main core. Secondary I/O buffers are allocated to slow core. Large buffers are used to maintain efficiency and therefore 96K of slow core is required. This requirement is flexible however, since it is defined by job control input rather than the programs themselves. The efficiency of the program (central processor time, CPU, versus real or system residence time) depends on the number of electrons in a molecule, the number of primitive basis functions, and the number of contracted functions. For example, in typical runs on H_3^+ and Na_3^+ in the B bases, the total Real:CPU time ratios were 5.2:1 and 1.3:1 respectively. The efficiency increases as the amount of information processed increases. Table 80 includes more complete information on the program performance. Processing time is given for all molecule calculations for both integral and SCF steps and total run time is the sum of these. Integral time is highly dependent on the number of primitive basis functions used, and is therefore different for the A and B bases. The SCF times depend on the numbers of contracted functions and the number of iterations needed for convergence. Since both bases contain equal numbers of contracted functions and since the numbers of SCF iterations were essentially the same for both bases, the SCF times dif-

fer little.

The integrals section of the program has some restrictions on the amount of input data which can be processed, e.g. number of atom centers, total number of contraction coefficients, and total number of primitive orbital zetas. Information for each kind of atom is entered only once unless an inhomogeneity is required. Redimensioning certain small arrays alleviates this with little change in core requirements. The main restrictions however are the maximum number of primitives per contracted orbital (ten), and the maximum number of contracted orbitals per symmetry (six) in one atom. These restrictions highly influence the size of core needed for processing integrals and changes here will require drastic changes in core requirements. These restrictions may be avoided by the trick of including dummy atoms with nuclear charge zero, at the site of real atoms in order to increase the basis size in an artificial way. However, contraction of functions on real atoms with those on dummy atoms is not possible.

Because of the practical restriction of the integrals to a 128K region, the SCF step was adjusted to admit a maximum of 54 variational basis functions. The source of this restriction is not the integral program since many more contracted integrals than this could be computed using it with no change. In order to do larger basis calculations, only

the SCF segment dimensions need be changed. If necessary, all the size restrictions are easily modified but the integral program ought to be made still more efficient. Work on more efficient integrals programs which make heavier use of even-tempered basis simplifications is under way.

Special Features

The programs have a number of optional features which may be selected. Two of the most important are the restarts available during integral computation and during the SCF iterations. Essential information may be copied to a peripheral device if the run terminates because of a time underestimate. Options are then chosen to take up where the calculation left off.

Using the integral restart feature one may do efficient brute force geometry optimizations. Atoms in a molecule to be left stationary are treated first as a separate molecule or molecule fragment. These integrals are calculated and stored separately. Then the molecule data is augmented by addition of the atoms which are to be moved during a series of SCF runs. The stored integrals are copied into the two-integral data set and the integral restart is opted for. The program will automatically add the new integrals involving the fragment and the new atoms to the end of the set which was already computed for just the molecule fragment. The SCF

step is subsequently carried out and one energy is obtained. The same process may be repeated using the same separate fragment integrals as often as the new atoms need be moved. Rearrangement of the fragment nuclei of course requires re-computation of the fragment integrals.

An advantage is gained if the molecule has a particular orientation in the external coordinate system with respect to which atom coordinates are designated. The orientation is that one in which the most pairs of atom x-, y-, and z-coordinates are equal (especially for those atoms with more basis functions). The consequence of this is that more blocks of integrals are zero by symmetry. Since it is often easier to find atom coordinates which do not conform to the above criterion, a facility has been incorporated whereby the molecule may be rotated in order to reorient it to gain efficiency.

Other options are mainly print options, allowing one to increase the amount of printed output for debugging purposes. The standard punched output may be suppressed if desired.

Table 80. Molecule calculation times

Molecule	Num. Orbi- tals	Num. Contr. Func- tions	Basis A			Basis B			Num. SCF Itera- tions
			Prim. Func- tions	Inte- grals	SCF	Prim. Func- tions	Inte- grals	SCF	
H ₂	1	18	26	0.6	0.3	36	1.8	0.3	6
H ₃ ⁺	1	27	39	4.5	1.0	54	13.7	1.0	4
Li ⁺	1	11	18	0.2	0.1	23	0.4	0.1	6
LiH	2	20	31	1.4	0.6	41	3.1	0.5	9
LiH ₂ ⁺	2	29	44	7.2	2.0	59	19.2	2.0	7
Li ₂	3	22	36	2.1	0.7	46	4.5	0.7	10
Li ₂ H ⁺	3	31	49	6.0	2.0	64	15.1	1.8	8
Li ₃ ⁺	4	33	54	13.0	3.5	69	31.5	3.4	8

Table 80. (Continued)

Molecule	Num. Orbi- tals	Num. Contr. Func- tions	Basis A			Basis B			Num. SCF Itera- tions
			Prim. Func- tions	Inte- grals	SCF	Prim. Func- tions	Inte- grals	SCF	
Na ⁺	5	18	23	0.5	0.3	31	1.3	0.4	11
NaH	6	27	36	2.4	1.8	49	5.9	1.9	17
NaH ₂ ⁺	6	36	49	10.7	8.2	67	29.9	9.0	16
NaLi	7	29	41	3.1	2.6	54	8.4	2.9	20
NaLi ₂ ⁺	8	40	59	18.3	16.6	77	46.1	16.7	21
Na ₂	11	36	46	5.2	4.5	62	12.0	4.3	15
Na ₂ H ⁺	11	45	59	11.6	10.1	80	31.3	10.6	16
Na ₂ Li ⁺	12	47	64	23.4	23.2	85	57.3	21.9	18
Na ₃ ⁺	16	54	69	32.7	33.4	93	70.5	32.0	15

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